

Chapter Two

Deductive Reasoning

Objectives



- A. Use the terms defined in the chapter correctly.
- B. Properly use and interpret the symbols for the terms and concepts in this chapter.
- C. Appropriately apply the properties and theorems in this chapter.
- D. Correctly interpret the information contained in a conditional.

Section 2-1

If-Then Statements; Converses
Homework Page 35:
2-30 even

Objectives



- A. Identify and properly use conditional statements.
- B. Identify the hypothesis and conclusion of conditionals.
- C. Identify and state the converse of a conditional.

- D. Provide correct counterexamples to prove statements false.
- E. Identify and use biconditional statements.

Conditional Statements

- We use conditional statements in our everyday language, as well as in our mathematical language.
- The common form of a conditional statement, or conditional, is:
 - If *hypothesis*, then *conclusion*.

Conditional Statements

- **Hypothesis**
 - According to Merriam-Webster dictionary \rightarrow a tentative assumption made in order to draw out and test its logical or empirical consequences
 - More simply \rightarrow a set of pre-conditions from which we **attempt** to reach a conclusion.
 - Or \rightarrow the information which must be known in order to apply the conclusion to a problem.
 - In geometry, it is common for the hypothesis to describe a diagram, or to be part of a diagram.
- **Conclusion** \rightarrow Information which can be added to a problem when the criteria of the hypothesis have been met.

Conditional Statements

- Conditional statements can be TRUE or FALSE:
 - If they are considered TRUE, they must be TRUE in ALL cases.
 - If there is a SINGLE case where the statement is false, then the ENTIRE conditional is considered FALSE!
- Examples:
 - If I do not eat, then I will eventually starve.
 - If I live in Bexley, then I live in Ohio.
 - If I add 3 to 4, then I will have 7.
 - If you cheat on homework, then you won't do well in this class.
 - If you want the freedom of an adult, then you must accept adult responsibilities.

Other Forms of Conditional Statements

- Thanks to the English language, you have several other ways of expressing a conditional statement:
 - **IF** hypothesis, **THEN** conclusion.
 - hypothesis **IMPLIES** conclusion.
 - hypothesis **ONLY IF** conclusion.
 - conclusion **IF** hypothesis.

Equivalent Conditionals: Examples

- **If you live in Ohio, then** you live in the United States.
- **You live in Ohio implies** that you live in the United States.
- **You live in Ohio only if** you live in the United States.
- **You live in the United States if** you live in Ohio.

“Say it ain’t so!”

- A major outcome of your work in this class will be your ability to prove or disprove conditionals.
- Remember, a conditional is always true or it is false, there is no “sometimes this, sometimes that”.
- To prove a conditional or theorem to be true usually takes a number of steps.
 - The proof MUST show that the statement to be true for ALL cases.
- To prove something is false we need ONLY ONE example where the hypothesis contradicts the conclusion.
 - Such an example is known as a counterexample:
 - A counter example proves a conditional false by agreeing with the hypothesis but disagreeing with the conclusion.

Counterexamples

- If it is a week night, then you have geometry homework.
 - August 4th is a week night, but you don’t have geometry homework during summer break.
- This statement agrees with the hypothesis, but disagrees with the conclusion.
- Since we have found one counterexample for the conditional we say that it is false.
- It makes no difference how many examples we can find where it is true, because it was false once it may be false again so it has no value in predicting the future or judging the present.

Converse of a Conditional

- ★ converse: The converse of a conditional is another if-then statement formed by interchanging the hypothesis and the conclusion of a given statement.
 - ★ Conditional statement \rightarrow If p then q
 - ★ Converse of above statement \rightarrow If q then p
- ★ Example:
 - ★ Conditional
 - ★ If tomorrow is Saturday, then today is Friday.
 - ★ Converse
 - ★ If today is Friday, then tomorrow is Saturday.

Converses of Conditionals

- **NOTE!** Just because the conditional statement is true does NOT make the converse of the statement true!
 - Likewise, just because the converse of a statement is true does not make the conditional true.
- Example:
 - Conditional \rightarrow If I have 2 dimes and a nickel, then I have 25 cents.
 - Converse \rightarrow If I have 25 cents, I have 2 dimes and a nickel.
- Remember, you need only ONE counterexample to prove a statement false.

