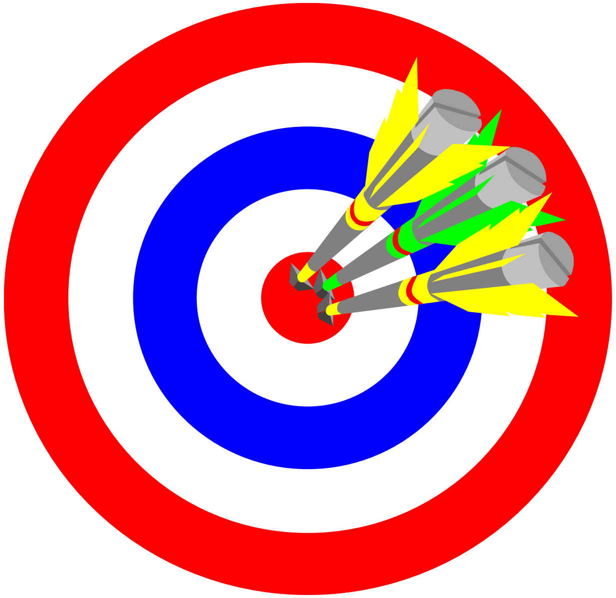
Chapter One

Points, Lines, Planes, and Angles

Objectives



A. Use the terms defined in the chapter correctly.
B. Properly use and interpret the symbols for the terms and concepts in this chapter.
C. Appropriately apply the postulates and theorems in this chapter.
D. Correctly interpret the information contained in two and three dimensional diagrams.
E. Correctly interpret and represent the information given in problems with either a two or three dimensional diagram.

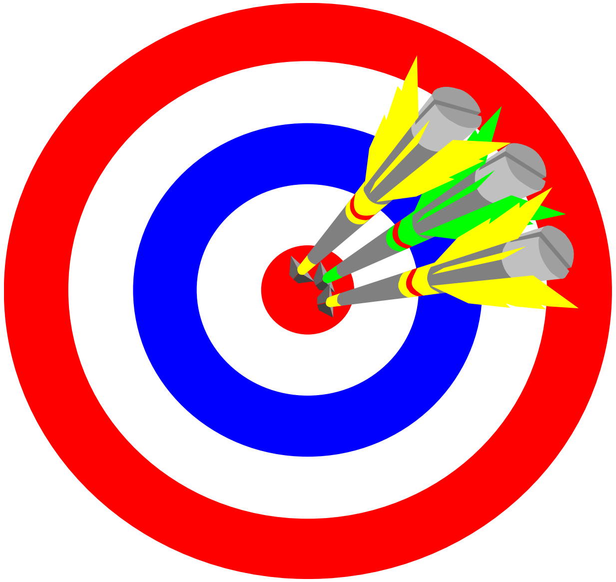
Section 1-1

A Game and Some Geometry

Homework Page 4:

#6, #8, #10

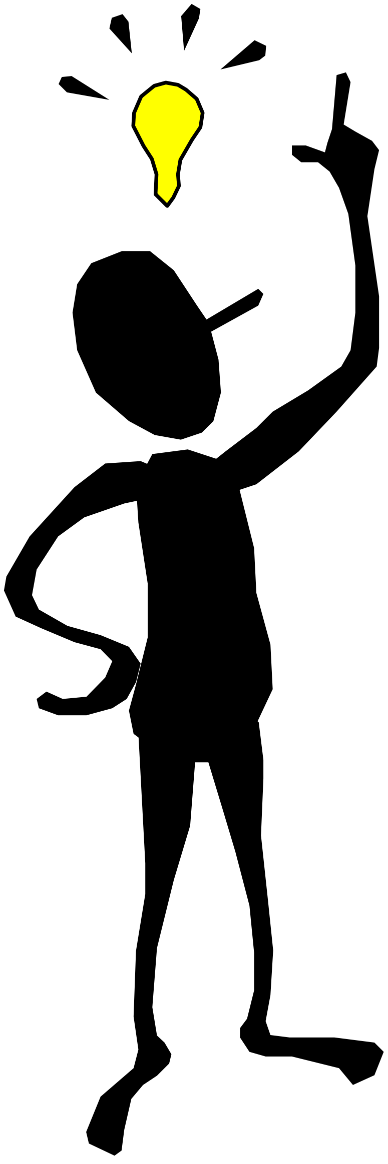
Objectives

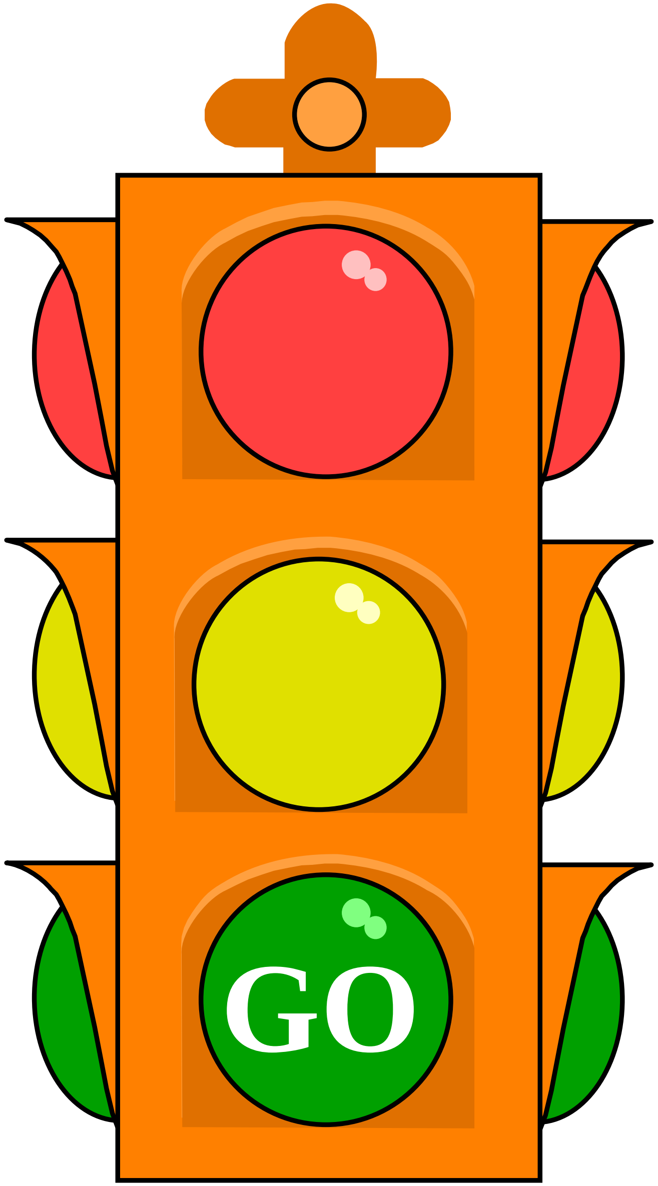


A. Use the terms equidistant correctly.
B. Understand the nature of math and science.
C. Distinguish between valid and invalid assumptions in geometry.

Nature of Math & Science

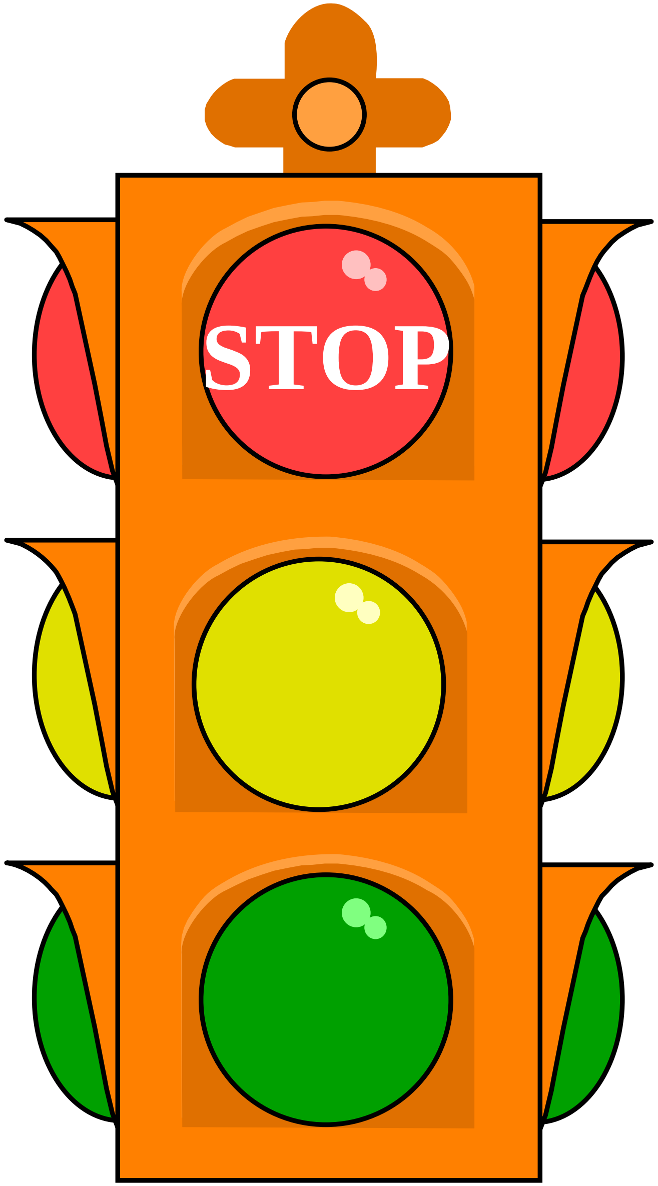
- Defines reality.
- Describes our sensory experiences.
- Impacts every life.
- Based on unproven assumptions.
- Does not conflict with religion.





Always assume that:

- Lines are straight.
- Angles exist.
- Points on a line are collinear.
- There are unlabeled points in the diagram.
- Points have the relative positions drawn.



Never assume that:

- Angles measure 90°.
- Angles are equal.
- Segments are equal.
- Angles and segments are as large or as small as they appear.
- An answer is correct because of the way it looks.

EQUIDISTANT

- Equidistant has the prefix ‘equi’, meaning equal, and a suffix of distant. Therefore, two locations that are *equidistant* from a third location are the same distance from the third location.
- Choose two locations in this classroom that you believe are equidistant from your desk.
- Check the distance to those locations.
- Are the distances *approximately* equal?
- Are the distances *exactly* equal?
- Why or why not?

Sample Problems Section 1-1

- Look at problem 5 on page 4 of your book. Which distance is appears to be greater:
  - The distance from R to S, or
  - The distance from T to U.
  - Check your answer!
- Look at problem 7 on page 4 of your book. What is the relationship of the inner square to the outer square?
  - Check using a square piece of paper.

