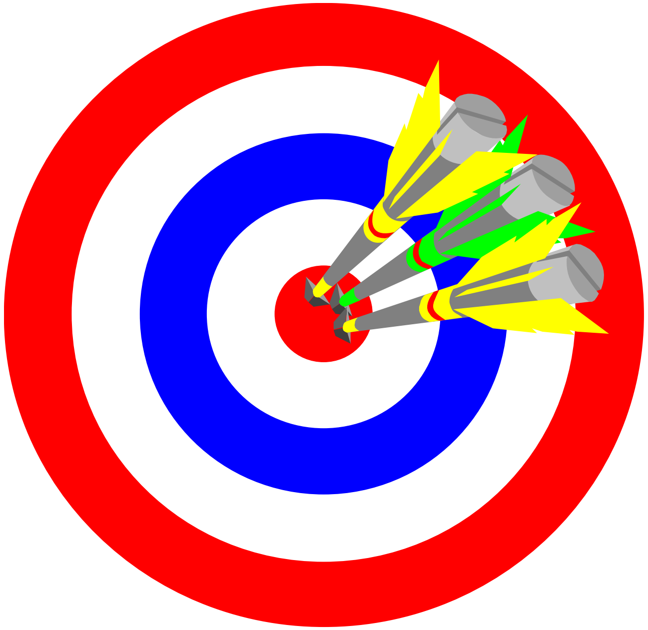
Chapter Three

Parallel Lines and Planes

Objectives



A. Use the terms defined in the chapter correctly.
B. Properly use and interpret the symbols for the terms and concepts in this chapter.
C. Appropriately apply the postulates and theorems in this chapter.
D. Identify the special pairs of angles created by lines and transversals.
E. Classify polygons according to sides and angles.
F. Find the measures of interior and exterior angles.
G. Compare/contrast inductive & deductive reasoning.
H. Use inductive reasoning to reach a conclusion.

HOMEWORK NOTE!

In homework problems, all proofs MUST be done as two-column proofs. Although various homework problems will ask you to write a 'paragraph proof', you are to write all proofs as two-column proofs.

Section 3-1

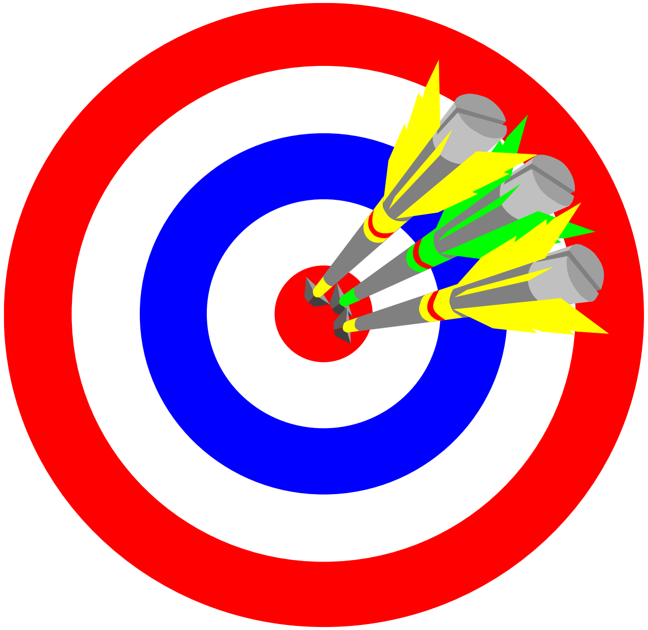
Definitions

Homework Pages 76-77:

2-38 evens

Excluding 18, 20, 22

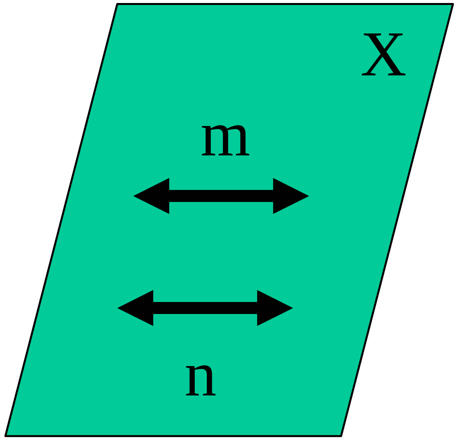
Objectives



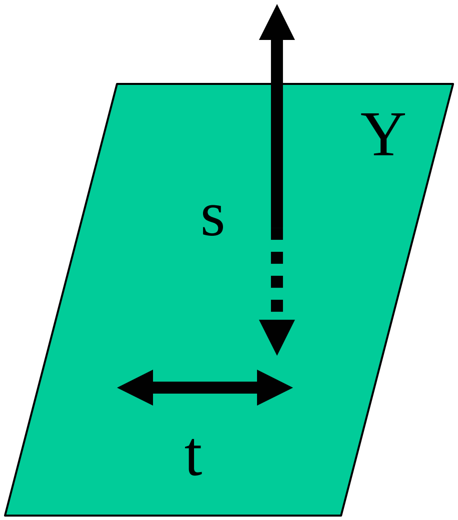
A. Use the terms parallel lines, skew lines, and parallel planes correctly.
B. Distinguish between parallel and skew lines.
C. Use the term transversal correctly.
D. Use the terms alternate interior angles, same-side interior angles, and corresponding angles correctly.
E. Distinguish between alternate interior, same-side interior, and corresponding angles.
F. Apply the terms in theorems and proofs.

Concepts of a 'parallel universe' …

- Parallel lines → *coplanar* lines that do not intersect.
  - The symbol for parallel lines is ||
- Skew lines → *non-coplanar* lines that do not intersect.
- Parallel planes → planes that do not intersect.
- A line is parallel to a plane if they do not intersect.



m||n

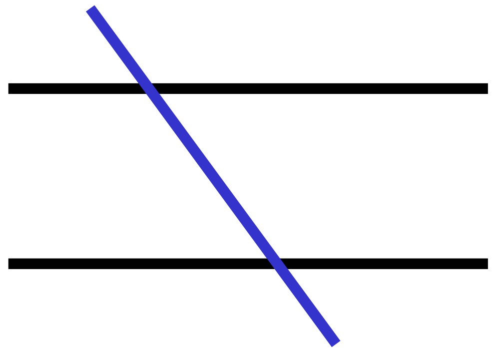


s is skew to t

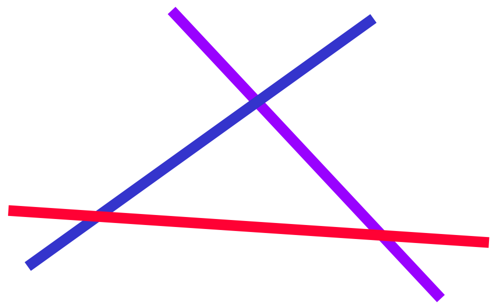
‘Transversing’ the road of geometry …

- Transversal → A line that intersects two or more coplanar lines at ***different*** points.

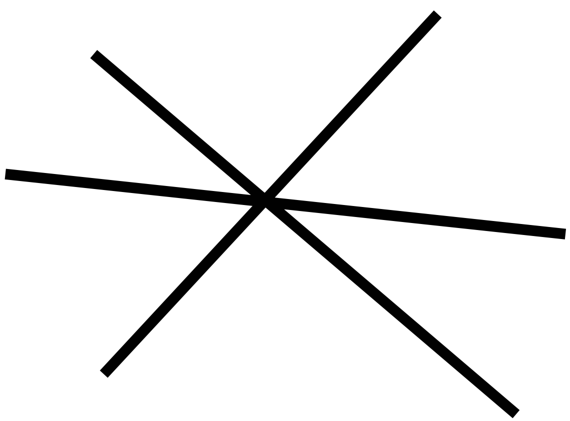
Transversals



Blue line is a transversal.



Blue is a transversal for purple and red
Purple is transversal for blue and red
Red is transversal for blue and purple



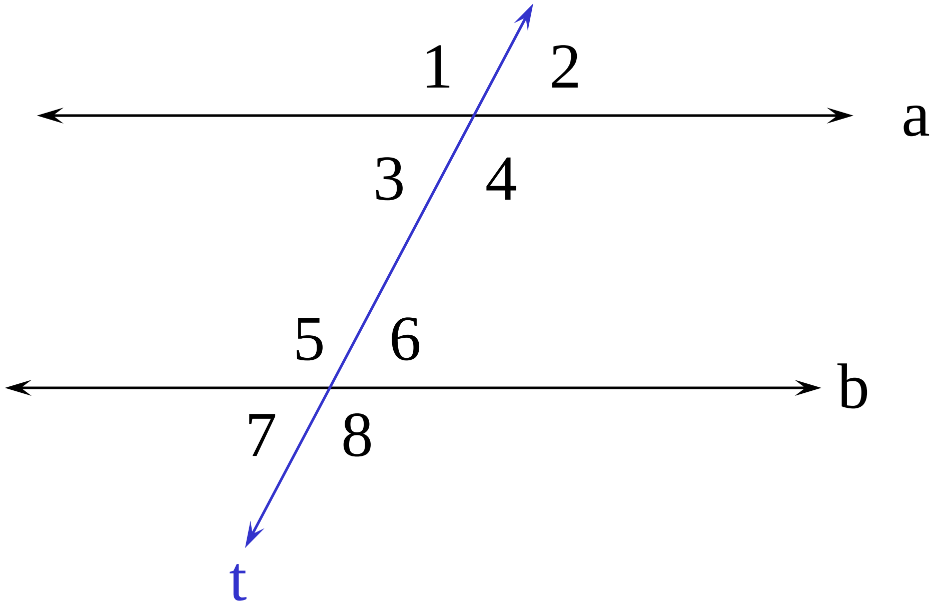
None of the lines are transversals.

Angular Relationships

- ★ Alternate interior angles → Two angles that are between a pair of lines (interior) and on opposite sides of a transversal (alternate).
- ★ Corresponding angles → Two angles that are in the same position relative to a line and a transversal eg. above-left, above-right, below-left, below-right.
- Same side interior angles → two angles that are between a pair of lines and on the same side of a transversal.
- So what do you think would be the definition of alternate exterior angles?

★Transversals & Special Pairs of Angles

Given lines a and b and transversal t:



Alternate Interior Angles:

$\angle 3$ & $\angle 6$

$\angle 4$ & $\angle 5$

Corresponding Angles:

$\angle 1$ & $\angle 5$

$\angle 2$ & $\angle 6$

$\angle 3$ & $\angle 7$

$\angle 4$ & $\angle 8$

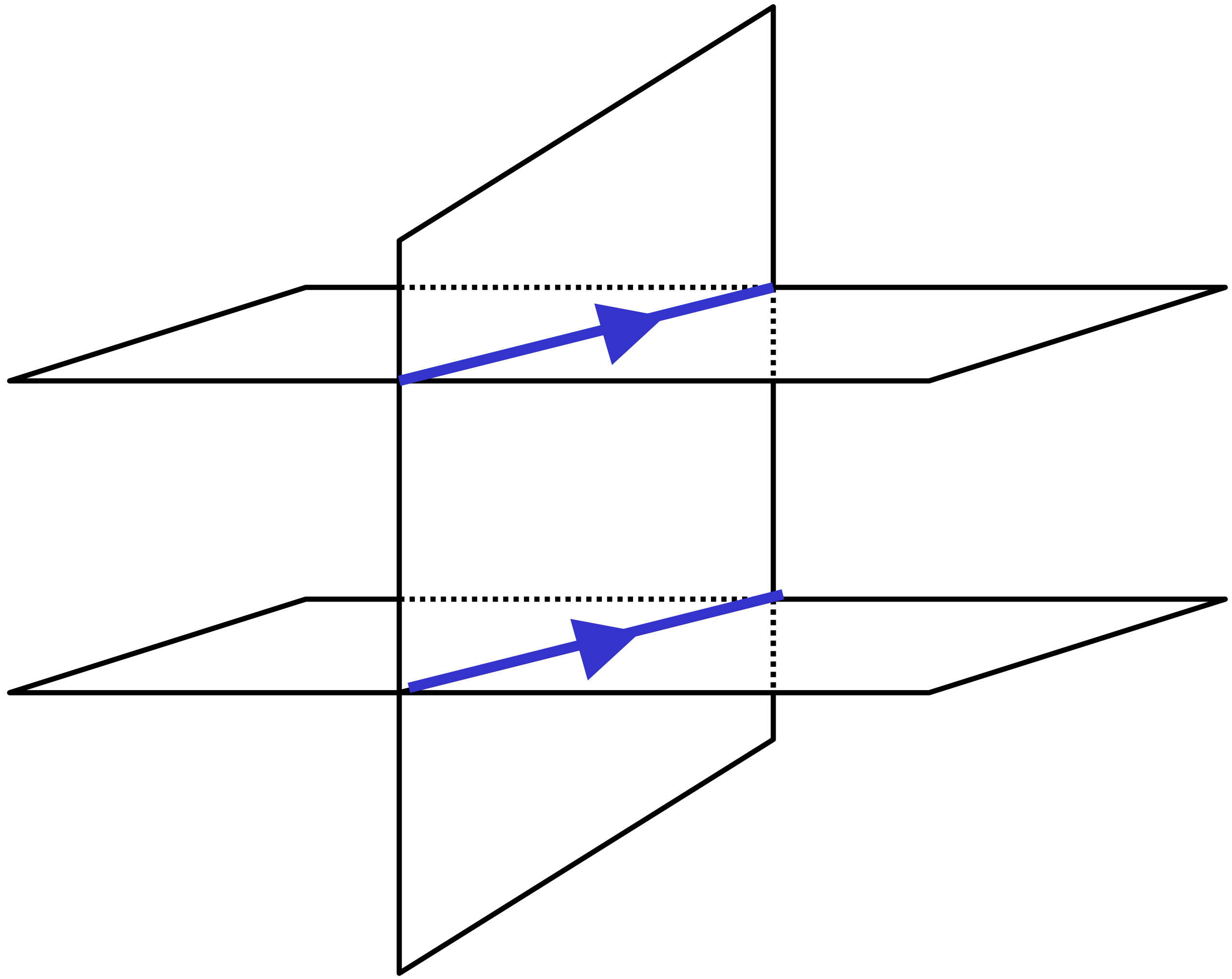
Same Side Interior Angles:

$\angle 4$ & $\angle 6$

$\angle 3$ & $\angle 5$

Theorem 3-1

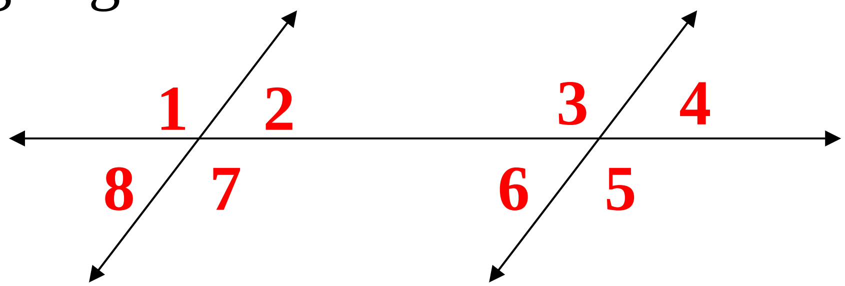
If two parallel planes are cut by a third plane, then the lines of intersection are parallel.



Sample Problems Section 3-1

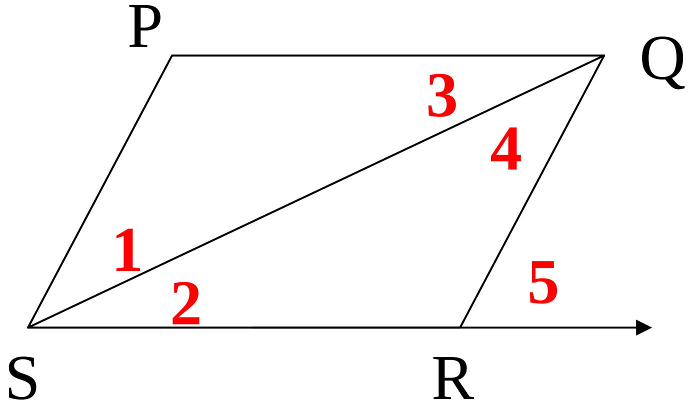
Classify each pair of angles as alternate interior, same-side interior, or corresponding angles.

1. ∠ 2 & ∠ 6
3. ∠ 2 & ∠ 3
5. ∠ 5 & ∠ 7



Name the two lines and the transversal that form each pair of angles.

7. ∠ 2 & ∠ 3
9. ∠ P & ∠ PSR
11. ∠ 5 & ∠ PQR



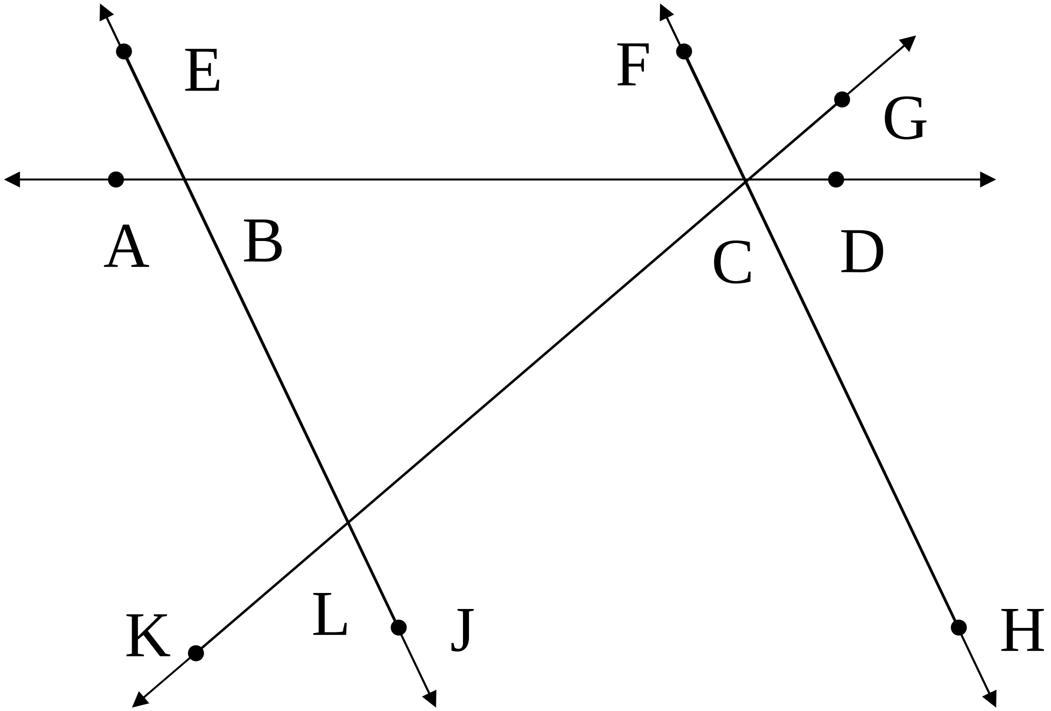
Sample Problems Section 3-1

Classify each pair of angles as alternate interior, same-side interior, or corresponding angles.

13. ∠ DCH & ∠ CBJ

15. ∠ FCL & ∠ BLC

17. ∠ GCH & ∠ GLJ



Try This …

On graph paper, carefully draw a LARGE figure with the following characteristics:

- Two parallel lines approximately 2 inches apart
- A slanted transversal through the approximate center of the page

Measure and label each angle.

What do you find when you:

Compare the alternate interior angles?

Compare the corresponding angles?

Compare the same-side interior angles?

Sample Problems Section 3-1

19. Measure a pair of alternate interior angles drawn on a piece of lined notebook paper. What appears to be true?
21. Draw a large diagram showing three transversals intersecting two nonparallel lines l and n. Number three pairs of same-side interior angles on the same side of the transversals.

a. Find $m\angle 1 + m\angle 2$.

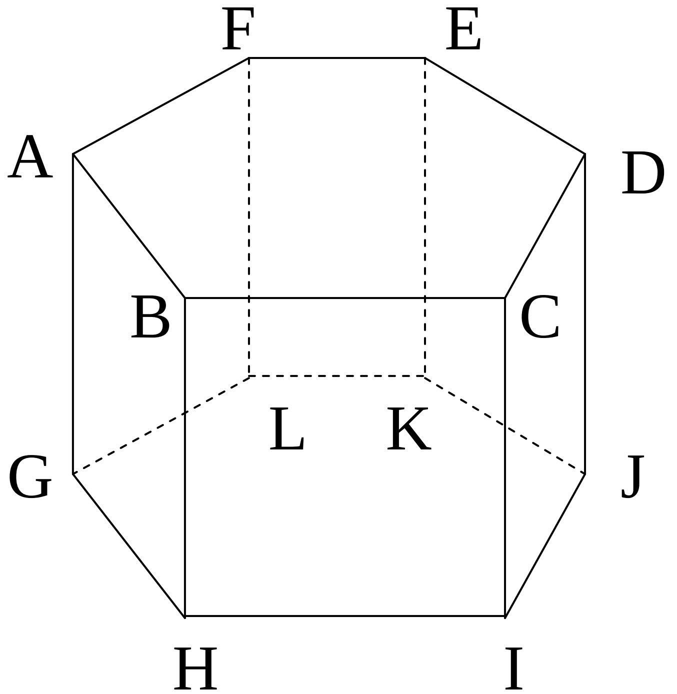
b. Find $m\angle 3 + m\angle 4$.

c. Predict the value of $m\angle 5 + m\angle 6$.

d. What do you conclude?

Sample Problems Section 3-1

23. Name five lines that appear to be parallel to AG.
25. Name four lines that appear to be skew to AB.
27. Name four planes parallel to FL.



Sample Problems Section 3-1

Complete each statement with always, sometimes or never.

31. Three lines intersecting in one point are ______ coplanar.

33. Two lines parallel to a third line are ______ parallel to each other.

35. Two lines perpendicular to a third line are ______ perpendicular to each other.

37. Two planes parallel to the same plane are ______ parallel to each other.

39. Two lines parallel to the same plane are ______ parallel to each other.

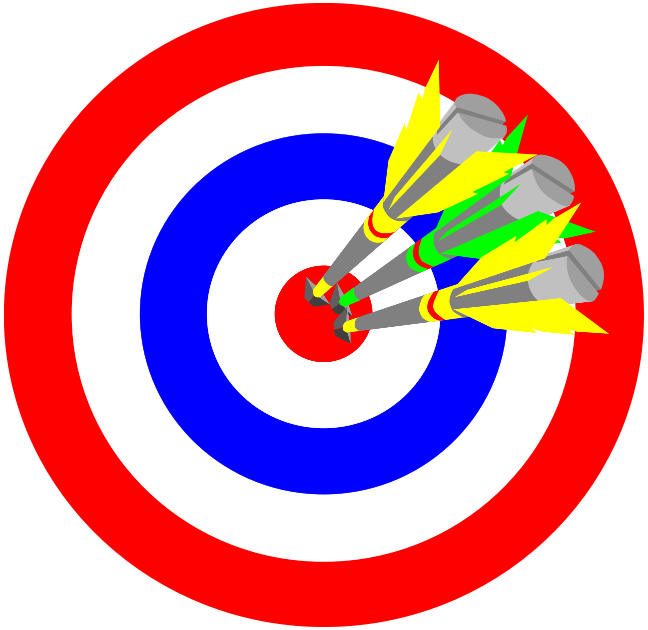
Section 3-2

Properties of Parallel Lines

Homework Pages 80-82:

2-22 evens

Objectives



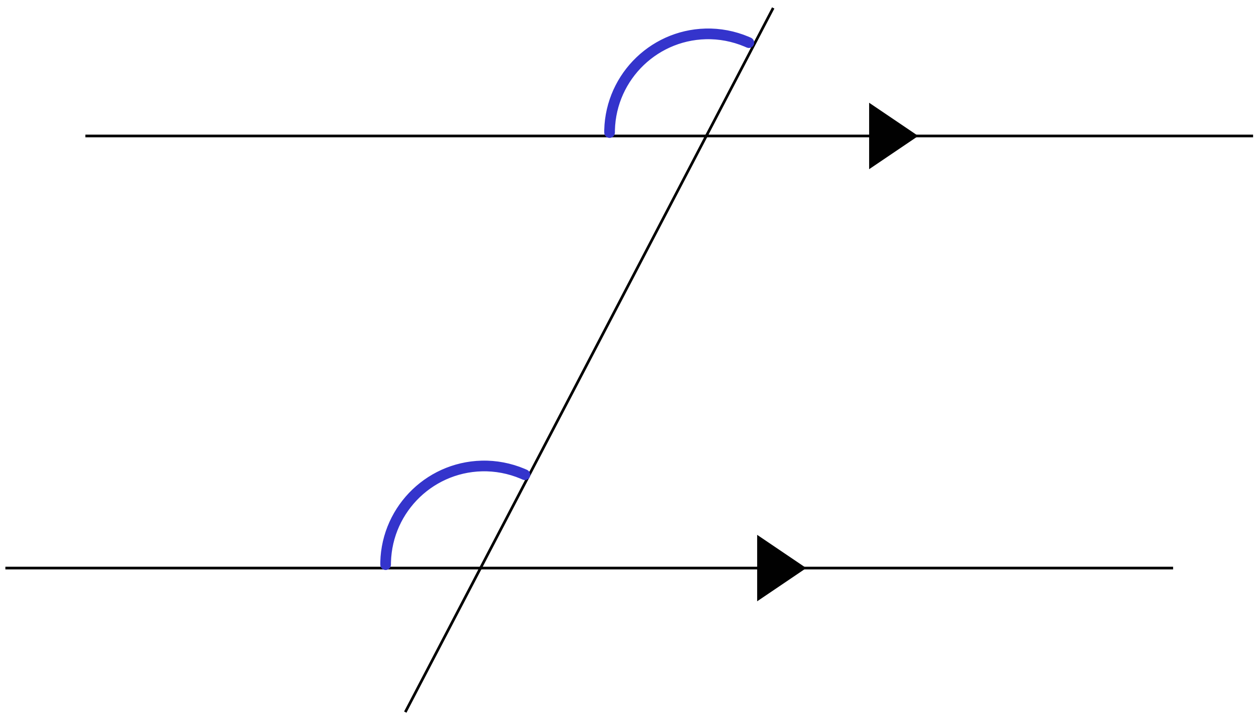
A. Use the postulate relating to parallel lines and transversals correctly.
B. Apply the postulate to prove theorems relating to parallel lines.
C. Use the parallel lines theorems correctly.

Remember the results of your experiment?

- When two parallel lines were cut by a transversal, what angles did you find to be congruent? Supplementary?
- Looking at the definitions, postulates, and theorems you have so far, do you believe you have sufficient information to perform a deductive proof of your findings?

★ Postulate 10

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

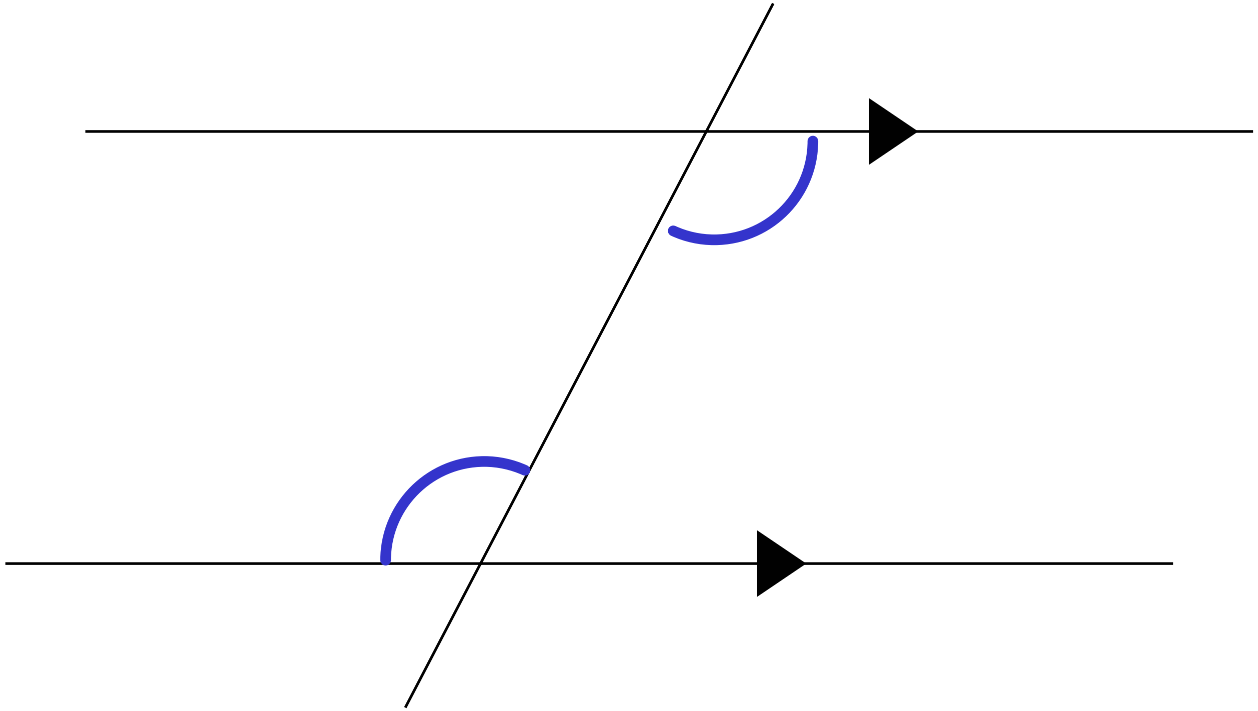


A New ‘Symbol’ for Diagrams

- Note in the previous slide the appearance of an arrowhead in the middle of a line segment.
- These are used to indicate parallel lines.
- A line with a single internal arrowhead is parallel to another line with a single internal arrowhead.

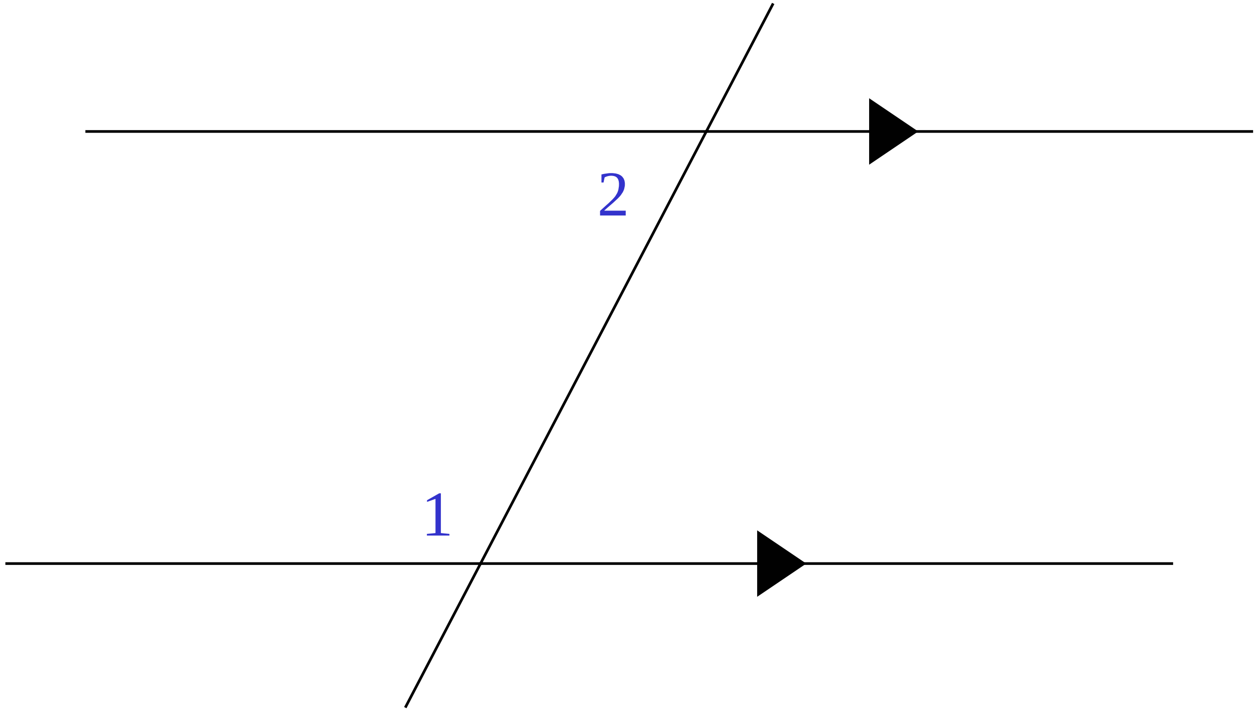
★ Theorem 3-2

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.



★ Theorem 3-3

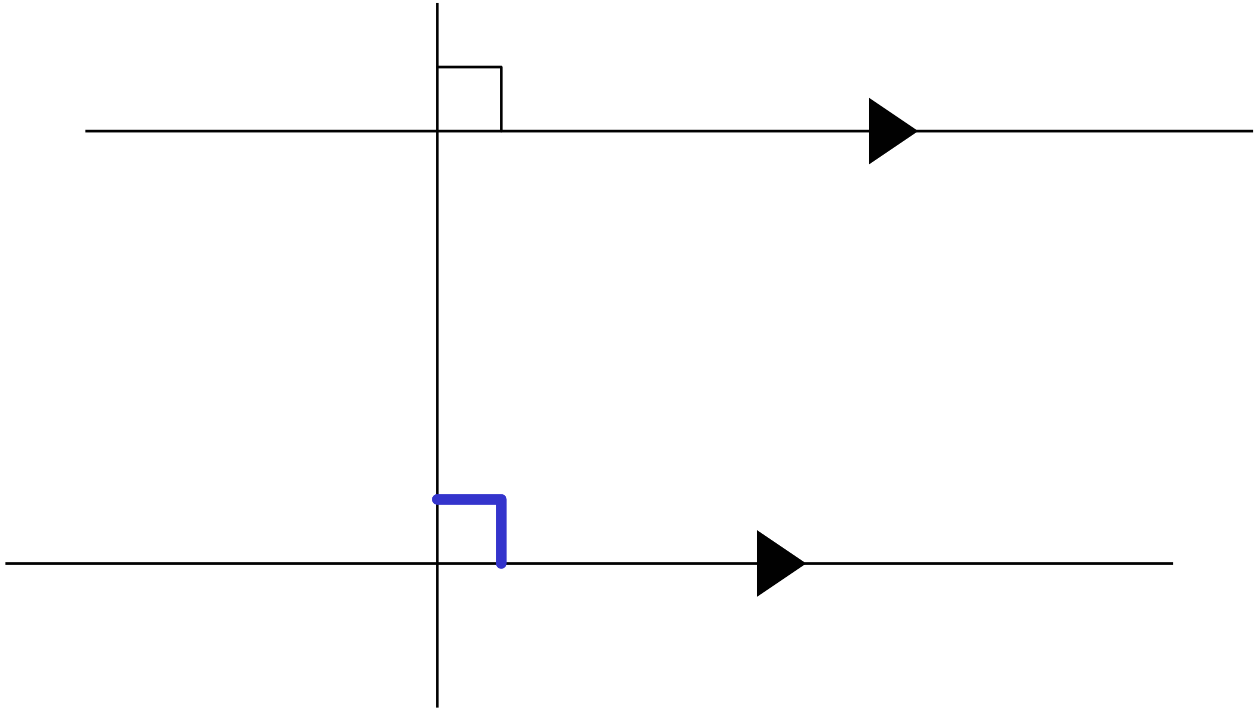
If two parallel lines are cut by a transversal, then same side interior angles are supplementary.