Biconditionals

- For a statement to be biconditional, both the original conditional (statement) and its converse must be true.
- One sign that you have a biconditional statement is the key phrase “if and only if” to connect the parts of the statement.
- In a biconditional, the order of the phrases can be switched without changing the meaning.

Biconditional Example

- Conditional \rightarrow If I draw a right angle, then I draw a 90 degree angle.
- Converse \rightarrow If I draw a 90 degree angle, then I draw a right angle.
- Biconditional \rightarrow I draw a right angle if and only if I draw a 90 degree angle.

Definitions Written as Biconditionals

- An angle is acute if and only if it measures between 0° and 90° .
 - An angle measures between 0° and 90° if and only if the angle is acute.
- A ray bisects an angle if and only if it divides the angle into two congruent adjacent angles.
 - A ray divides the angle into two congruent adjacent angles if and only if the ray bisects an angle.
- Points are collinear if and only if the points lie on one line.
 - Points lie on one line if and only if the points are collinear.

Section 2-1 Sample Problems

Write the hypothesis and the conclusion of each conditional.

1. If $3x - 7 = 32$, then $x = 13$
3. I'll try if you will.
5. $a + b = a$ implies $b = 0$.

Rewrite each pair of conditionals as a biconditional.

7. If B is between A and C, then $AB + BC = AC$.
If $AB + BC = AC$, then B is between A and C.

Write each biconditional as two conditionals that are converses of each other.

9. Points are collinear if and only if they all lie on one line.

Sample Problems Section 2-1

Provide a counterexample to show that each statement is false. You may use words or diagrams.

11. If $ab < 0$, then $a \leq 0$. \rightarrow
13. If point G is on AB, then G is on BA.
15. If a four sided figure has four right angles, then it has four congruent sides.

Tell whether each statement is true or false. Then write the converse and tell whether it is true or false.

17. If $x = -6$, then $|x| = 6$.
19. If $b > 4$, then $5b > 20$.
21. If Pam lives in Chicago, then she lives in Illinois.
23. $a^2 > 9$ if $a > 3$.

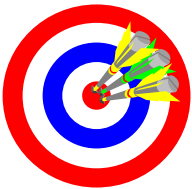
Sample Problems Section 2-1

- 25. $n > 5$ only if $n > 7$.
- 27. If points D, E and F are collinear, then $DE + EF = DF$.
- 29. Write a definition of congruent angles as a biconditional.

Section 2-2

Properties from Algebra
Homework Pages 41-42:
2-14 evens

Objectives



- A. Properly use and describe algebraic properties.
- B. Relate the algebraic properties to geometric properties.
- C. Properly apply geometric properties.

Algebraic Properties of Equality: Transformations

- Addition: You may add the same value to both sides of an equation.
 - If $a = b$, then $a + c = b + c$.
- Subtraction (add a negative): You may subtract the same value from both sides of an equation.
 - If $a = b$, then $a - c = b - c$.
- Multiplication: You may multiply the same value to both side of an equation.
 - If $a = b$, then $a * c = b * c$.

Algebraic Properties of Equality: Transformations

- Division (multiply by a reciprocal): You may divide the same value into both sides of an equation.
 - If $a = b$, then $a / c = b / c$.
 - HOWEVER: c cannot be zero!
- Distribution: You may multiply a factor next to a grouping symbol to every term inside the grouping symbol.
 - If $a(b + c + d) = e$, then $ab + ac + ad = e$.
- Substitution: Left and right sides of an equation are interchangeable. Either statement may be used in another equation.
 - If $a + b = c$ AND $d - e = c$, then:
 - $a + b = d - e$

Algebraic Properties of Equality: Transformations

- Reflexive: A value must equal itself.
 - $a = a$
- Symmetric: The left and right sides of an equation can be switched.
 - If $a = b$, then $b = a$.
- Transitive: Any two values in a chain of equality are equal.
 - If $a = b$ AND If $b = c$, then $a = c$.