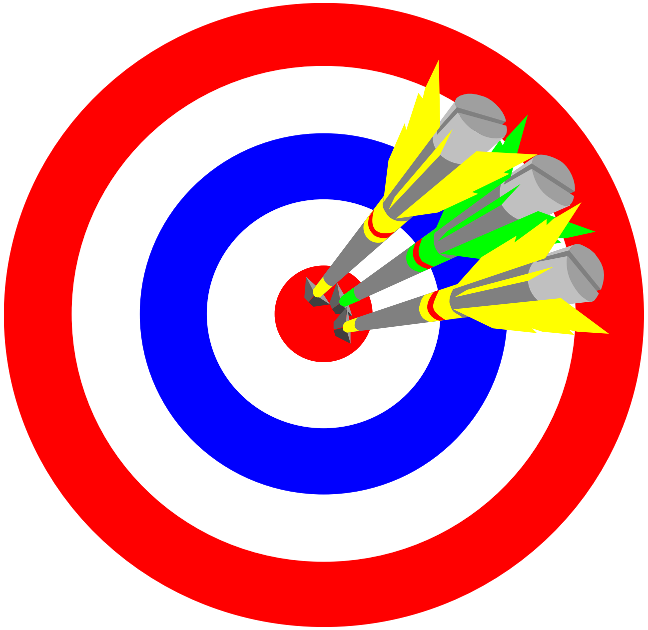
Section 1-2

Points, Lines and Planes

Homework Pages 7-9:

2-32 even

Objectives



A. Understand the meaning of, and the reasons for, undefined terms.
B. Use the undefined terms point, line and plane correctly.
C. Use the undefined terms to define the terms space, collinear points, and coplanar points.
D. Use the correct naming conventions for points, lines, and planes.
E. Create proper graphic representations of points, lines, planes, and intersections, including the use of dotted lines.

UNDEFINED TERMS

- Undefined terms are considered intuitive ideas.
  - You have a grasp of the concept represented by the term.
  - These terms are ‘accepted on faith’ and are used to define other terms.
- point: any location having no dimension.
  - A point is labeled with a single capital letter.
  - Graphically, we represent a point as a dot on a page.

UNDEFINED TERMS

- line: is a set of points extending infinitely in two directions.
  - Has one dimension => length.
    - Since a line extends infinitely in two directions, the length of any line is infinite.
  - Has NO width.
    - We will see why we say this later.
  - A line is labeled either:
    - with a single lower case letter (*l*), or
    - with the labels of two points on the line with a line symbol (←→) above them.
  - Remember to assume that all lines are straight!
  - Graphically, we represent it as a straight line with arrows on both ends.

UNDEFINED TERMS

- Plane: is a set of points that make up a flat surface. The surface extends infinitely in two directions.
  - Has two dimension => length and width.
    - Since a plane extends infinitely in two directions, the length and width of any plane is infinite.
  - Has NO thickness.
    - We will see why we say this later.
  - A plane is labeled either with a single capital letter not representing the label of a point or with the labels of three non-collinear points on the plane.
  - Graphically, we represent a plane as having four edges.

TERMS BASED ON UNDEFINED TERMS

Now that we have some basic terms, we can use these to build other definitions.

- collinear points: points all on one line
- coplanar points: points all on one plane
- space: the set of all points
- intersection: the set of points common to two or more figures
  - You will see that the actual intersection of two figures could be a point, a line, a plane, or other figures.

GRAPHICAL REPRESENTATIONS

We use graphical representations of points, line, and planes to convey the meaning of the object or figure.

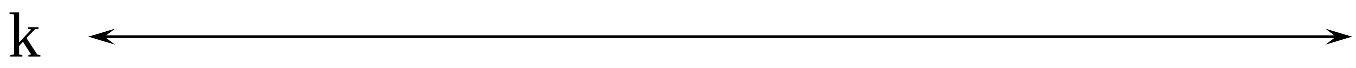
- Point → Represented as a dot.
- Line → Represented as a straight line with arrows on each end.
- Plane → Represented as a surface with four edges.
- Dotted lines → Used in three dimensional figures and indicate the parts of the figure which would be hidden from view if the figure were solid.

BE CAREFUL!

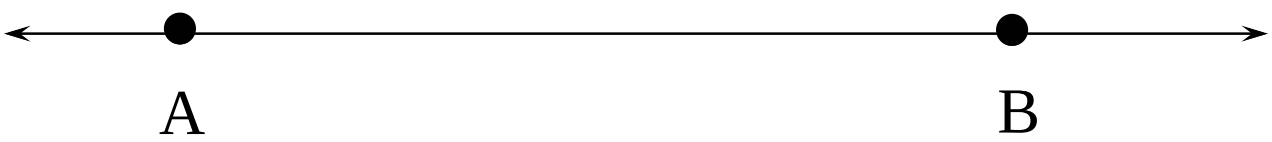
- Remember that these are REPRESENTATIONS and are not exact.

Naming Lines

Lines may be labeled with a single lower-case letter.

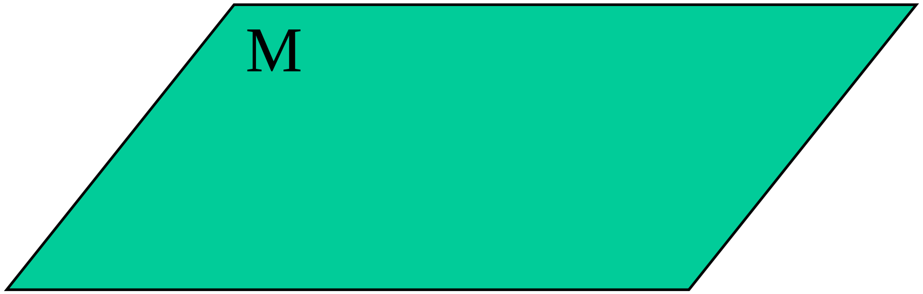


Lines may be labeled using two points on the line: $\overleftrightarrow{AB}$

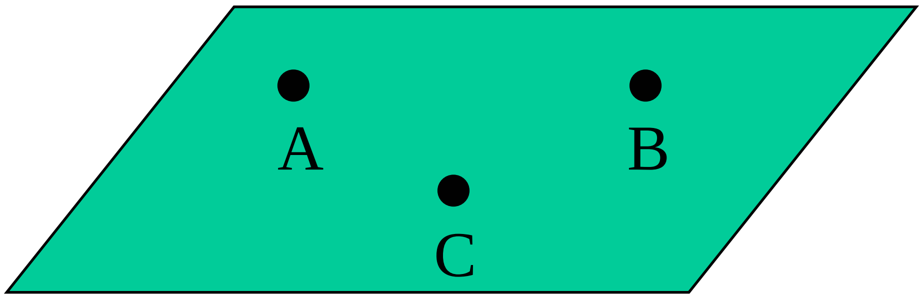


Naming Planes

Planes may be labeled with a single capital letter not associated with a point in the plane: plane M



Planes may be labeled using three non-collinear points in the plane: plane ABC



Sample Problems Section 1-2

Classify each statement as true or false.

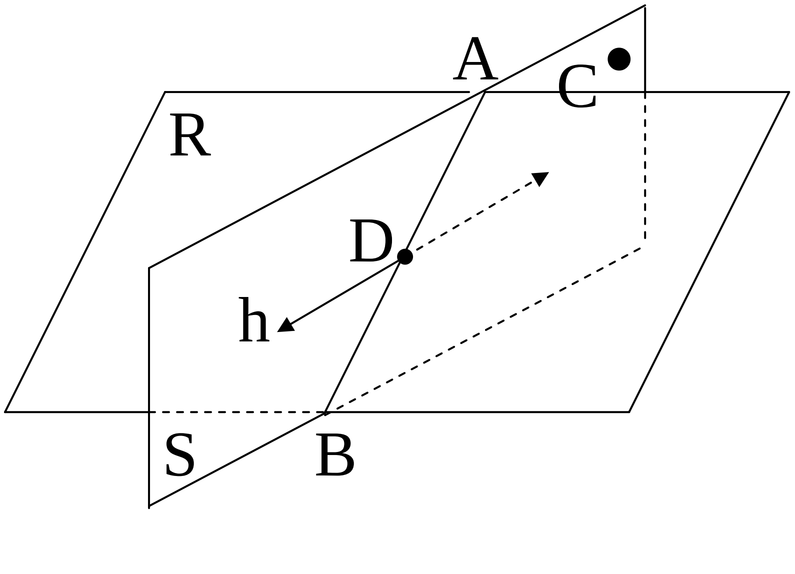
1. $\overleftrightarrow{AB}$ is in plane R.

3. Planes R and S contain D.

5. h is in plane S.

7. Plane R intersects plane S in $\overleftrightarrow{AB}$

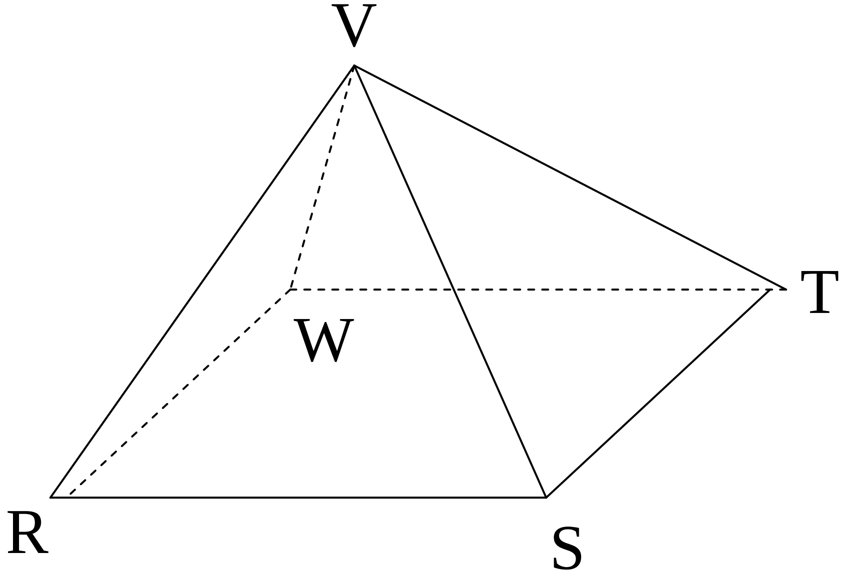
9. A, B and C are collinear.



Sample Problems Section 1-2

11. Make a sketch showing four coplanar points such that three, but not four, of them are collinear.

13. Name five planes that contain sides of the pyramid.
15. Name three lines that intersect at point R.
17. Name three planes that intersect at point S.