$m\angle 1 + m\angle 2 = 180$

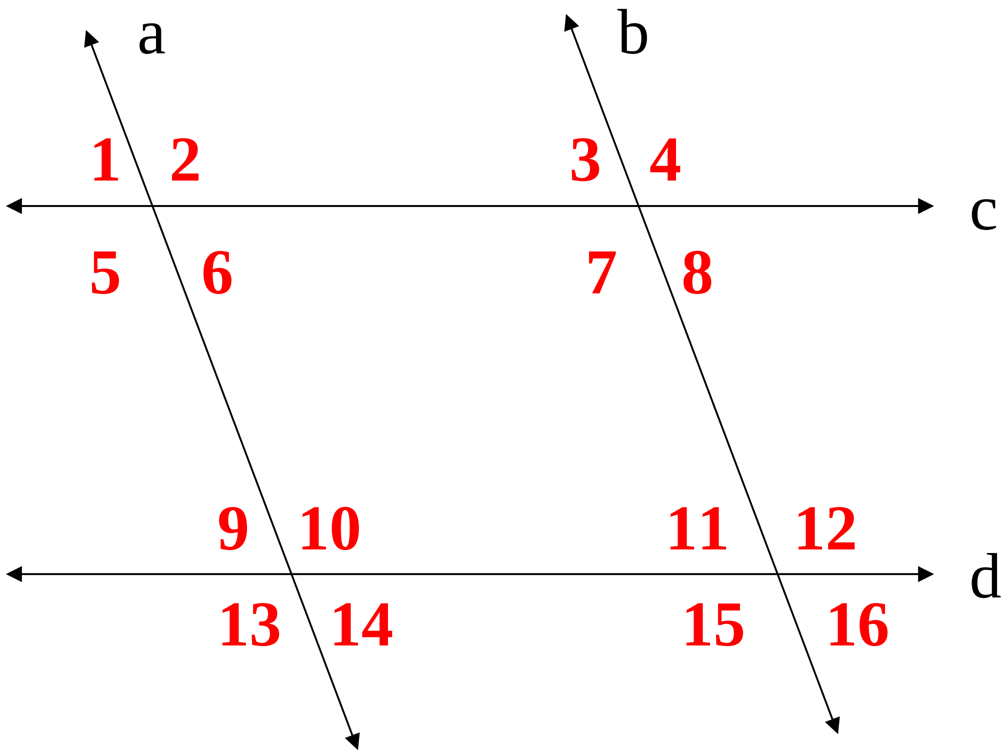
Theorem 3-4

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one also.



Sample Problems Section 3-2

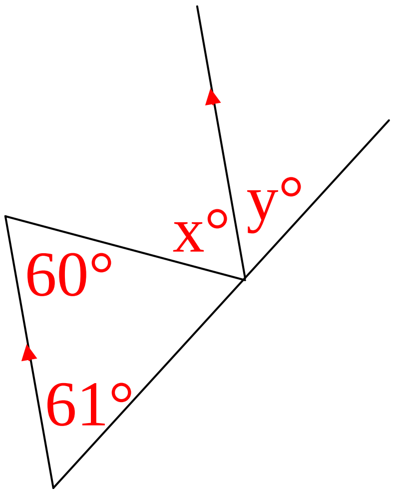
1. If a | | b, name all of the angles that must be congruent to ∠ 1. Assume that a | | b and c | | d.
3. Name all the angles congruent to ∠ 2.
5. If m ∠ 13 = 110, then m ∠ 15 = ? and m ∠ 3 = ?



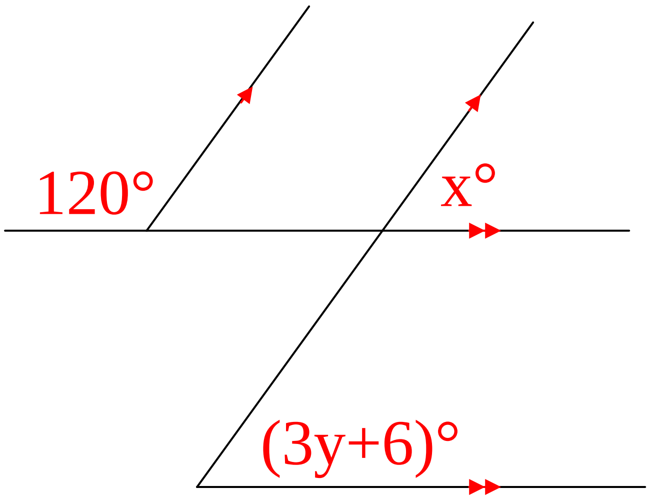
Sample Problems Section 3-2

Find the values of x and y.

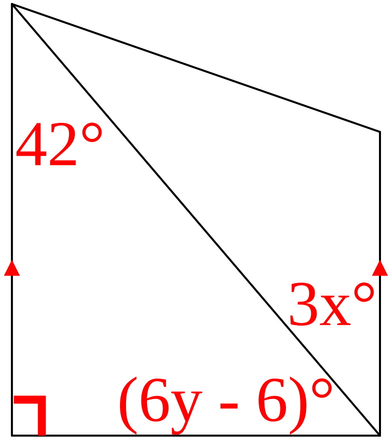
7.



9.



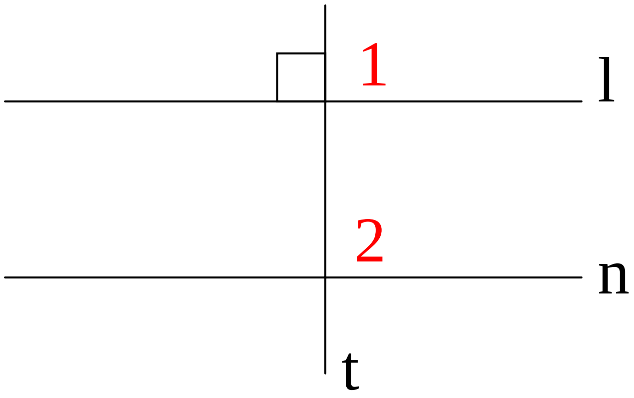
11.



Sample Problems Section 3-2

13. Given: $t \perp l$; $l \parallel n$

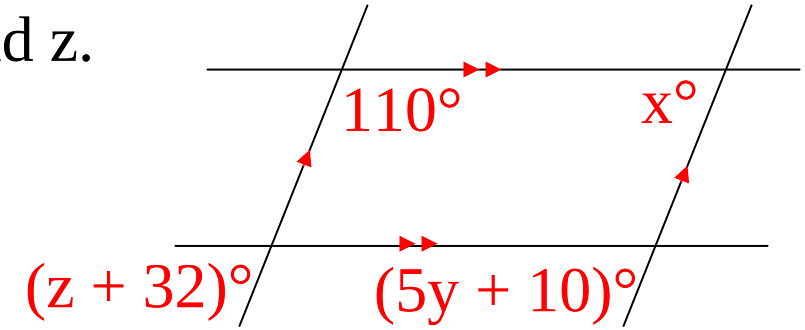
Prove: $t \perp n$



1. $t \perp l$	1. ?
2. $m\angle 1 = 90$	2. ?
3. ?	3. Given
4. $m\angle 2 = m\angle 1$	4. ?
5. ?	5. Substitution
6. $t \perp n$	6. ?

Sample Problems Section 3-2

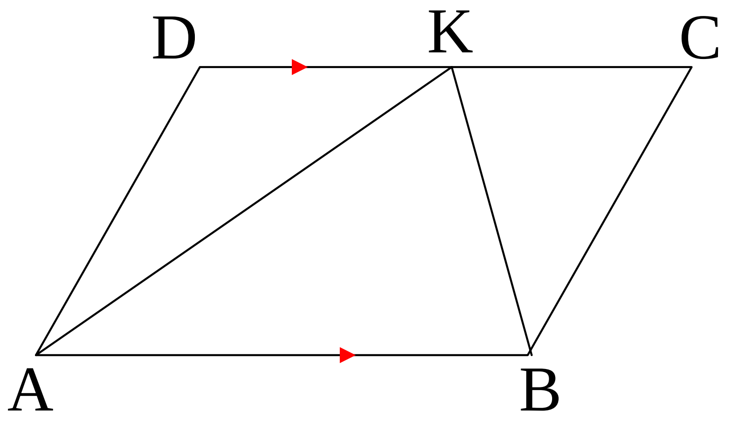
15. Find the values of x, y and z.



17. Given: $m\angle D = 116$; $\overrightarrow{AK}$ bisects $\angle DAB$; $\overline{AB} \parallel \overline{CD}$

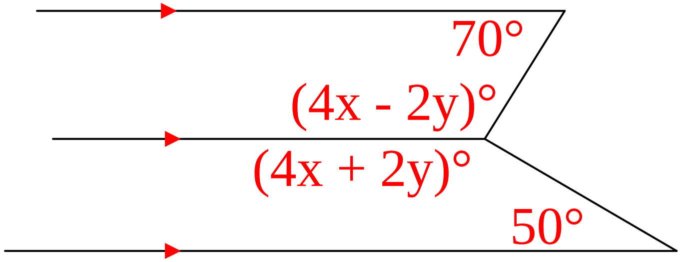
a. Find measures of $\angle DAB$, $\angle KAB$, and $\angle DKA$.

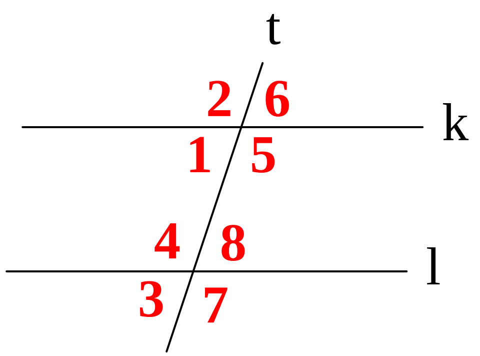
b. Is there enough information for you to conclude that $\angle D$ and $\angle C$ are supplementary, or is more information needed?



Sample Problems Section 3-2

19. Find the values of x and y.





21. Given: k | | l

Prove: ∠ 1 is supplementary to ∠ 7

23. Draw a four-sided figure ABCD with $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$

a. Prove that ∠ A ≅ ∠ C

b. Is ∠ B ≅ ∠ D

Section 3-3

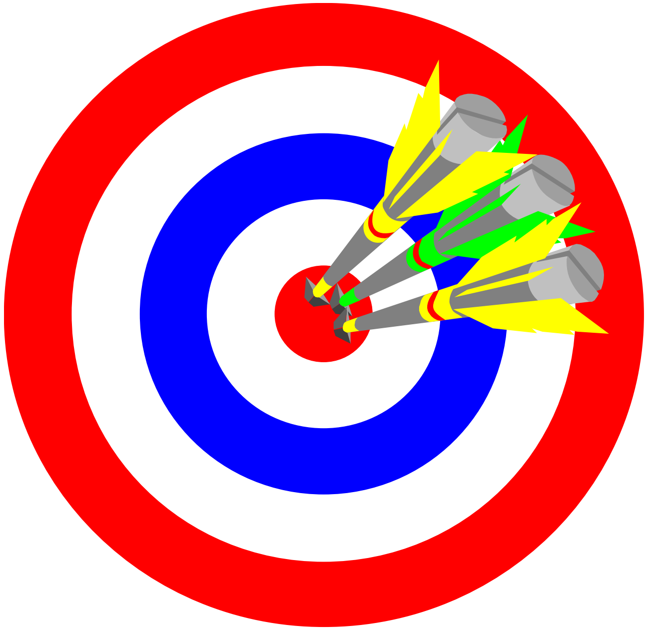
Proving Lines Parallel

Homework Pages 87-88:

2-28 evens

Excluding 16, 20, 26

Objectives



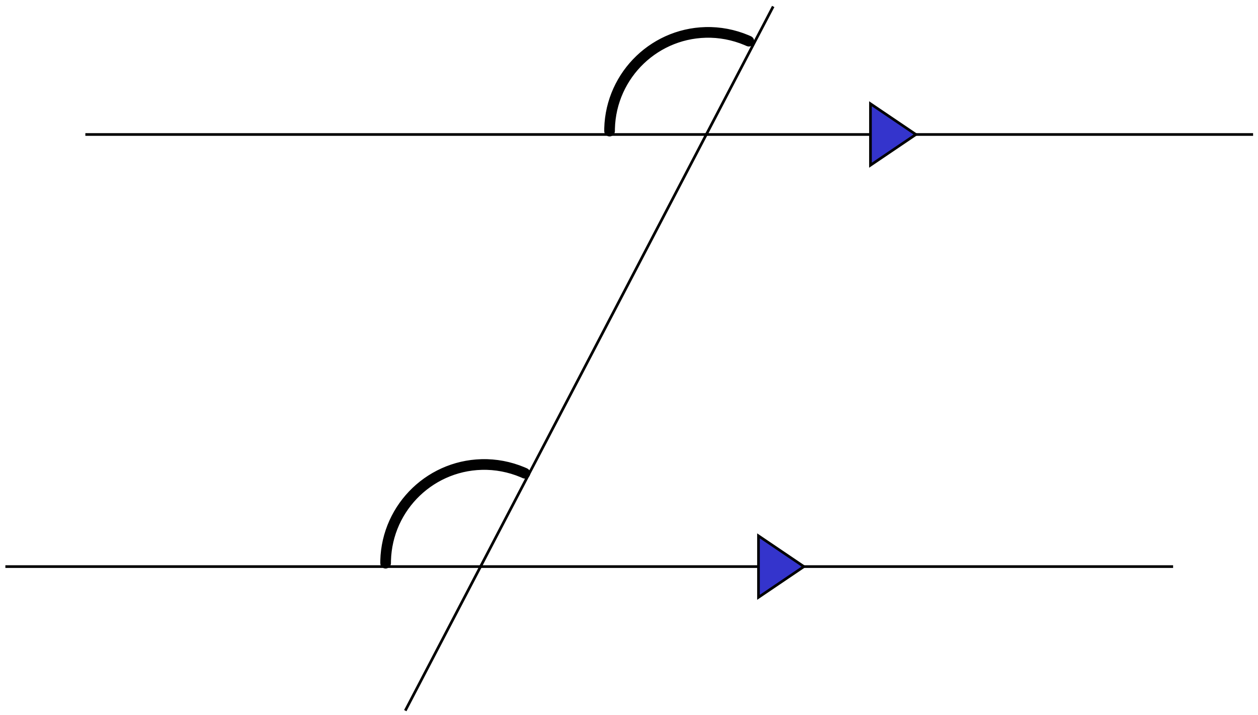
A. Use the postulate relating to determining if lines are parallel lines correctly.
B. Apply the postulate to prove lines are parallel.
C. Use the proof of parallel lines theorems correctly.
D. Use the theorem of the existence of a parallel line through a point correctly.
E. Use the theorem of the existence of a perpendicular line through a point correctly.