But what about GEOMETRIC properties?

- Remember, we cannot talk about two FIGURES being EQUAL!
 - Two geometric figures can be CONGRUENT.
 - So certain properties, such as the Addition Property of equalities, cannot be applied to figures.
 - SOME properties can be applied to figures.
- However, lengths of line segments and measures of angles are real numbers.
 - Therefore, we can apply algebraic properties of equalities (such as the Addition Property) to these real numbers.

Properties of Congruence

•Reflexive: Any object must be congruent (same size and shape) to itself. $\overline{DE} \cong \overline{DE}$ $\angle D \cong \angle D$

•Symmetric: The objects on the left and right sides of a congruence statement may be switched. If $\overline{DE} \cong \overline{FG}$ then $\overline{FG} \cong \overline{DE}$.
If $\angle D \cong \angle E$ then $\angle E \cong \angle D$.

•Transitive: Any two objects in a chain of congruence statements are congruent (same size and shape).

If $\overline{DE} \cong \overline{FG}$ and $\overline{FG} \cong \overline{JK}$ then $\overline{DE} \cong \overline{JK}$.
If $\angle D \cong \angle E$ and $\angle E \cong \angle F$ then $\angle D \cong \angle F$.

So, what do we do with these properties?

- We use these algebraic and geometric properties to prove statements.
- For example:

If $3X + 5 = 17$, then $X = 4$.	Given.
$3X + 5 (-5) = 17 (-5)$ $3X = 12$	Subtraction Property of Equality.
$3X (/3) = 12 (/3)$ $X = 4$	Division Property of Equality.

Sample Problems Section 2-2

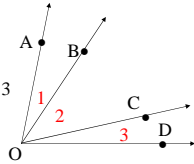
Justify each step.

- | | |
|--------------------------|----------------------------|
| 1. $4x - 5 = -2$ | 5. $\frac{2}{3}b = 8 - 2b$ |
| $4x = 3$ | $2b = 3(8 - 2b)$ |
| $x = \frac{3}{4}$ | $2b = 24 - 6b$ |
| 3. $\frac{z+7}{3} = -11$ | $8b = 24$ |
| $z+7 = -33$ | $b = 3$ |
| $z = -40$ | |

Sample Problems Section 2-2

Given: $\angle AOD$ as shown

Prove: $m\angle AOD = m\angle 1 + m\angle 2 + m\angle 3$



Statements

Reasons

- | | |
|--|----|
| 1. $m\angle AOD = m\angle AOC + m\angle 3$ | 1. |
| 2. $m\angle AOC = m\angle 1 + m\angle 2$ | 2. |
| 3. | 3. |

Sample Problems Section 2-2

Given: $DW = ON$

Prove: $DO = WN$



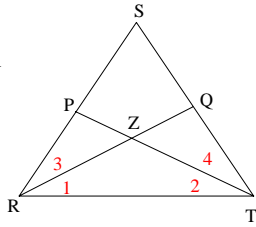
Statements

Reasons

- | | |
|--|-----------------|
| 1. $DW = ON$ | 1. |
| 2. $DW = DO + OW$ | 2. |
| $ON = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ | 3. Substitution |
| 3. $\underline{\hspace{1cm}}$ | 4. |
| 4. $OW = OW$ | 5. |
| 5. $\underline{\hspace{1cm}}$ | |

Sample Problems Section 2-2

Given: $m \angle 1 = m \angle 2$;
 $m \angle 3 = m \angle 4$
Prove: $m \angle SRT = m \angle STR$

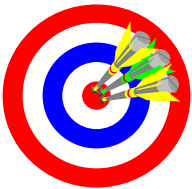


Given: $RQ = TP$
 $ZQ = ZP$
Prove: $RZ = TZ$

Section 2-3

Proving Theorems
Homework Page 46:
2-16 evens

Objectives



- A. Use the Midpoint Theorem and the Angle Bisector Theorem correctly.
- B. Understand the valid reasons used in proofs.
- C. Apply valid reasons to prove theorems and conditionals.