Sample Problems Section 1-2

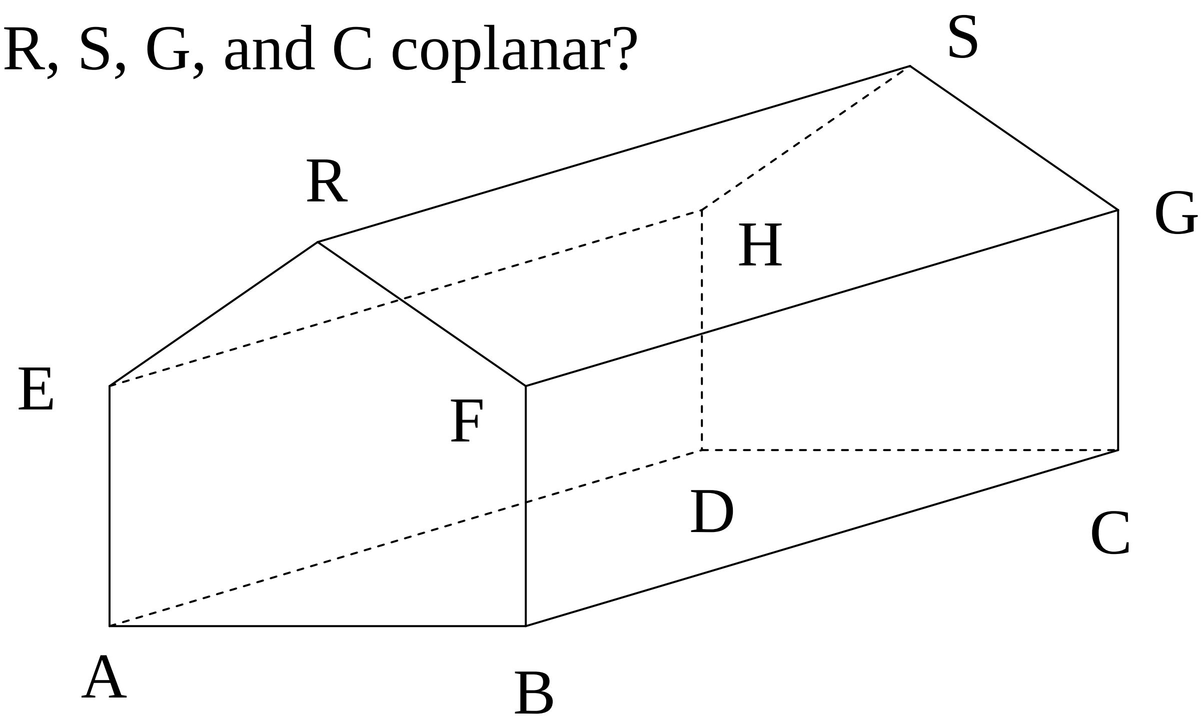
19. Draw a rectangular solid box.

21. Name two planes that intersect in $\overleftrightarrow{FG}$

23. Name three planes that intersect at point B.

25.a. Are points R, S, G, and F coplanar?

b. Are points R, S, G, and C coplanar?



Sample Problems Section 1-2

27. Can two horizontal planes intersect?

Sketch and label the figure described. Use dashed lines for hidden parts.

29. Vertical line l intersects a horizontal plane M at point O.
31. Horizontal plane Q and vertical plane N intersect.
33. Point P is not in plane N. Three lines through point P intersect plane N in points A, B, and C.

Section 1-3

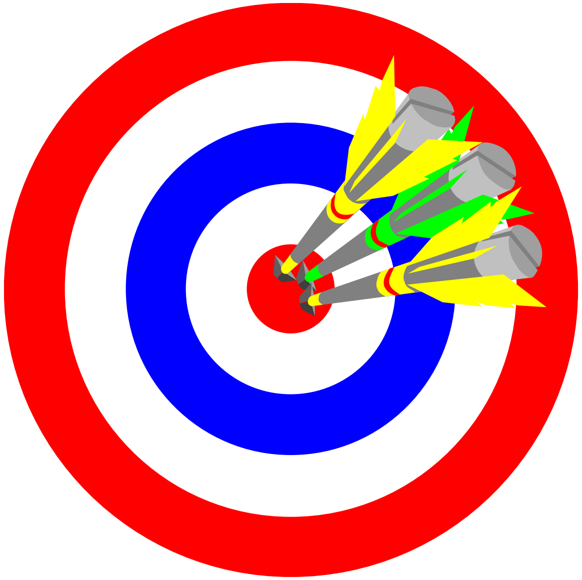
Segments, Rays and Distance

Homework Pages 15-16:

2-46 evens

Excluding 14, 44

Objectives



A. Understand the concept of congruence.
B. Use the term congruence correctly.
C. Use the terms segment, ray, opposite ray, and segment length correctly.
D. Use the terms midpoint and bisector correctly.
E. Understand the meaning and use of postulates or axioms.
F. Apply the Ruler and Segment Addition Postulates correctly.

CONGRUENCE

- ★ congruent: to be the same size and shape. The symbol for congruence is ≅
- congruent segments: segments of equal length
- Note that congruence requires same size AND SHAPE.
  - Example: Two circles have the same shape. Two circles with identical radii lengths are congruent
    - Same shape → Circle
    - Same size → equal radii

LINEAR 'PARTS'

- Remember a **line** is an undefined term that is understood to mean the set of all points extending infinitely in two directions.
- Points are **collinear** if they are all part of the same line.
- A **line segment**, or **segment**, is a set of points consisting of two points (**endpoints**) on a line and all the points in between.
  - A segment is labeled with a bar over its endpoints.
- A **ray** is a set of points on a line beginning with an endpoint and traveling infinitely in one direction.
  - A ray is labeled with its endpoint first and another point on the ray with an arrow above the letters. (ORDER)
- **Opposite rays** are two coplanar rays with the same endpoint traveling in opposite directions

LINEAR CHARACTERISTICS

- **coordinate**: the real number assigned to a point on a number line
- **length**: the distance between the endpoints of a segment; the absolute value of the difference of the coordinates of the endpoints
  - The length of a line segment is represented by the two endpoints of the segment WITHOUT a bar over them.
    - $\overline{AB}$ indicates the line segment.
    - AB = 5 indicates the length of the line segment.
- **midpoint of a segment**: a point dividing a segment into two congruent segments
- **bisector of a segment**: a line, segment, ray or plane that intersects a segment at its midpoint

Naming Rays

Rays are named using the endpoint and one other point on the ray: $\overrightarrow{AB}$

The tip of the arrow must be over the point on the ray not the endpoint.

$\overrightarrow{BA}$

Postulates & Theorems

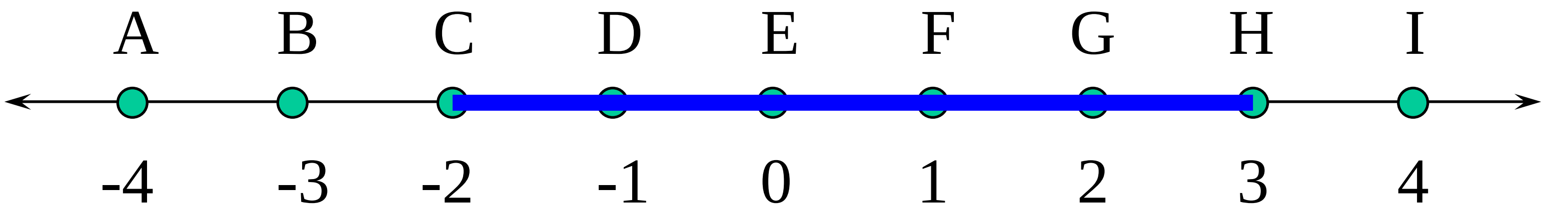
- Tell you how to solve every problem in this book.
- Are broken into two parts:
  - a statement that tells you when to use the rule; usually a description of a picture
  - a statement of how to use the rule; usually an equation or a description of what to add to the picture

POSTULATES

- **postulate** (axiom): statements which are accepted as true without proof.
- Postulates are similar to undefined terms as we accept both on faith, without proof.

Postulate 1 (Ruler Postulate)

- the points on a number line can be paired with the real numbers in such a way so that any two points can have coordinates 0 and 1

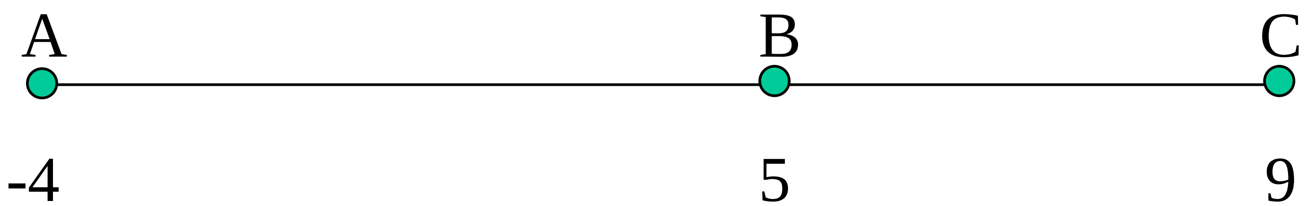


- once a coordinate system has been chosen in this way, the distance between any two points equals the absolute value of the difference of their coordinates

$$CH = |-2 - 3| = |-5| = 5$$

★Postulate 2 (Segment Addition Postulate)

If B is between A & C, then AB + BC = AC.



$AB = |-4 - 5| = |-9| = 9$

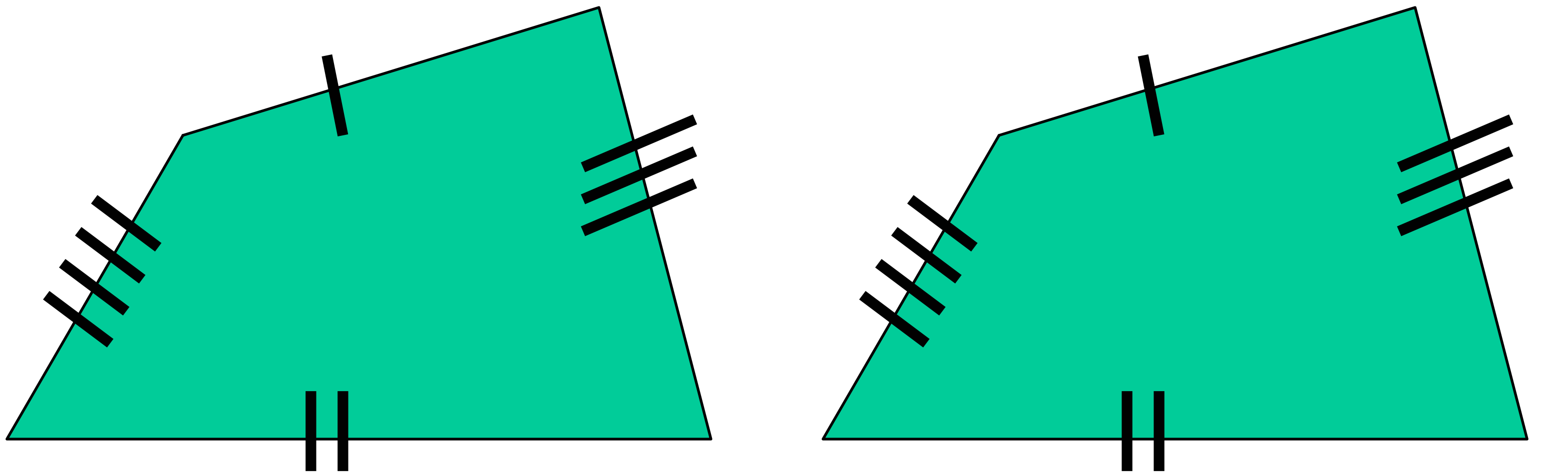
$BC = |5 - 9| = |-4| = 4$

$AC = |-4 - 9| = |-13| = 13$

$AB + BC = 9 + 4 = 13 = AC$

Congruent Segments in Diagrams

Congruent segments are marked with slashes. One slash for the first pair of congruent segments, two slashes for the second pair, etc.