Are the Lines Parallel?

- Thus far we have used postulates and theorems that REQUIRE lines to be parallel.
  - The hypothesis statement in each of these postulates and theorems requires parallel lines to exist.
  - Based on the fact that parallel lines exist (and a transversal also exists) we can prove relationships between alternate interior, same side interior, and corresponding angles.
- But how do we prove that such parallel lines exist?

Postulate 11

If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

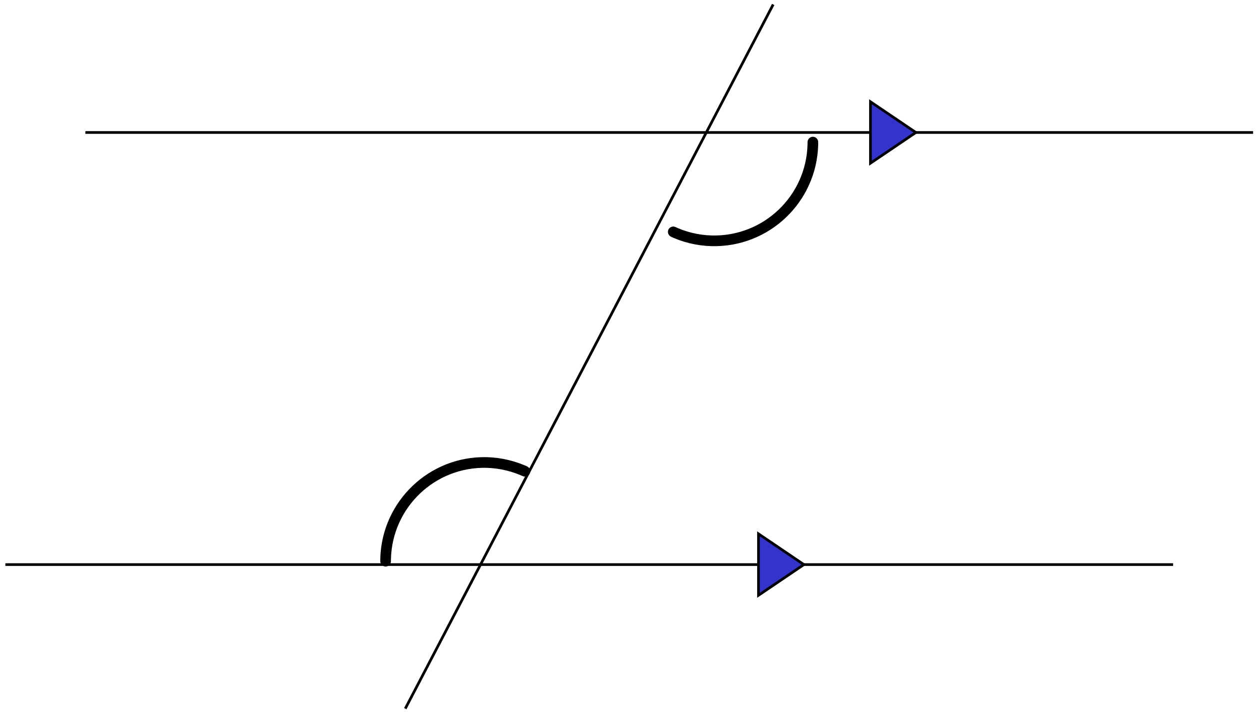


Postulate or Theorem?

- Note that both Postulate 10, which requires parallel lines and a transversal to conclude the relationship of CORRESPONDING angles, and Postulate 11, which requires the existence of relationship between CORRESPONDING angles to conclude that lines are parallel, both deal with CORRESPONDING angles!
- Theorems give us the relationships between parallel lines with a transversal, same side interior angles, and alternate interior angles.
- Further, what is the relationship between Postulate 10 and Postulate 11?

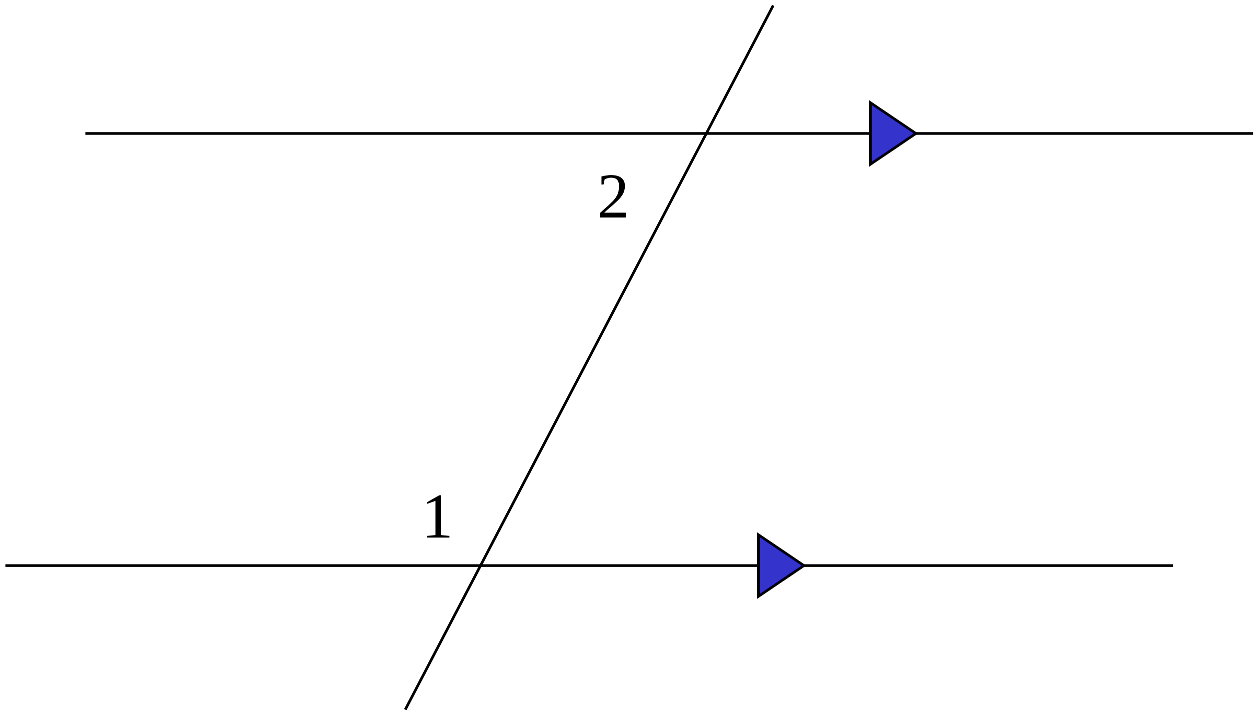
Theorem 3-5

If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.



Theorem 3-6

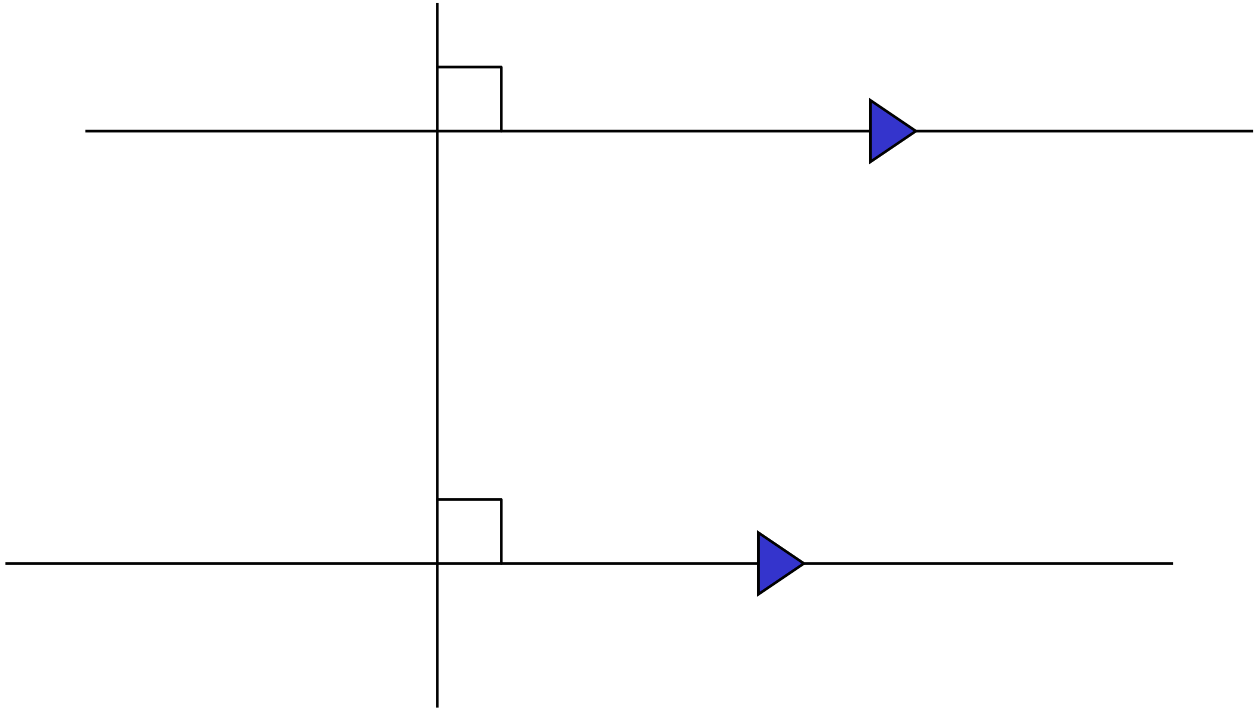
If two lines are cut by a transversal and same side interior angles are supplementary, then the lines are parallel.