Deductive Reasoning

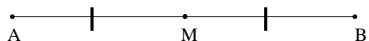
- Also known as direct proof
- Deductive reasoning is one logical process used to prove conditionals true by building an argument based upon valid reasoning.

Valid Reasons Used in Proofs

- The items you may use in a proof are:
 - Information given in the hypothesis,
 - Accepted postulates,
 - Algebraic and geometric properties,
 - Definitions,
 - Previously proven or accepted theorems, and
 - Previously proven or accepted corollaries.

★Theorem 2-1 (Midpoint Theorem)

If M is the midpoint of segment AB,



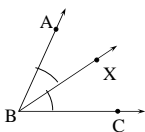
then $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$.

Proof Of Midpoint Theorem

Statements	Reasons
M is the midpoint of \overline{AB}	Given Information
$\overline{AM} \cong \overline{MB}$	Definition of Midpoint.
$AM = MB$	Definition of Congruence.
$AM + MB = AB$	Segment Addition Postulate.
$AM + AM = AB$ $2AM = AB$	Substitution Property. Simple addition.
$AM = \frac{1}{2} AB$	Division Property of Equalities.

★ Theorem 2-2 (Angle Bisector Theorem)

If ray BX is the bisector of $\angle ABC$,



then $m\angle ABX = \frac{1}{2}m\angle ABC$ and $m\angle XBC = \frac{1}{2}m\angle ABC$.

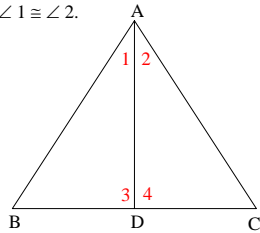
Proof Of Angle Bisector Theorem

Statements	Reasons

Sample Problems Section 2-3

Name the definition, postulate or theorem that justifies the statement about the diagram.

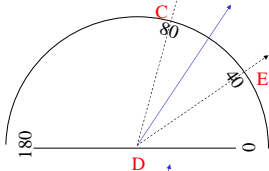
1. If D is the midpoint of \overline{BC} , then $\overline{BD} \cong \overline{DC}$
3. If \overrightarrow{AD} bisects $\angle BAC$, then $\angle 1 \cong \angle 2$.
5. If $\overline{BD} \cong \overline{DC}$, then D is the midpoint of
7. $m\angle 1 + m\angle 2 = m\angle BAC$



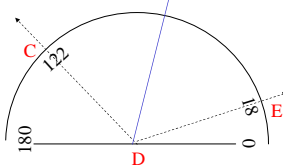
Sample Problems Section 2-3

Write the number that is paired with the bisector of $\angle CDE$.

9.

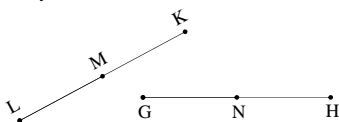


11.



Sample Problems Section 2-3

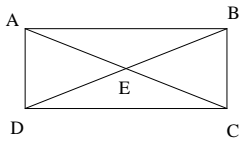
13. The coordinates of points L and X are 16 and 40, respectively. N is the midpoint of LX, and Y is the midpoint of LN. Sketch a diagram and find:
 - a. LN
 - b. the coordinate of N
 - c. LY
 - d. the coordinate of Y
- 15.a. Suppose M and N are the midpoints of LK and GH, respectively. What segments are congruent?
 - b. What additional information about the picture would enable you to deduce that $LM = NH$.



Sample Problems Section 2-3

What can you deduce from the given information?

17. Given: $AE = DE$;
 $CE = BE$



Section 2-4

Special Pairs of Angles
Homework Pages 52-54:
2-32 evens

Objectives



- A. Use the terms complementary, supplementary, and vertical angles correctly.
- B. Apply these terms to proofs.
- C. Use the Vertical Angle Theorem correctly.

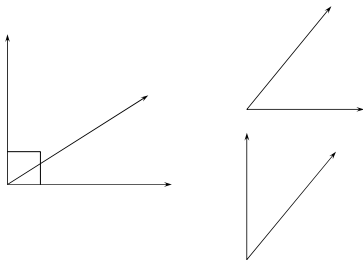
Definitions

complementary angles: A pair of coplanar angles, called complements, whose measurements add up to be 90° .