Sample Problems Section 1-3

The numbers given are the coordinates of the two points on a number line. State the distance between the points.

1. - 6 and 9

3. - 1.2 and - 5.7

Sample Problems Section 1-3

In the diagram, $\overline{HL}$ and $\overleftrightarrow{KT}$ intersect at the midpoint of $\overline{HL}$

Classify each statement as true or false.

5. $\overline{LM} \cong \overline{MH}$

7. $\overline{MT}$ bisects $\overline{LH}$

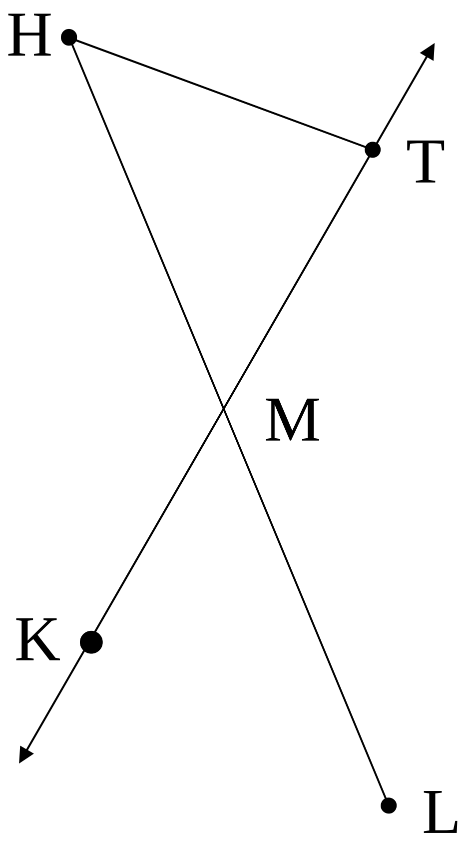
9. $\overrightarrow{MT}$ and $\overrightarrow{TM}$ are opposite rays

11. $\overleftrightarrow{LH}$ is the same as $\overline{HL}$

13. $\overleftrightarrow{KT}$ is the same as $\overleftrightarrow{KM}$

15. HM + ML = HL

17. T is between H and M.



Sample Problems Section 1-3

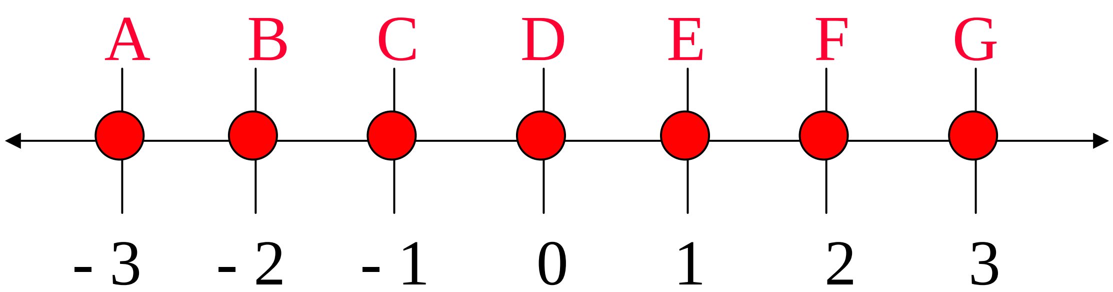
Name each of the following.

19. The point on $\overrightarrow{DA}$ whose distance from D is 2.

21. Two points whose distance from E is 2.

23. The midpoint of $\overline{BF}$

25. The coordinate of the midpoint of $\overline{AE}$



Sample Problems Section 1-3

Draw $\overline{CD}$ and $\overline{RS}$ so that the conditions are satisfied.

27. $\overline{CD}$ and $\overline{RS}$ intersect, but neither segment bisects the other.

29. $\overline{CD}$ bisects $\overline{RS}$, but $\overline{RS}$ does not bisect $\overline{CD}$

Sample Problems Section 1-3

31. In the diagram, $\overline{PR} \cong \overline{RT}$ S is the midpoint of $\overline{RT}$

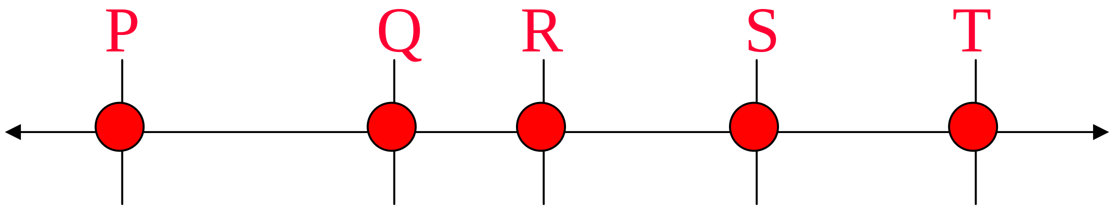
QR = 4, and ST = 5. Complete.

31.a. RS =

31.b. RT =

31.c. PR =

31.d. PQ =

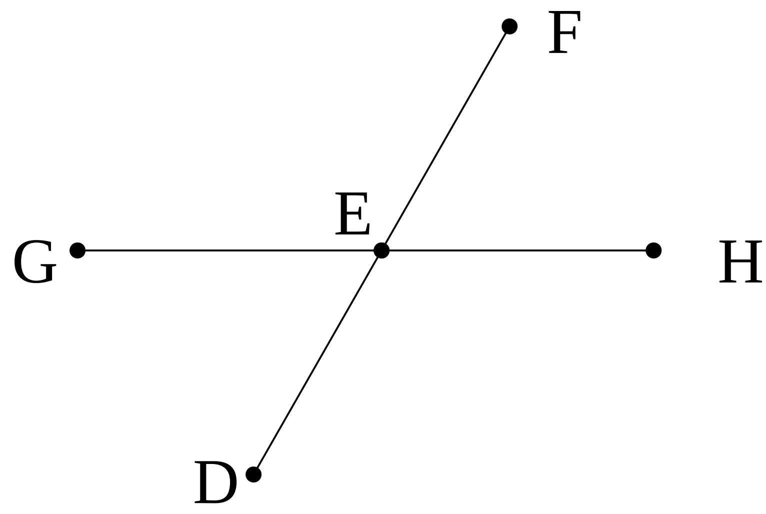


Sample Problems Section 1-3

E is the midpoint of $\overline{DF}$. Find the value of x.

33. DE = 5x + 3, EF = 33

35. DE = 3x, EF = x + 6



Find the value of y.

37. GE = y, EH = y - 1, GH = 11

Find the value of z. Then find GE and EH and state whether E is the midpoint of $\overline{GH}$

39. GE = z + 2, GH = 20, EH = 2z - 6

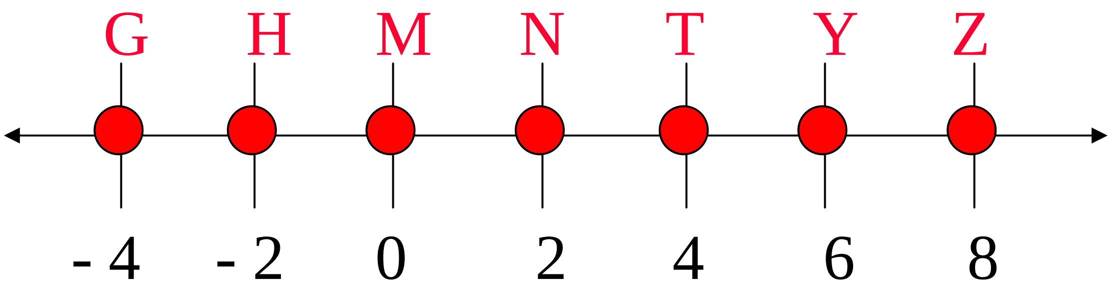
Sample Problems Section 1-3

Name the graph of the given equation or inequality.

41. $-2 \le x \le 2$

43. $|x| \le 4$

45. $|x| = 0$



Section 1-4

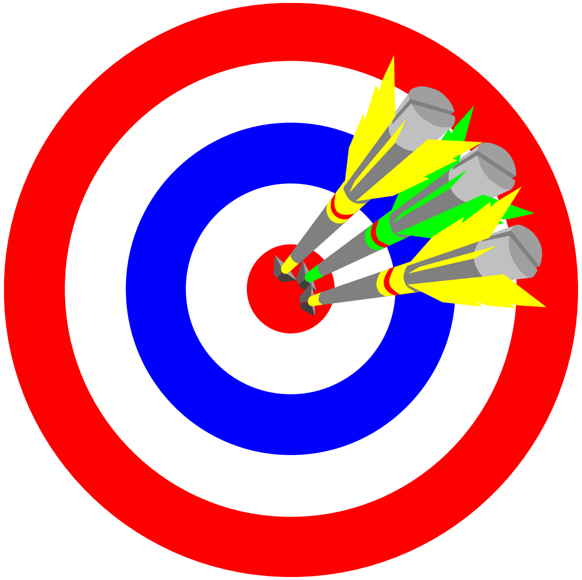
Angles

Homework Pages 21-22:

2-34 evens

Excluding 8, 22

Objectives



A. Understand how angles are constructed, labeled, represented graphically, and measured.
B. Use the terms angle, sides, and vertex correctly.
C. Understand congruent and adjacent angles.
D. Identify and define acute, right, obtuse, and strait angles.
E. Use the term bisector of an angle correctly.
F. Apply the Protractor and Angle Addition Postulates correctly.

THE ‘MAKING’ OF AN ANGLE

- An **angle** is a figure formed by two rays (sides) with a common endpoint (vertex).
- The two rays that make up the angle are known as **sides** of the angle.
- The common endpoint of the angle is the **vertex**.
- An angle is labeled with the angle symbol ($\angle$) and either:
  - A number from the interior,
  - The label of the vertex point, if it is the only angle at that vertex, or
  - Three points on the angle--two from the rays and one from the vertex (ORDER MATTERS).