$m\angle 1 + m\angle 2 = 180$

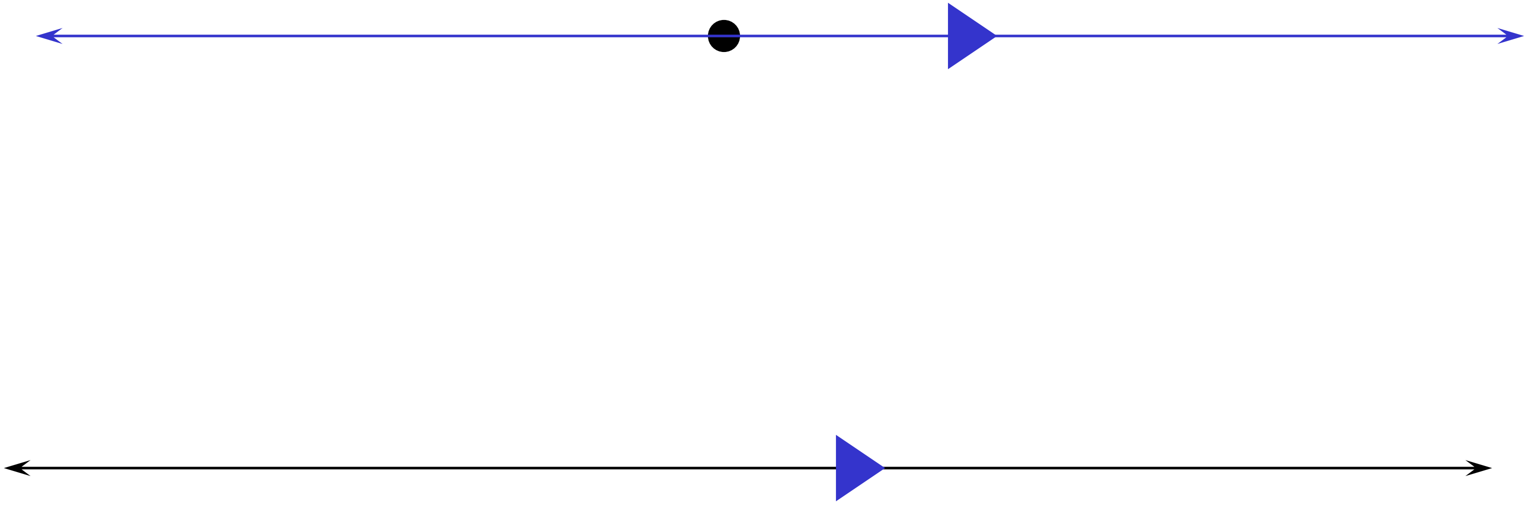
Theorem 3-7

In a plane if two lines are perpendicular to the same line, then those lines are parallel.



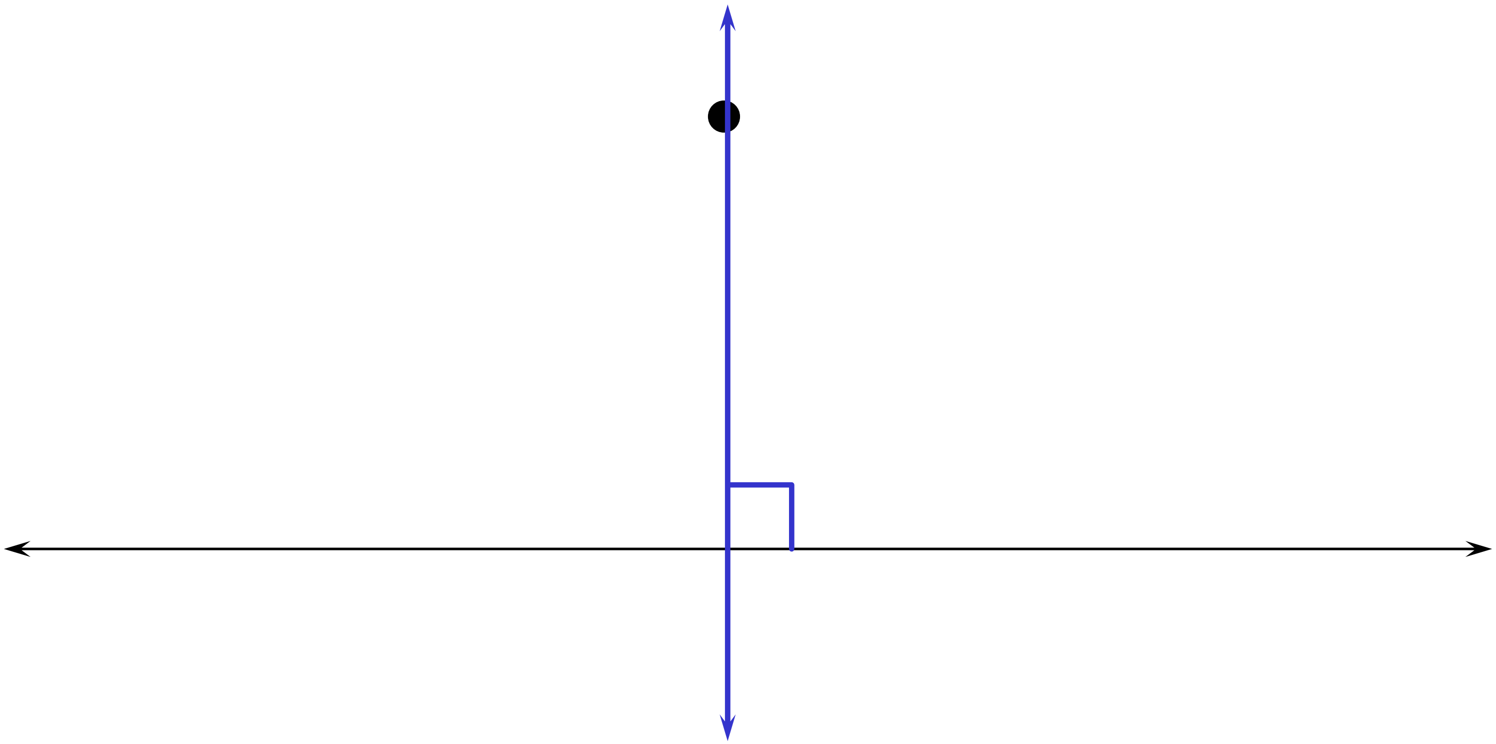
Theorem 3-8

Through a point outside a line, there is exactly one line parallel to the given line.



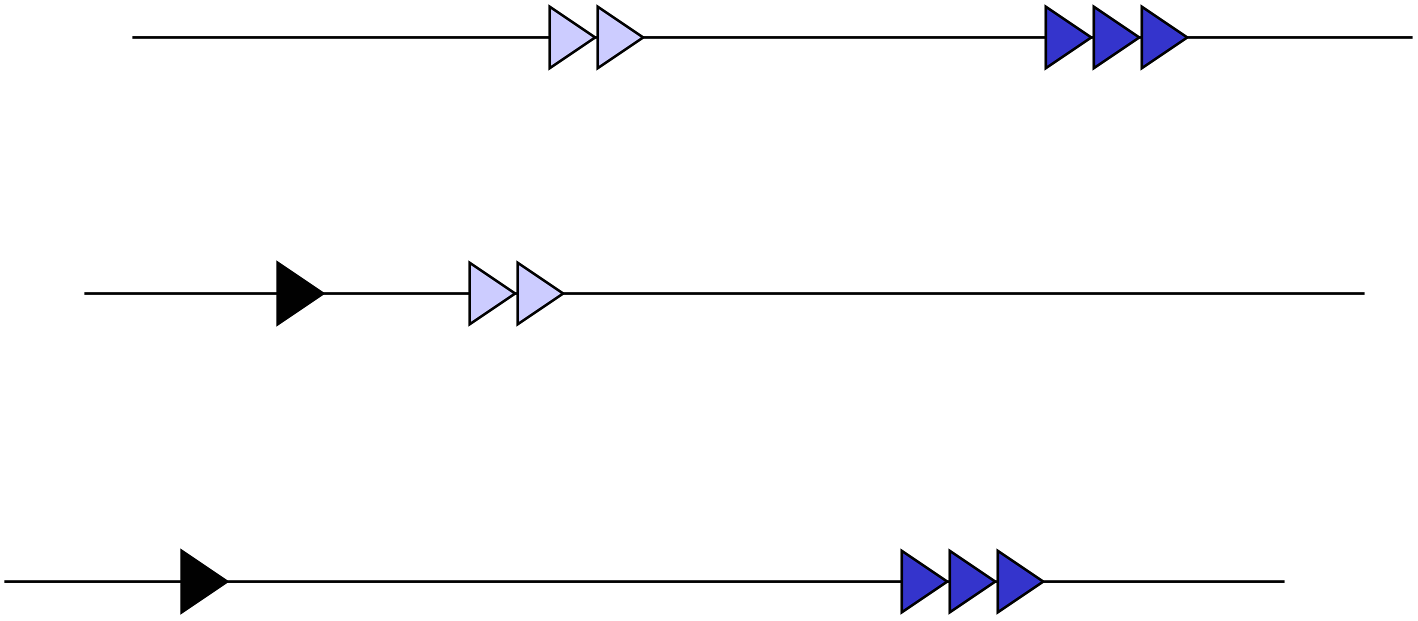
Theorem 3-9

Through a point outside a line, there is exactly one line perpendicular to the given line.



Theorem 3-10

If two lines are parallel to a third line, then those lines are parallel to each other.

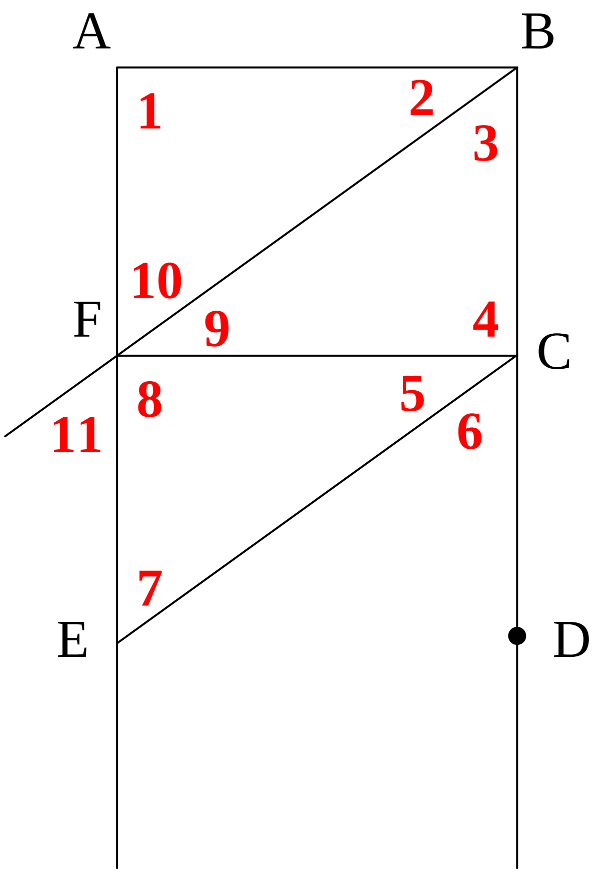


★ Methods to Prove Lines Parallel

- Show that a pair of corresponding angles are congruent (Postulate 11).
- Show that a pair of alternate interior angles are congruent (Theorem 3- 5).
- Show that a pair of same-side interior angles are supplementary (Theorem 3- 6).
- In a plane show that both lines are perpendicular to a third line (Theorem 3- 7).
- Show that both lines are parallel to a third line (Theorem 3- 10).
- Note that Postulate 11 and Theorems 3-5, 3-6, and 3-7 show the existence of parallel lines by relating angles.

Sample Problems Section 3-3

Use the information to decide which segments, if any, are parallel. If no segments are parallel, then write none.