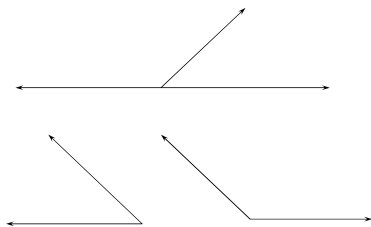
supplementary angles: A pair of coplanar angles whose measurements add up to be 180 degrees. A supplement of an angle is another angle that when added to the first makes 180 degrees.

vertical angles: A pair of coplanar angles such that the sides of one angle are opposite rays to the sides of the other angle.

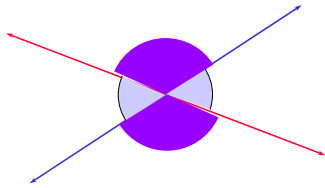
Complementary Angles



Supplementary Angles

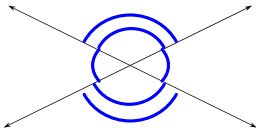


Vertical Angles



★ Theorem 2-3

Vertical angles are congruent.



Proof Of Theorem 2-3 (Vertical Angles Theorem)

Statements	Reasons

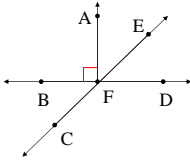
Sample Problems Section 2-4

Find the measures of a complement and a supplement of $\angle K$.

1. $m \angle K = 20$
3. $m \angle K = x$
5. Two complementary angles are congruent. Find their measures.

In the diagram, $\angle AFB$ is a right angle. Name the figures described.

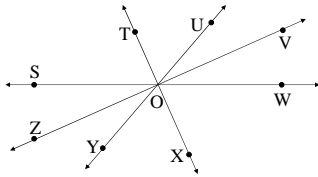
7. Another right angle.
9. Two congruent supplementary angles.
11. Two acute vertical angles.



Sample Problems Section 2-4

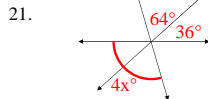
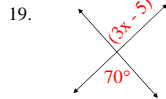
In the diagram, \overrightarrow{OT} bisects $\angle SOU$, $m \angle UOV = 35$, and $m \angle YOW = 120$. Find the measure of each angle.

13. $m \angle ZOY$
15. $m \angle VOW$
17. $m \angle TOU$



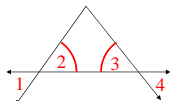
Sample Problems Section 2-4

Find the value of x .



23. Given: $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$

1. $\angle 2 \cong \angle 3$
2. $\angle 1 \cong \angle 2$
3. $\angle 3 \cong \angle 4$
- 4.



- 1.
- 2.
- 3.
4. Transitive Property

Sample Problems Section 2-4

If $\angle A$ and $\angle B$ are supplementary, find the value of x , $m\angle A$ and $m\angle B$.

25. $m\angle A = x + 16$, $m\angle B = 2x - 16$

If $\angle C$ and $\angle D$ are complementary, find the value of y , $m\angle C$ and $m\angle D$.

27. $m\angle C = y - 8$, $m\angle D = 3y + 2$

Use the information given to write an equation and solve the problem.

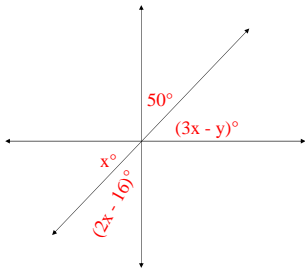
29. Find the measure of an angle that is half as large as its complement.

31. A supplement of an angle is six times as large as a complement of the angle. Find the measures of the angle, its supplement and its complement.

Sample Problems Section 2-4

Find the values of x and y for each diagram.

33.