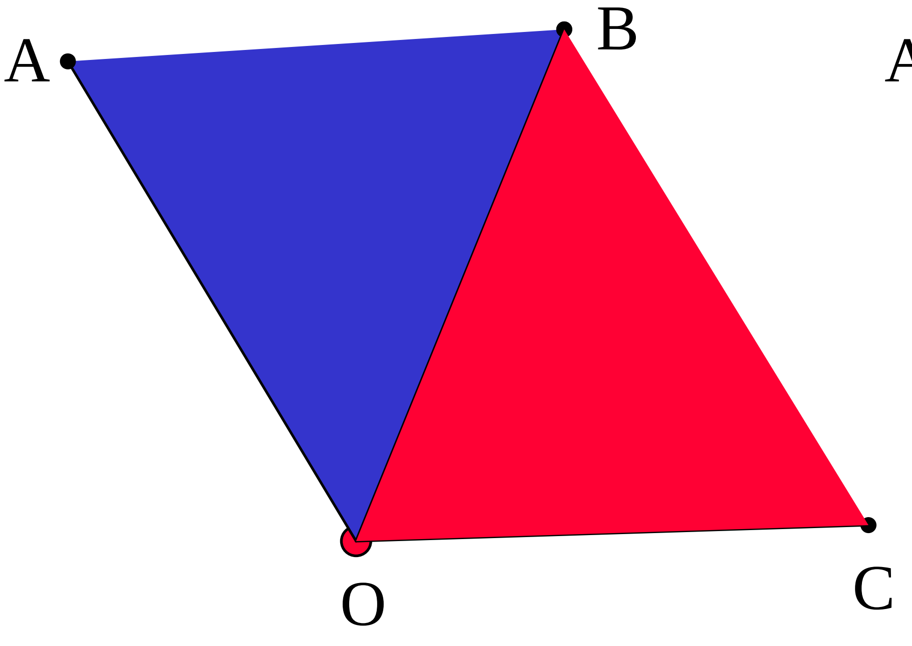
‘TYPES’ OF ANGLES

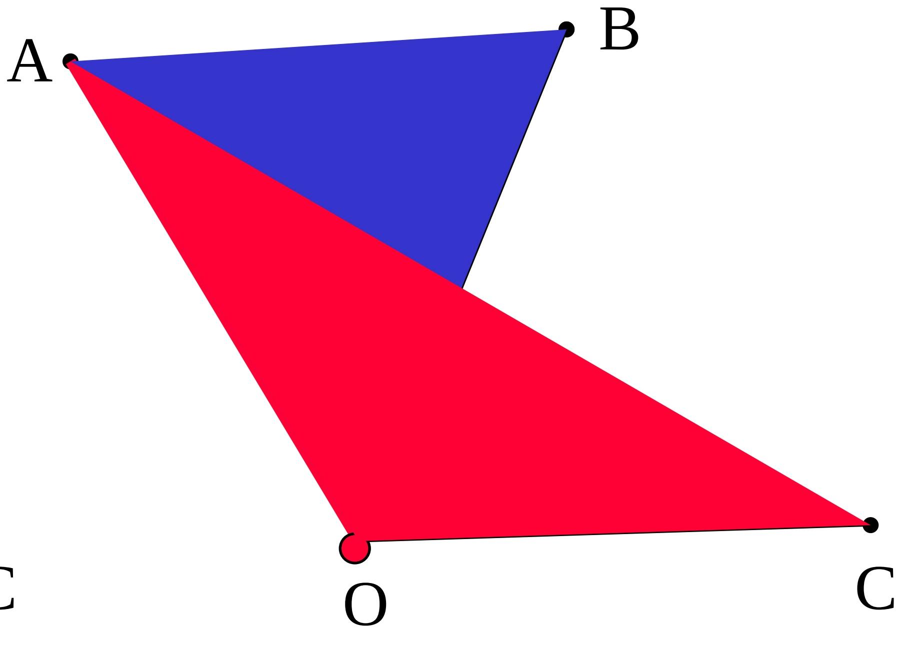
- ★ acute angle: measures between 0° and 90°
- ★ right angle: measures exactly 90°
- ★ obtuse angle: measures between 90° and 180°
- ★ straight angle: measures exactly 180°

ANGLE RELATIONSHIPS

- **congruent angles**: are angles with the same measurement
- **bisector of an angle**: is a ray that divides an angle into two congruent adjacent angles
- **adjacent angles**: are two coplanar angles with a common vertex and a common side but no common interior points.

Adjacent Angles





∠ AOB & ∠ BOC are adjacent ∠ 's

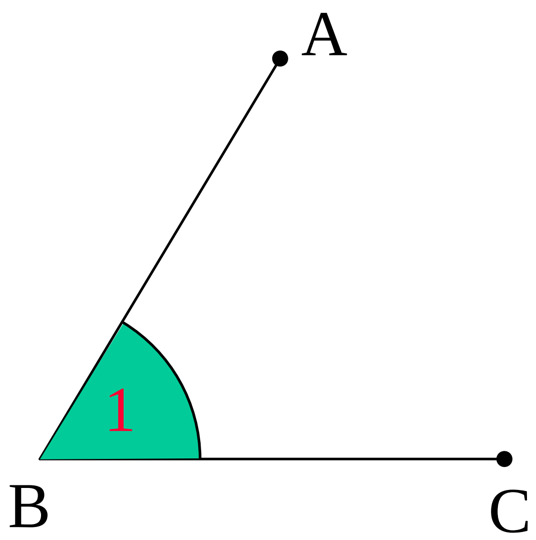
∠ AOB & ∠ AOC are not adjacent ∠ 's

Naming Angles

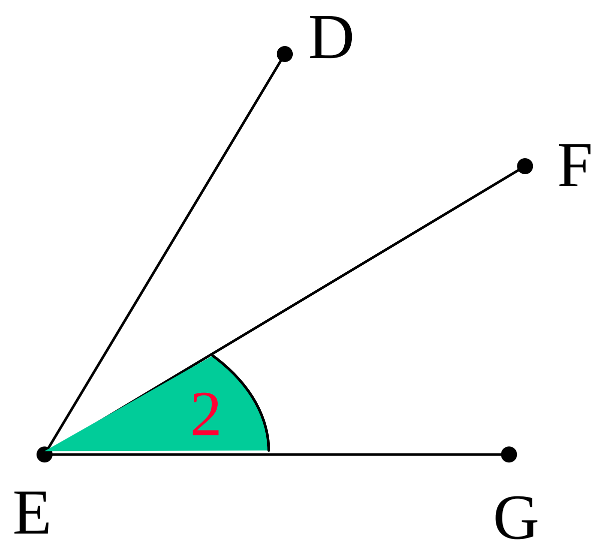
If there is one angle at the vertex, then you can use the letter of the vertex to name the angle.

If there are two or more angles at the vertex, then you must use three letters to name the angle.

You may use the number inside the angle anytime.



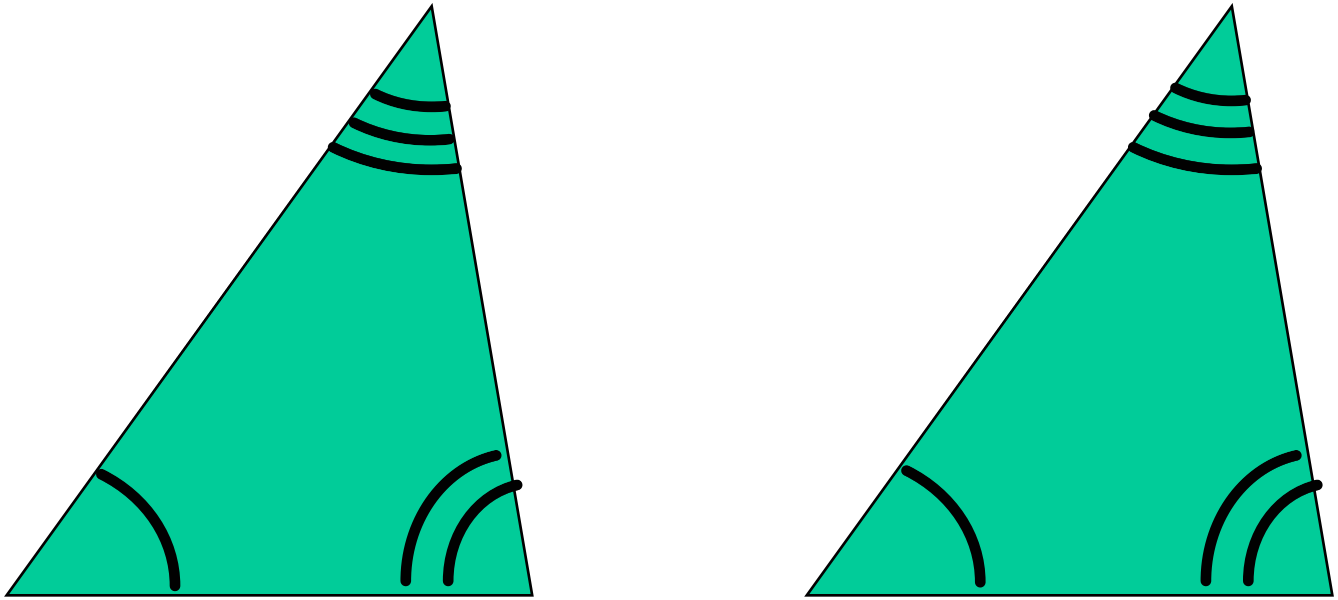
$\angle$ B or $\angle$ 1



$\angle$ FEG or $\angle$ GEF or $\angle$ 2

Congruent Angles in Diagrams

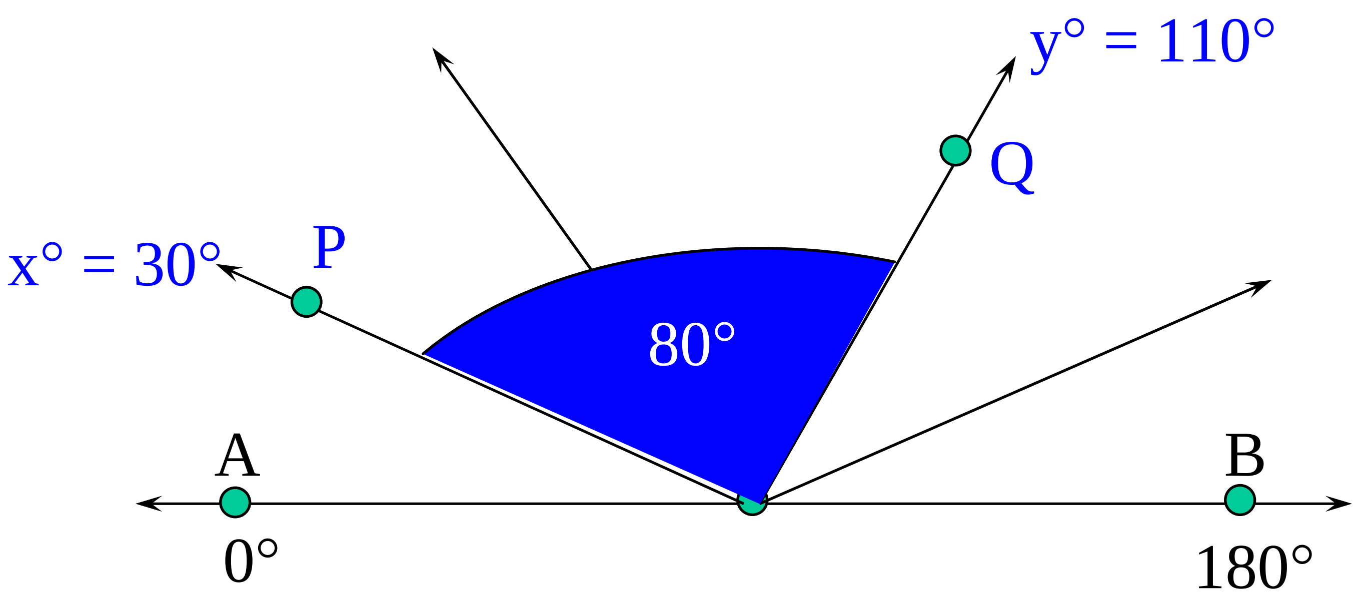
Congruent angles are marked with arcs. One arc for the first pair of congruent angles, two arcs for the second pair, etc.