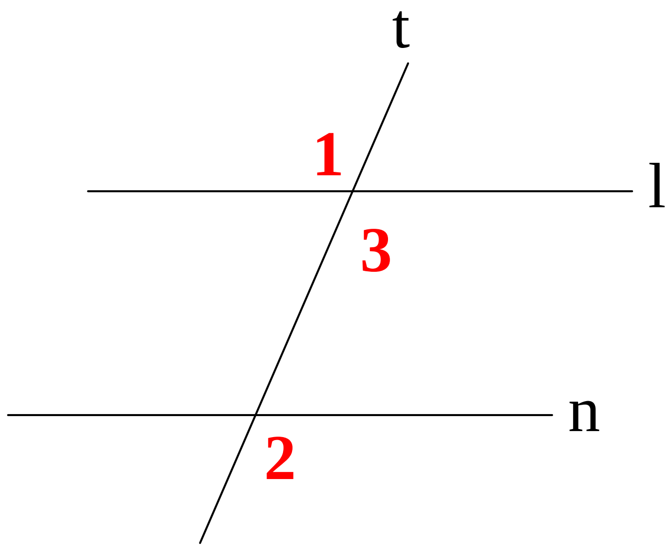
1. $\angle 2 \cong \angle 9$
3. $m\angle 1 = m\angle 8 = 90$
5. $m\angle 2 = m\angle 5$
7. $m\angle 1 = m\angle 4 = 90$
9. $m\angle 8 + m\angle 5 + m\angle 6 = 180$
11. $m\angle 5 + m\angle 6 = m\angle 9 + m\angle 10$
13. $\angle 2$ & $\angle 3$ are comp and $m\angle 1 = 90$
15. $m\angle 7 = m\angle 3 = m\angle 10$

Sample Problems Section 3-3

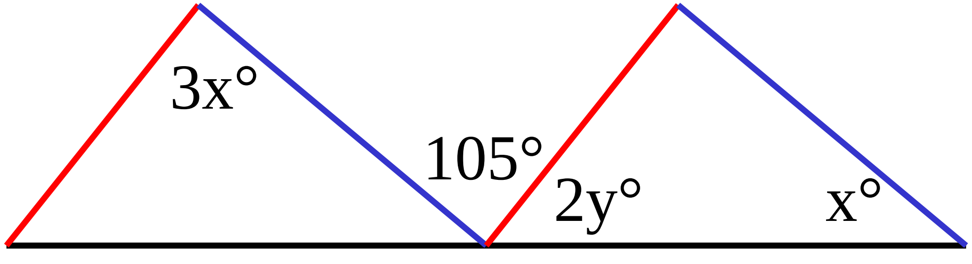
17. Given: $\angle 2 \cong \angle 1$

Prove: $l \parallel n$

1. $\angle 2 \cong \angle 1$	1. ?
2. $\angle 1 \cong \angle 3$	2. ?
3. $\angle 2 \cong \angle 3$	3. ?
4. $l \parallel n$	4. ?

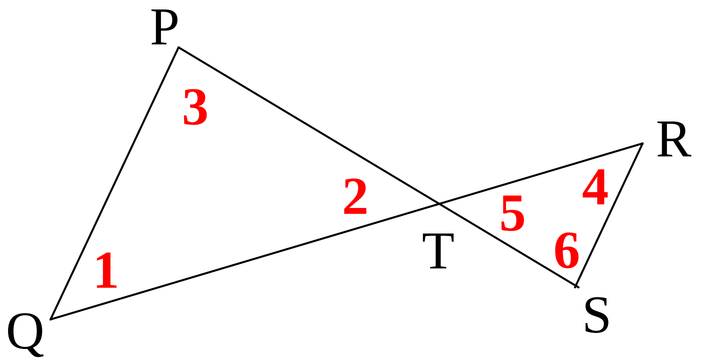


19. Find the values of x and y that make the red and blue lines parallel.



Sample Problems Section 3-3

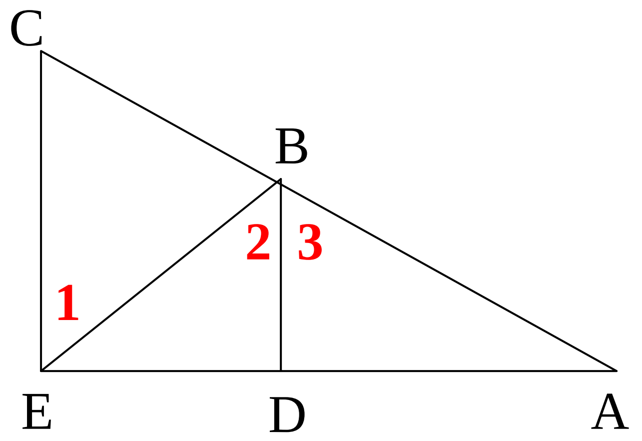
21. Given: $\angle 3 \cong \angle 6$. What can you prove about the other angles?



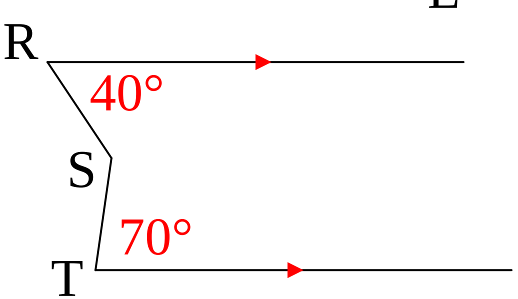
23. Copy what is shown for Theorem 3-7 on p 84. Then write a proof.

25. Given: BE DA; CD DA

Prove: $\angle 2 \cong \angle 1$



27. Find the measure of $\angle$ RST.



Section 3-4

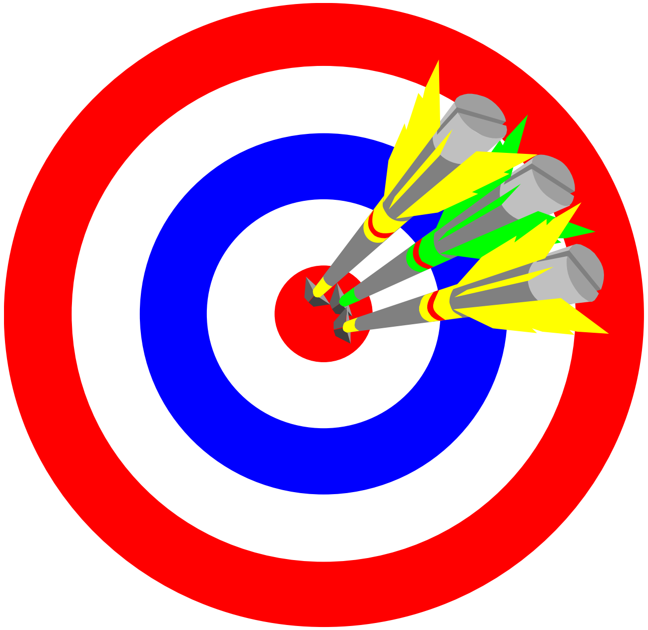
Angles of a Triangle

Homework Pages 97-99:

6-30 evens

Excluding 26

Objectives



A. Classify triangles according to sides and angles.

B. State and apply the theorems and corollaries about the sum of the measures of the angles of a triangle.

C. State and apply the theorems about the measure of an exterior angle of a triangle.

D. Use the definitions relating to triangles correctly.

The Birth of a Triangle

- triangle: is a figure formed by three segments joining three non-collinear points
- sides: the segments that make up a polygon
- vertex: the intersection of two consecutive sides of a polygon
- exterior angle: an angle formed by extending one side of a polygon beyond the vertex
- remote interior angle: two angles inside of a triangle that are not adjacent to a specific exterior angle

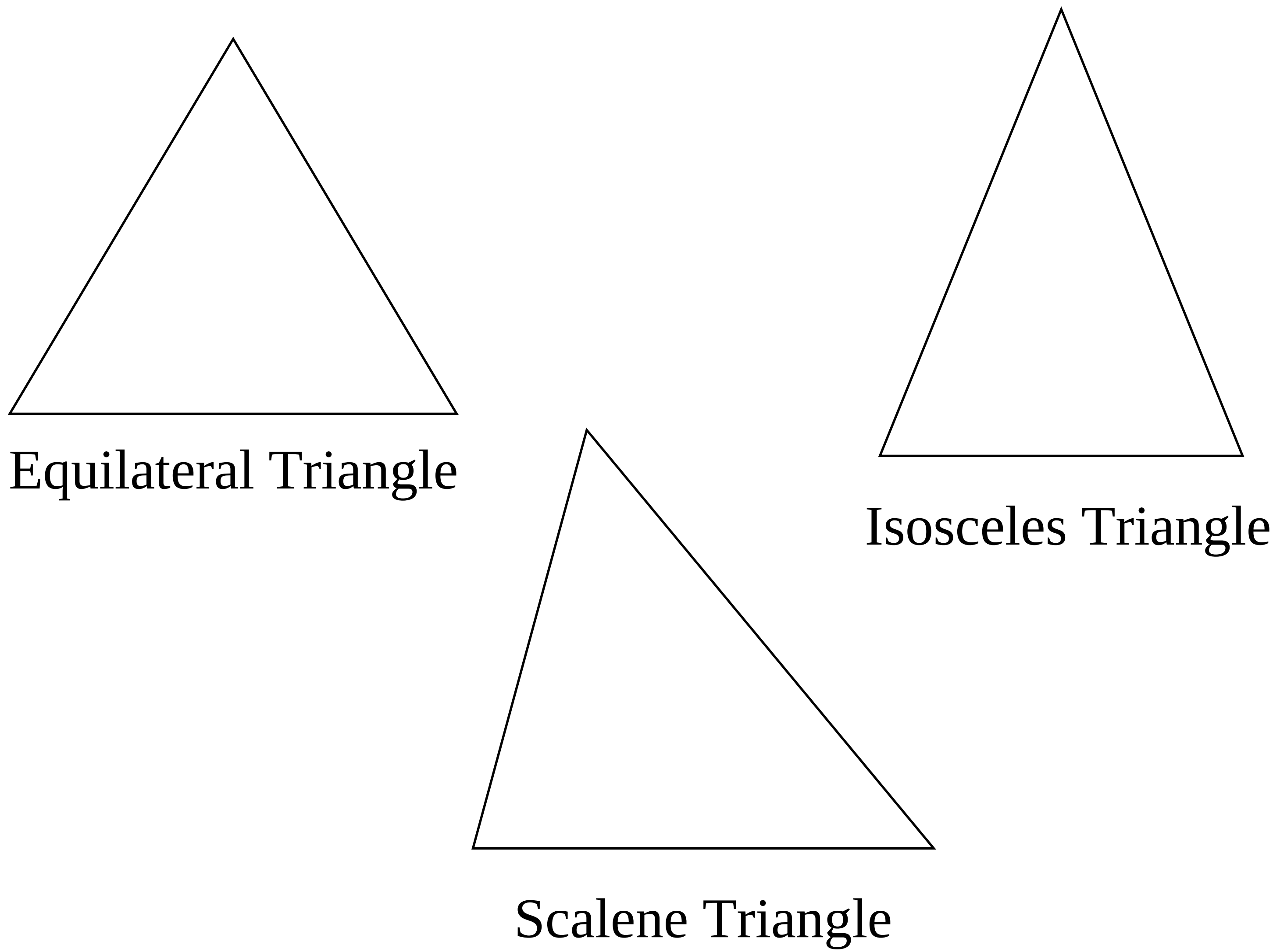
Types of Triangles

- acute triangle: a triangle with three acute angles
- equiangular triangle: a triangle in which all of the angles are congruent
- equilateral triangle: a triangle in which all of the sides are congruent
- isosceles triangle: a triangle in which at least two sides are congruent
- obtuse triangle: a triangle with one obtuse angle
- right triangle: a triangle with one right angle
- scalene triangle: a triangle in which no sides are congruent

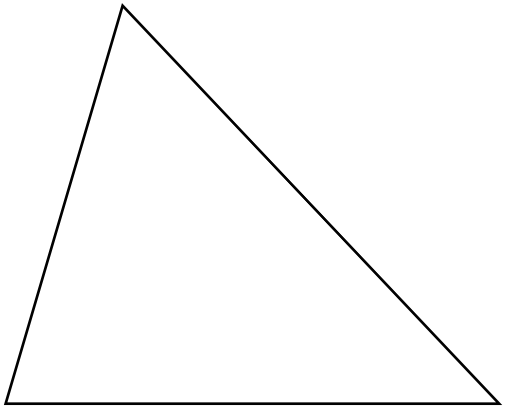
Other Definitions

- ★ auxiliary line: a line added to a diagram to help solve a problem or write a proof
- ★ corollary: a statement proven easily by applying a theorem; a baby theorem