Section 2-5

Perpendicular Lines
Homework Pages 58-60:
2-28 evens
Excluding 26

Objectives



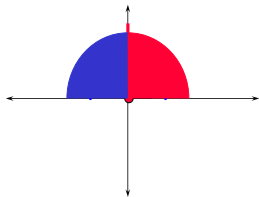
- A. Use the term perpendicular lines correctly.
- B. Apply the term perpendicular lines to theorems.
- C. Use the theorems associated with perpendicular lines (Theorems 2-4, 2-5, and 2-6) correctly.

Definition of Perpendicular Lines

- Perpendicular lines are two lines that intersect to form right angles.
 - Notice how we are using previous terms (defined and undefined) to build new terms.

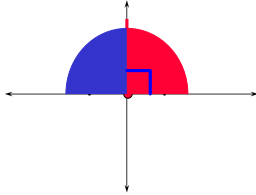
Theorem 2-4

If two lines are perpendicular, then they form congruent adjacent angles.



Theorem 2-5

If two lines form congruent adjacent angles, then the lines are perpendicular.

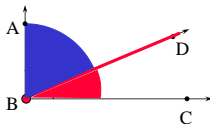


Comparing Theorems 2-4 and 2-5

- How is Theorem 2-4 (If two lines are perpendicular, then they form congruent adjacent angles) related to Theorem 2-5 (If two lines form congruent adjacent angles, then the lines are perpendicular)?
 - Are they the same?
 - Could they be written as a single statement?
 - What type of statement?

Theorem 2-6

If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.



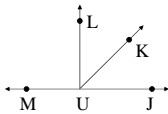
$$m\angle ABD + m\angle DBC = 90$$

Proof Of Theorem 2-6: If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.

Statements	Reasons

Sample Problems Section 2-5

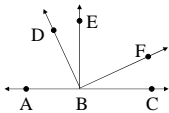
1. In the diagram, $UL \perp MJ$ and $m\angle JUK = x$. Express in terms of x the measures of the angles:
 a. $\angle LUK$ b. $\angle MUK$



Sample Problems Section 2-5

Name the definition or state the theorem that justifies the statement about the diagram.

3. If $\angle EBC$ is a right angle, then $BE \perp AC$.
 5. If $BE \perp AC$, then $\angle ABD$ and $\angle DBE$ are complementary.
 7. If $BE \perp AC$, then $m\angle ABE = 90$.



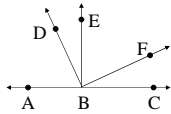
Sample Problems Section 2-5

In the diagram, $BE \perp AC$ and $BD \perp BF$.

Find the value of x .

9. $m \angle ABD = 2x - 15$, $m \angle DBE = x$

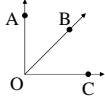
11. $m \angle ABD = 3x - 12$, $m \angle DBE = 2x + 2$,
 $m \angle EBF = 2x + 8$



13. Given: $OA \perp OC$

Prove: $\angle AOB$ and $\angle BOC$ are comp \angle s

- | | |
|---|------------------------|
| 1. $OA \perp OC$ | 1. |
| 2. $m \angle AOC = 90$ | 2. Def \perp |
| 3. $m \angle AOB + m \angle BOC = m \angle AOC$ | 3. |
| 4. | 4. Subst |
| 5. | 5. Def comp \angle s |



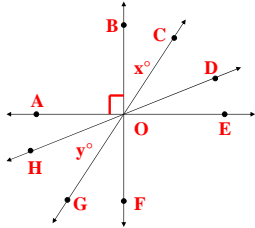
Sample Problems Section 2-5

In the figure $BF \perp AE$, $m \angle BOC = x$, and $m \angle GOH = y$.

Express the measures of the angles in terms of x and y .

15. $\angle COA$

17. $\angle DOE$



Sample Problems Section 2-5

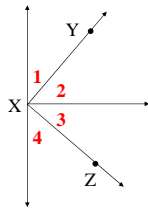
Can you conclude from the information given for each exercise that $XY \perp XZ$?