Postulate 3 (Protractor Postulate)

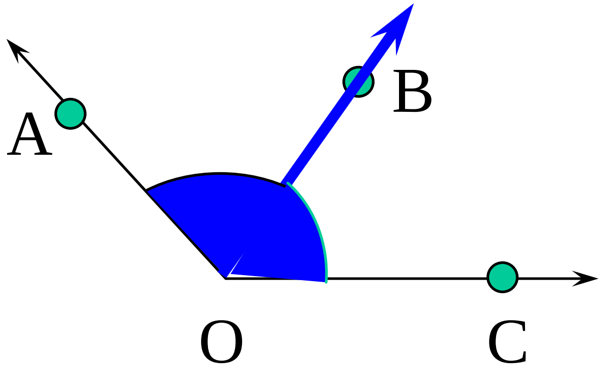
On line AB in a given plane, choose any point O between A & B. Consider ray OA and ray OB and all the rays that can be drawn from O on one side of the line AB. These rays can be paired with the real numbers such that ray OA is paired with 0, and ray OB is paired with 180.

If ray OP is paired with some number x, and ray OQ is paired with some other number y,
then the $m\angle POQ = |x - y| = |30 - 110| = |-80| = 80°$

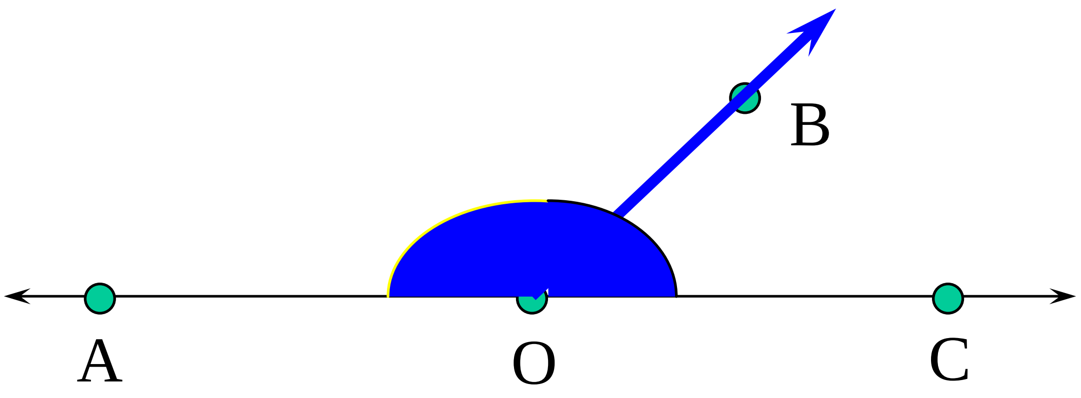


★Postulate 4 (Angle Addition Postulate)

If point B lies on the interior of of ∠AOC, then m∠AOB + m∠BOC = m∠AOC



If ∠AOC is a straight angle and B is any point not on line AC, then m∠AOB + m∠BOC = 180



Sample Problems Section 1-4

1. Name the vertex and sides of ∠ 5.

State another name for the angle.

3. ∠ 1

5. ∠ 5

7. ∠ AST

State whether the angle appears to be acute, right, obtuse, or straight.

9. ∠ 2

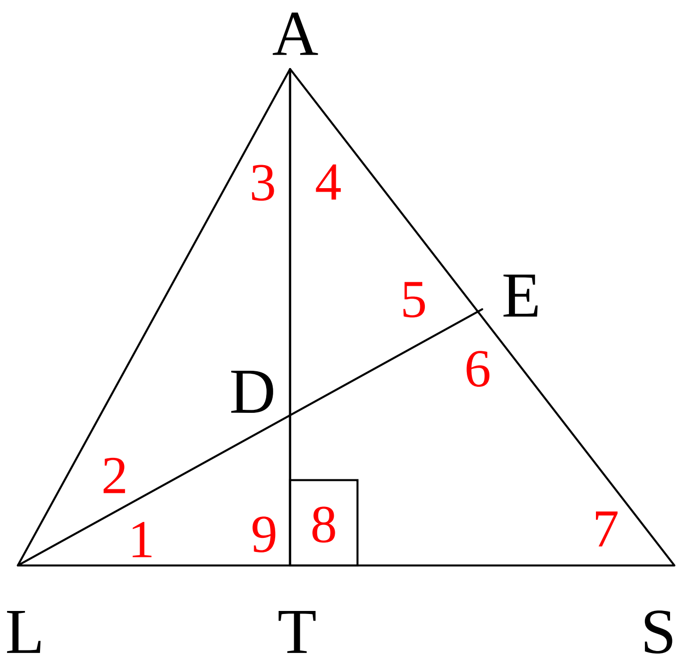
11. ∠ ATL

13. ∠ LTS

Complete.

15. m ∠ 3 + m ∠ 4 = m _____

17. If m ∠ 1 = m ∠ 2, then _____ bisects _____.



Sample Problems Section 1-4

Without measuring sketch each angle.

19. 90° angle

21. 150° angle

Draw a line, $\overleftrightarrow{AB}$. Choose a point O between A and B.

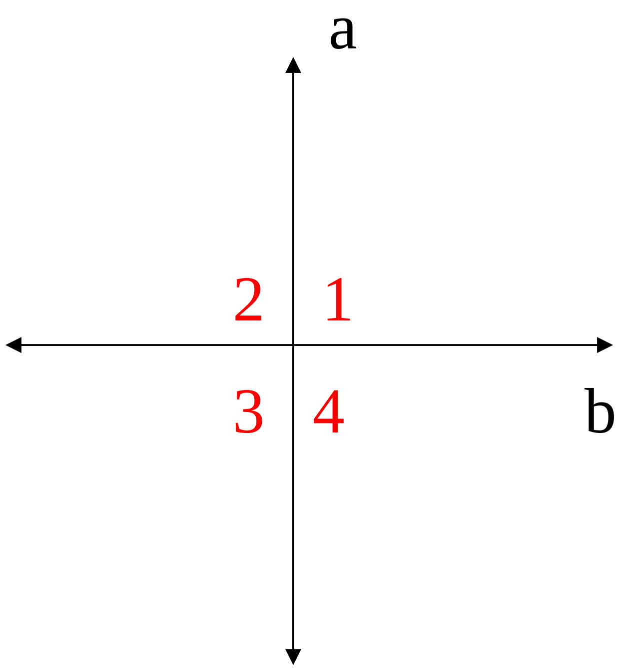
Investigate the following.

23. In the plane represented by your paper, how many lines can you draw through O that form a 30° angle with $\overrightarrow{OB}$?

25. Using a ruler, draw a large triangle. Then use a protractor to find the approximate measure of each angle and compute the sum of the three measures. Repeat this exercise for a triangle with a different shape. Did you get the same result?

Sample Problems Section 1-4

Express m ∠ 2, m ∠ 3 and m ∠ 4 in terms of t when m ∠ 1 = t.



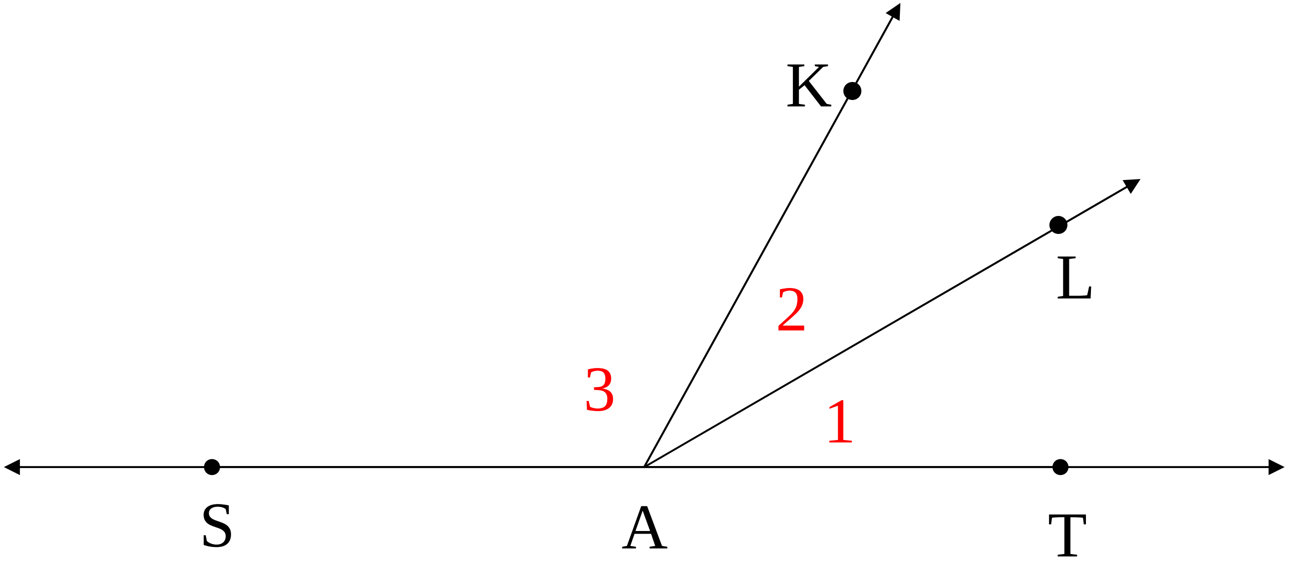
Sample Problems Section 1-4

$\overrightarrow{AL}$ bisects $\angle$ KAT. Find the value of x.

29. m $\angle$ 3 = 6x, m $\angle$ KAT = 90 - x

31. m $\angle$ 1 = 5x - 12, m $\angle$ 2 = 3x + 6

33. m $\angle$ 1 = 2x - 8, m $\angle$ 3 = 116



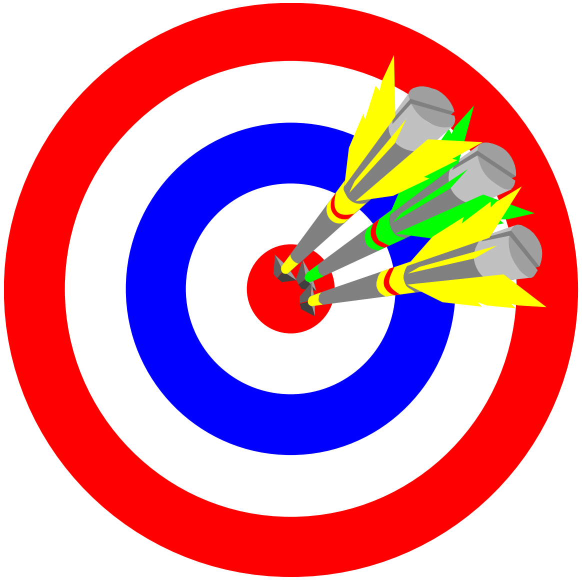
Section 1-5

Postulates and Theorems Relating Points, Lines and Planes

Homework pages 25-26:

2-18 even

Objectives



A. Use postulates 5 through 9 correctly.
B. Use theorems 1-1 through 1-3 correctly.
C. Understand the use of the phrases ‘exactly one’ and ‘one and only one’.