19. $\angle 1$ and $\angle 3$ are complementary

21. $m \angle 1 = m \angle 4$

23. $m \angle 1 = m \angle 2$ and $m \angle 3 = m \angle 4$

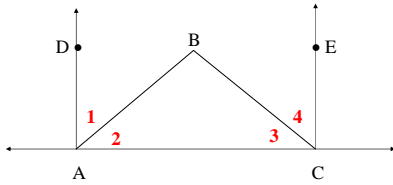
25. $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$



Sample Problems Section 2-5

What can you conclude from the information given.

27. Given: $AD \perp AC$; $CE \perp AC$; $m \angle 1 = m \angle 4$



Section 2-6

Planning a Proof
Homework Pages 63-64:
2-22 evens

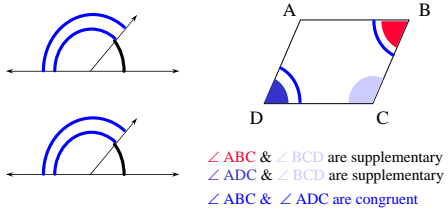
Objectives



- A. Understand and apply theorems 2-7 (supplementary angles) and 2-8 (complementary angles) correctly.
- B. Apply the two-column deductive proof method to prove statements, theorems, and/or corollaries.

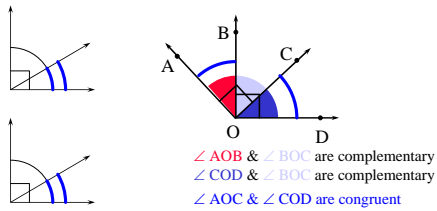
Theorem 2-7

If two angles are supplements of congruent angles (or the same angle), then the two angles are congruent.



Theorem 2-8

If two angles are complements of congruent angles (or of the same angle), then the two angles are congruent.



Parts of a Two Column Proof

Statements:

Each statement should be numbered and supported in the reason column.

The first statement is a list of the given information found in the hypothesis of the conditional or from information in a diagram.

The body of the proof consists of a logical series of statements and reasons flowing from the givens and from information that can be proved from the diagram.

The last statement must be what you were required to prove, found in the conclusion of the conditional.

Parts of a Two Column Proof

Reasons:

- Each reason should be numbered so that it matches the statement it supports.
- The first reason will be "given" provided that the first statement was a list of the given information.
- Only "given", definitions, postulates, algebraic properties, theorems and their corollaries are acceptable reasons for the body of the proof.
- The final reason will depend upon the logical structure of the whole proof, but it still must come from the list above (except that it cannot be "given").

Steps for Writing a Two Column Proof

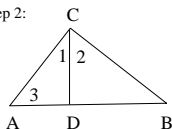
- Step 1: Identify the conditional you are required to prove.
- Step 2: Draw and label a diagram for the proof.
- Step 3: List from the conditional, in terms of the figure, what is given.
- Step 4: Determine from the conditional, in terms of the diagram, what is to be proven.
- Step 5: List from the diagram, in terms of the figure, what can be proven and why.
- Step 6: Select and arrange, in a logical order, those statements from steps 3, 4 & 5 that will allow you to move from the given information to the statement to be proved.

N.B. If you get stuck writing the proof from the top down, then work up from the conclusion by deciding what you had to know to make that statement.

Example of a Two Column Proof: #21 page 64

Step 1: If $AC \perp BC$ and $\angle 3$ is complementary to $\angle 1$, then $\angle 3 \cong \angle 2$.

Step 2:



Step 3: Given: $AC \perp BC$
 $\angle 3$ is complementary to $\angle 1$

Step 4: Prove: $\angle 3 \cong \angle 2$

Step 5: $m\angle ACB = 90^\circ$, def. \perp
 $m\angle 3 + m\angle 1 = 90$, def comp \angle 's
 $m\angle 2 + m\angle 1 = m\angle ACB$, post. 4
 $m\angle 2 + m\angle 1 = 90$, substitution
 $\angle 2$ is complementary to $\angle 1$, def comp \angle 's
 $m\angle 3 + m\angle 1 = m\angle 2 + m\angle 1$, transitive
 $m\angle 3 = m\angle 2$, subtraction
 $\angle 3 \cong \angle 2$, Th. 2-8

Example of a Two Column Proof: #21 page 64

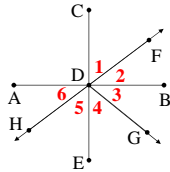
Step 5: $m\angle ACB = 90^\circ$, def \perp
 $m\angle 3 + m\angle 1 = 90$, def comp \angle 's
 $m\angle 2 + m\angle 1 = m\angle ACB$, post. 4
 $m\angle 2 + m\angle 1 = 90$, substitution
 $\angle 2$ is complementary to $\angle 1$, def comp \angle 's
 $m\angle 3 + m\angle 1 = m\angle 2 + m\angle 1$, transitive
 $m\angle 3 = m\angle 2$, subtraction
 $\angle 3 \cong \angle 2$, Th. 2-8

Statements	Reasons
1. $AC \perp BC$, $\angle 3$ is complementary to $\angle 1$	1. Given
2. $m\angle ACB = 90^\circ$	2. def \perp
3. $m\angle 2 + m\angle 1 = m\angle ACB$	3. post. 4
4. $m\angle 2 + m\angle 1 = 90$	4. substitution
5. $\angle 2$ is complementary to $\angle 1$	5. def comp \angle 's
6. $\angle 3 \cong \angle 2$	6. Th. 2-8

Sample Problems Section 2-6

Write the name or statement of the definition, postulate, property, or theorem that justifies the statement about the diagram.

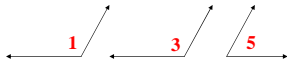
- $AD + DB = AB$
- $\angle 2 \cong \angle 6$
- If DF bisects $\angle CDB$, then $\angle 1 \cong \angle 2$.
- If $CD \perp AB$, then $m\angle CDB = 90$.
- If $m\angle 3 + m\angle 4 = 90$, then $\angle 3$ & $\angle 4$ are complementary.
- If $AB \perp CE$, then $\angle ADC \cong \angle ADE$.
- If $\angle FDG$ is a right angle, then $DF \perp DG$.



Sample Problems Section 2-6

15. Given: $\angle 1$ & $\angle 5$ are supplementary
 $\angle 3$ & $\angle 5$ are supplementary

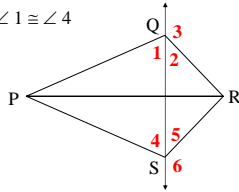
Prove: $\angle 1 \cong \angle 3$



- | | |
|---|----|
| 1. $\angle 1$ & $\angle 5$ are supplementary | 1. |
| $\angle 3$ & $\angle 5$ are supplementary | |
| 2. $m\angle 1 + m\angle 5 = 180$ | 2. |
| $m\angle 3 + m\angle 5 = 180$ | |
| 3. $m\angle 1 + m\angle 5 = m\angle 3 + m\angle 5$ | 3. |
| 4. $m\angle 5 = m\angle 5$ | 4. |
| 5. $m\angle 1 = m\angle 3$ or $\angle 1 \cong \angle 3$ | 5. |

Sample Problems Section 2-6

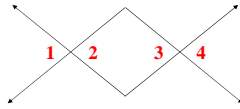
17. Given: $PQ \perp QR$, $PS \perp SR$, $\angle 1 \cong \angle 4$
 Prove: $\angle 2 \cong \angle 5$



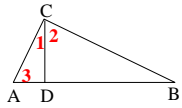
- | | |
|--|----|
| 1. $PQ \perp QR$, $PS \perp SR$, $\angle 1 \cong \angle 4$ | 1. |
| 2. $\angle 2$ is comp. to $\angle 1$ | 2. |
| $\angle 5$ is comp. to $\angle 4$ | |
| 3. $\angle 2 \cong \angle 5$ | 3. |

Sample Problems Section 2-6

19. Given: $\angle 2 \cong \angle 3$
 Prove: $\angle 1 \cong \angle 4$



21. Given: $AC \perp BC$
 $\angle 3$ is comp. to $\angle 1$
 Prove: $\angle 3 \cong \angle 2$



Chapter Two

Deductive Reasoning
 Review
 Homework Pages 68-69:
 2-12 evens