POSTULATE VERSUS THEOREM

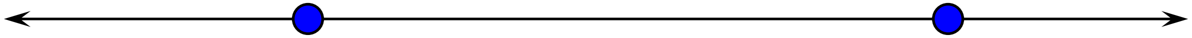
- Remember, a postulate is a statement which is accepted as true without proof.
- On the other hand, a **theorem** is a statement that is PROVEN using:
  - Undefined terms
  - Defined terms
  - Postulates
  - Other proven theorems

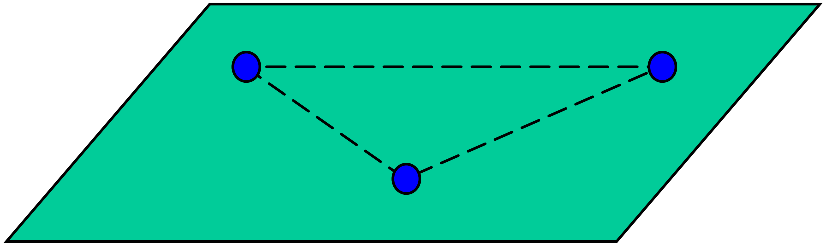
EXPRESSING UNIQUENESS

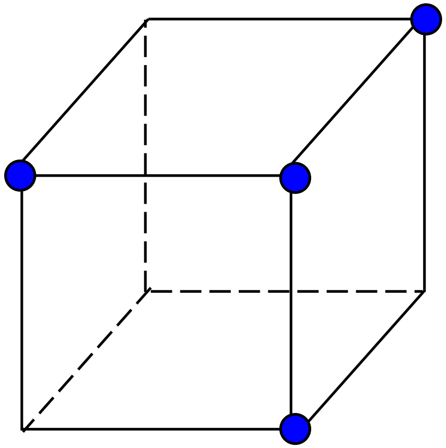
- To express the uniqueness of an object or expression in a postulate or theorem, we use the phrases:
  - Exactly one
  - One and only one

Postulate 5

A line contains at least two points; a plane contains at least three non-collinear points; space contains at least four non-coplanar points.







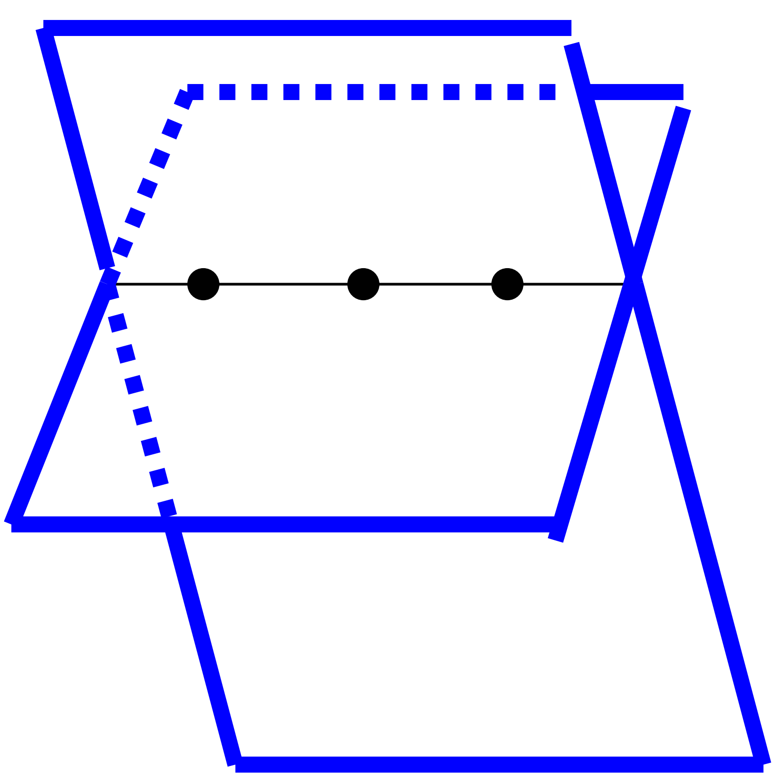
Postulate 6

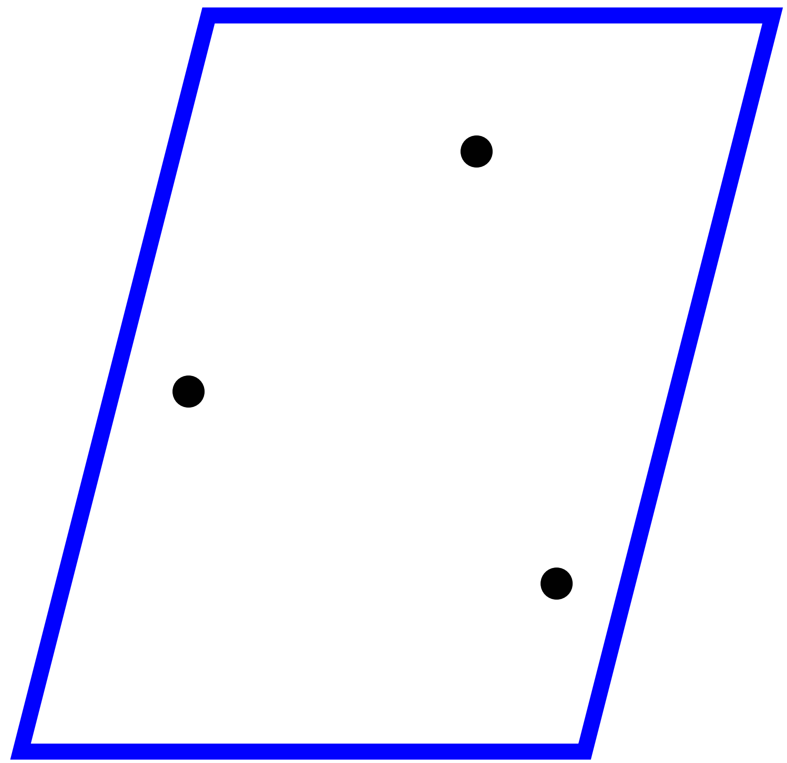
Through any two points there is exactly one line.

Now can you determine WHY a line can have no thickness?

Postulate 7

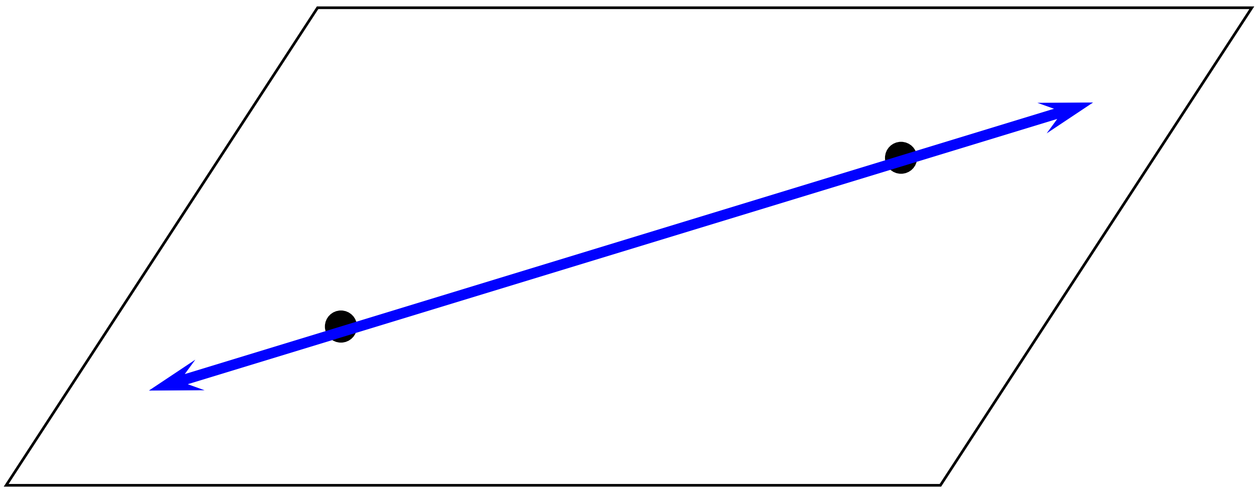
Through any three points there is at least one plane, and through any three non-collinear points there is exactly one plane.





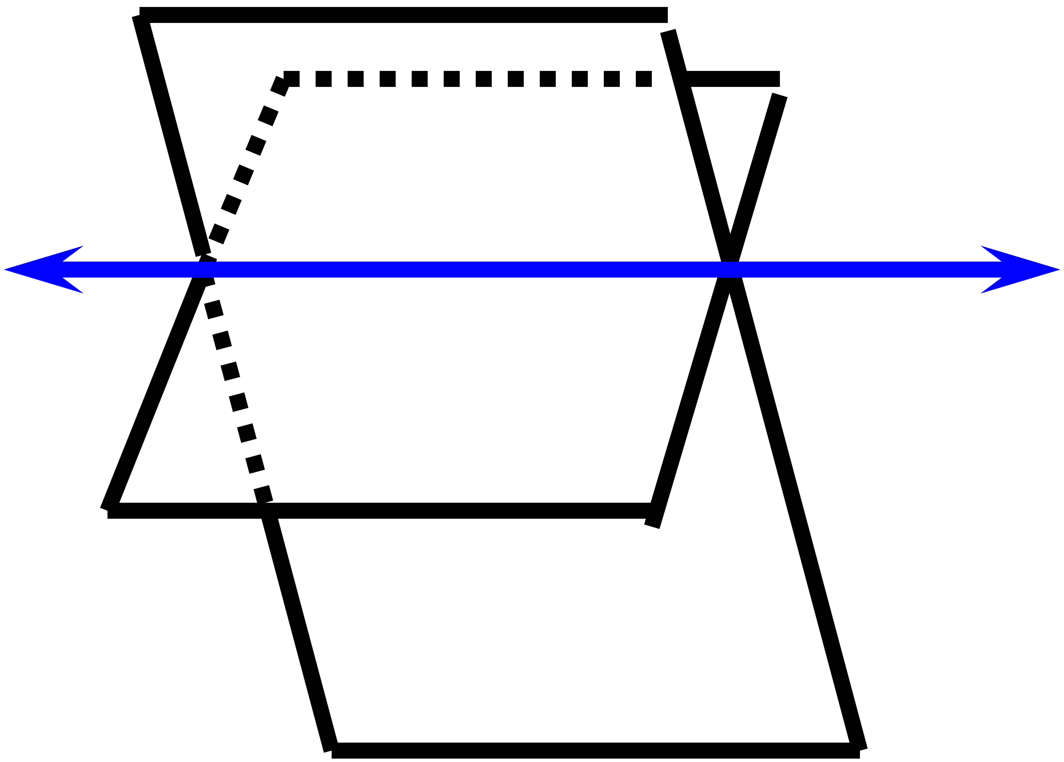
Postulate 8

If two points are in a plane, then the line that contains the points is in that plane.



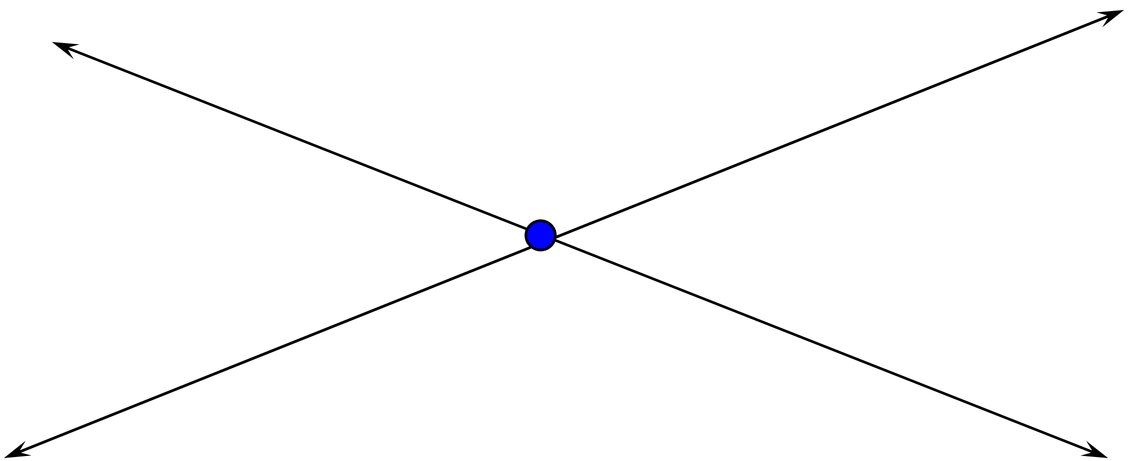
Postulate 9

If two planes intersect, then their intersection is a line.



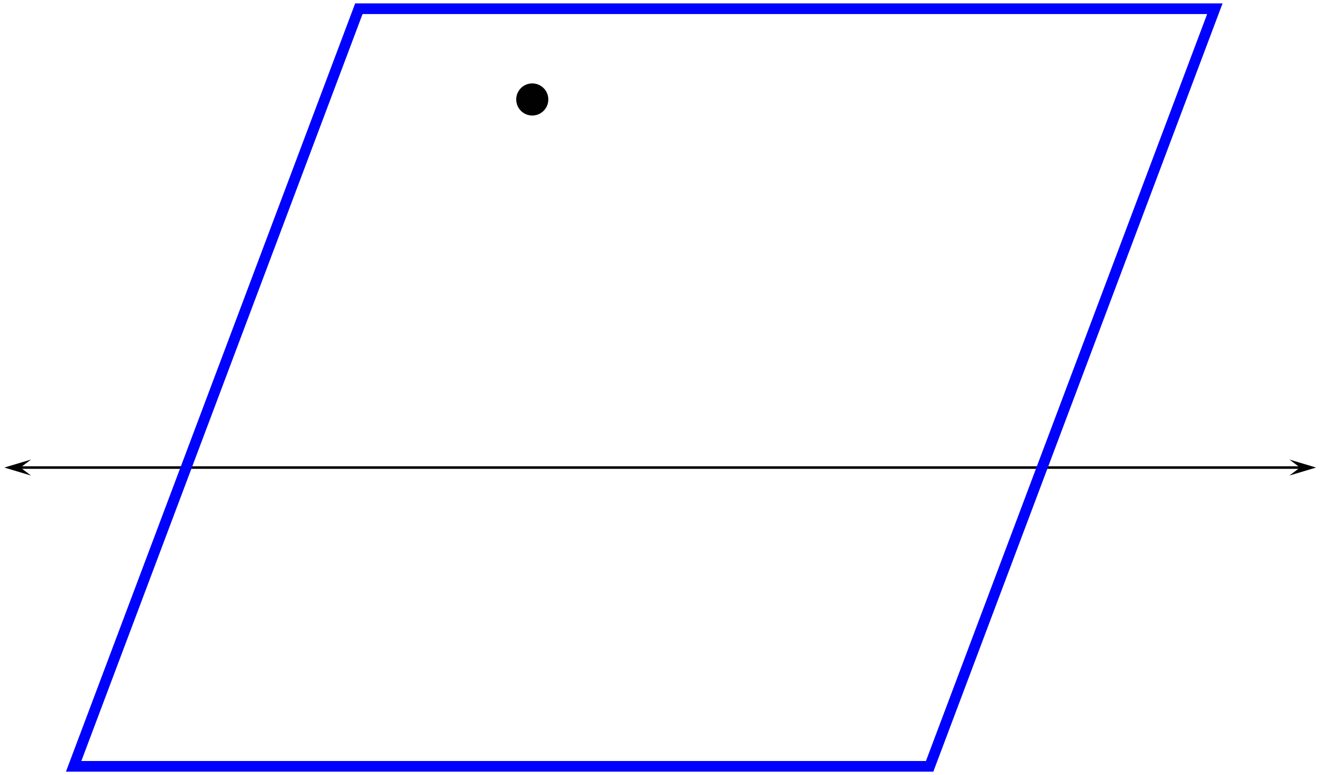
Theorem 1-1

If two lines intersect, then they intersect in exactly one point.



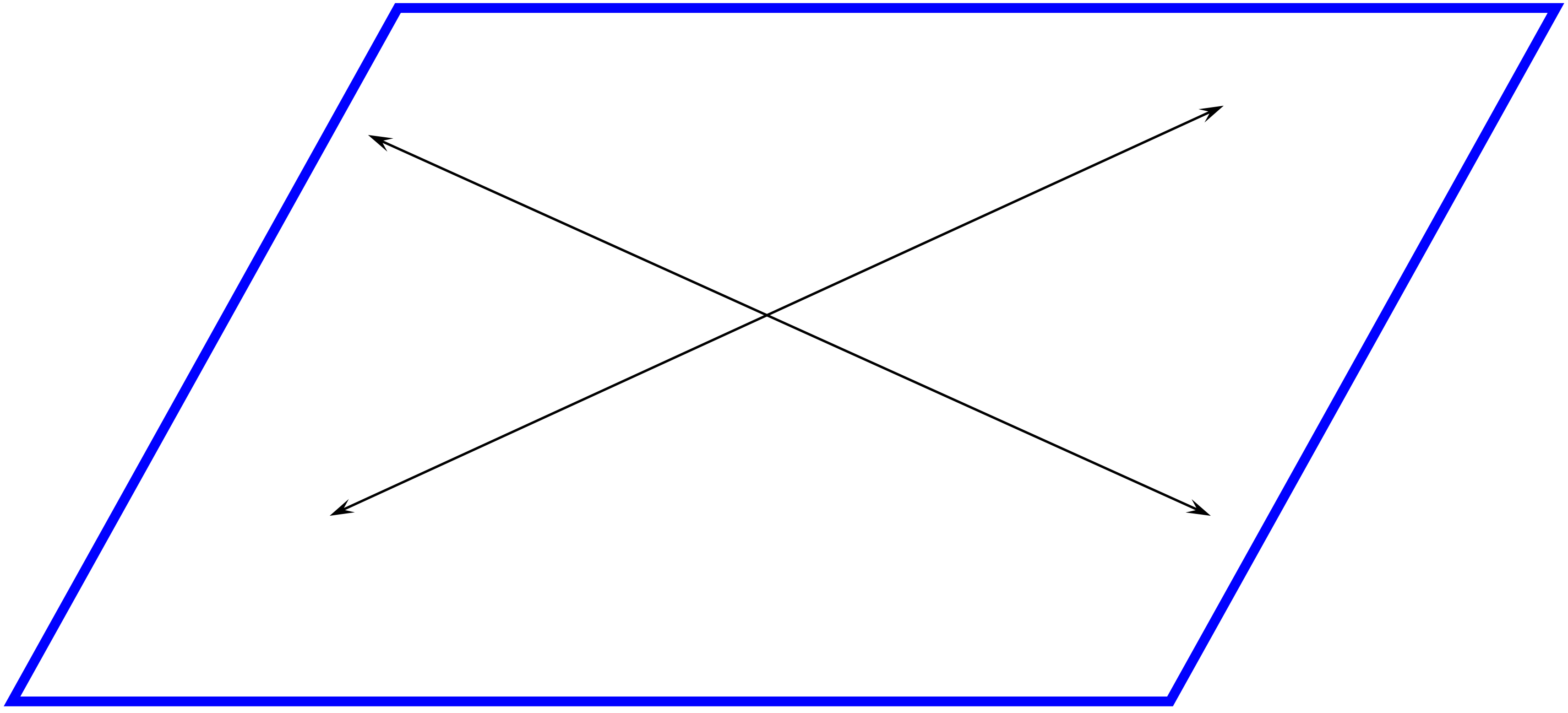
Theorem 1-2

Through a line and a point not on a line there is exactly one plane.



Theorem 1-3

If two lines intersect, then exactly one plane contains the lines.

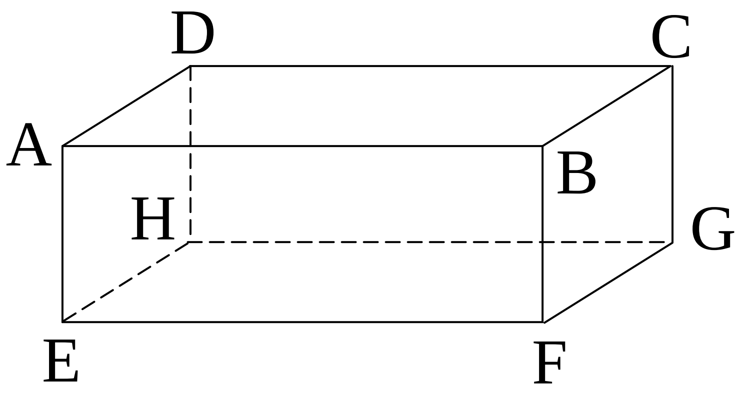


Sample Problems Section 1-5

1. State theorem 1-2 using the phrase one and only one.
3. Planes M and N are known to intersect.
   a. What kind of figure is the intersection of M and N?
   b. State the postulate that supports your answer to part a.

Sample Problems Section 1-5

5. Write the postulate that assures you that $\overleftrightarrow{AC}$ exists.
7. Name a plane that contains $\overleftrightarrow{AC}$ but is not shown in the diagram.
9. Name four line shown in the diagram that do not intersect plane EFGH.
11. Name three planes that do not intersect $\overleftrightarrow{EF}$ and don't contain $\overleftrightarrow{EF}$.



Sample Problems Section 1-5

State whether it is possible for the figure described to exist. Write yes or no.

13. Two points both lie in each of two lines.

15. Three noncollinear point all lie in each of two planes.

17. Points R, S, and T are noncollinear points.

    a. State the postulate that guarantees the existence of a plane X that contains R, S, and T.

    b. Draw a diagram showing plane X containing the noncollinear points R, S, and T.

    c. Suppose that P is any point of $\overleftrightarrow{RS}$ other than R and S. Does point P lie in plane X? Explain.

Sample Problems Section 1-5

17.d. State the postulate that guarantees that $\overleftrightarrow{TP}$ exists.

e. State the postulate that guarantees that $\overleftrightarrow{TP}$ is in plane X.

Chapter One

Points, Lines, Planes, and Angles

Review

Homework Page 31:

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