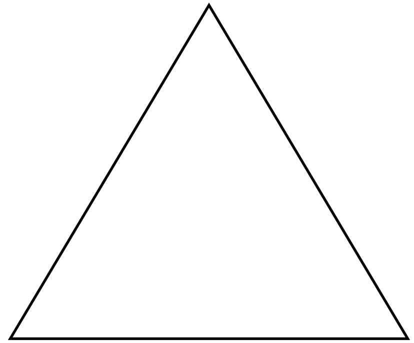
★ Triangles by Sides



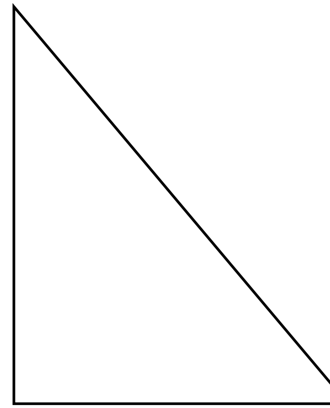
★ Triangles by Angles



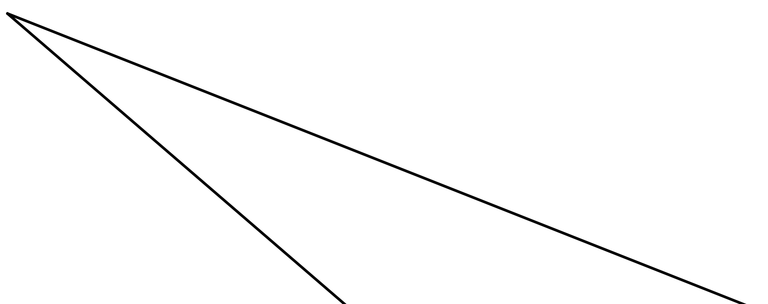
Acute Triangle



Equiangular Triangle

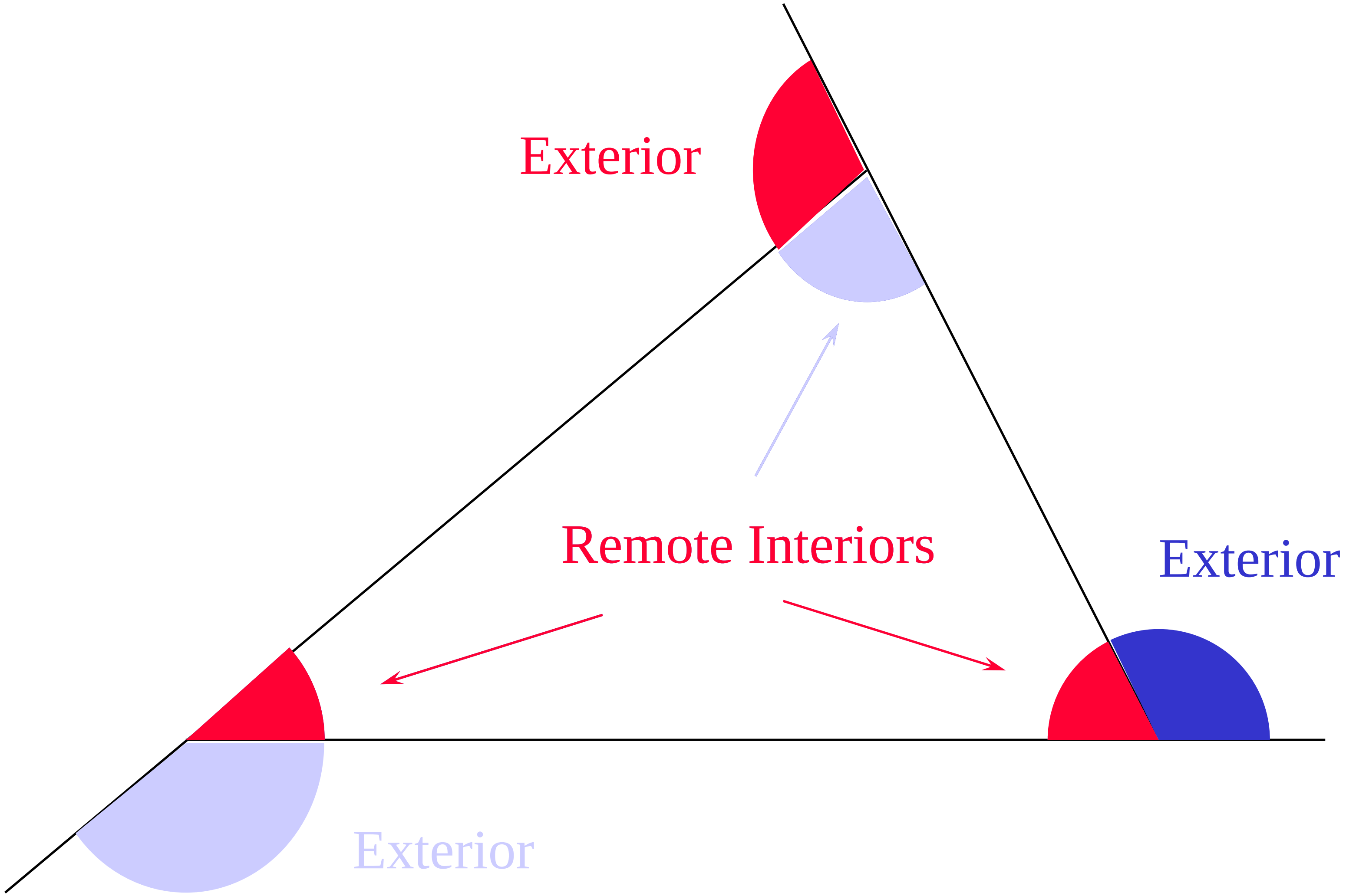


Right Triangle



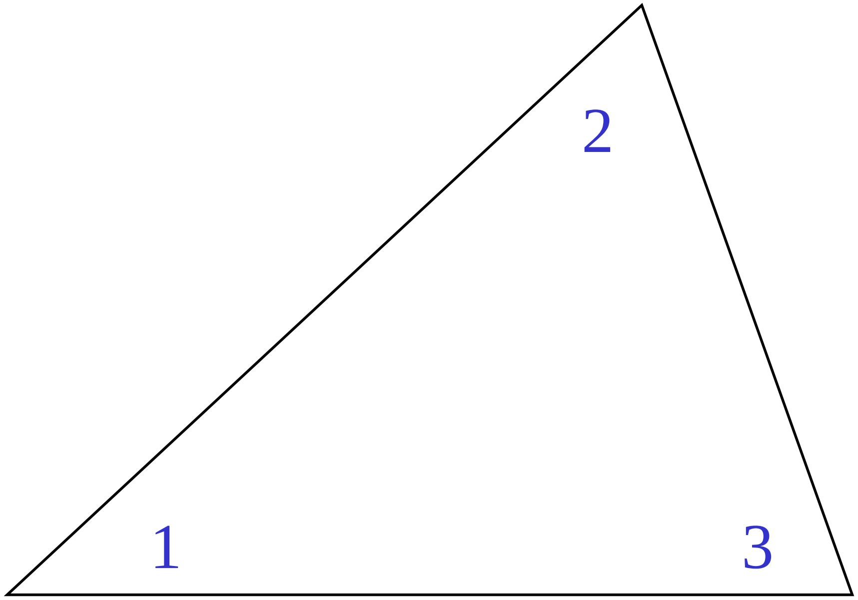
Obtuse Triangle

★ Exterior & Remote Interior Angles



★ Theorem 3-11

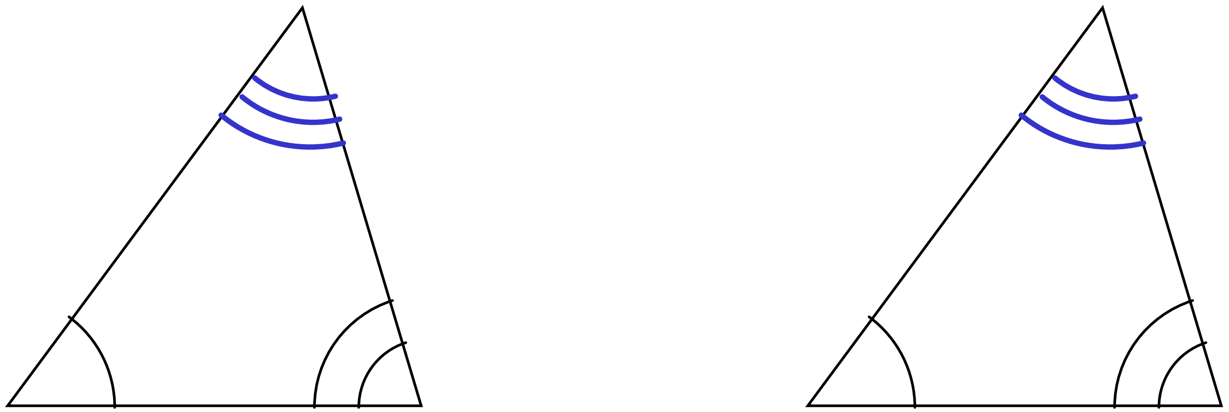
If a polygon is a triangle, then the sum of its angles is 180.



$$m\angle 1 + m\angle 2 + m\angle 3 = 180$$

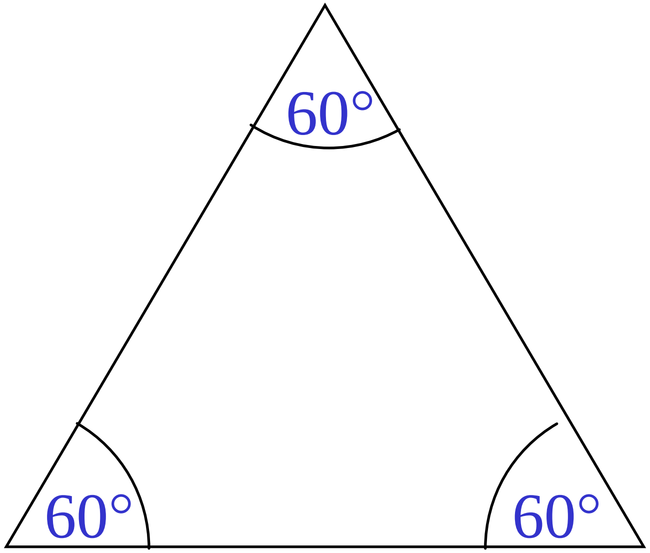
Corollary 1 Theorem 3-11

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.



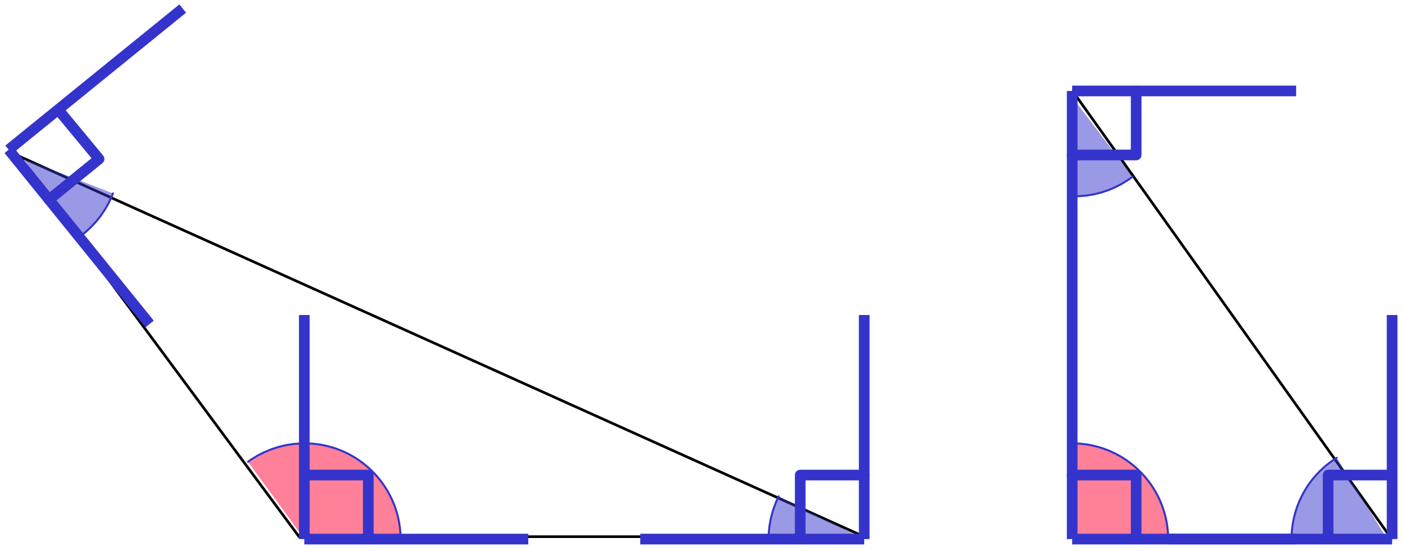
★ Corollary 2 Theorem 3-11

If a triangle is equiangular, then each angle measures 60°.



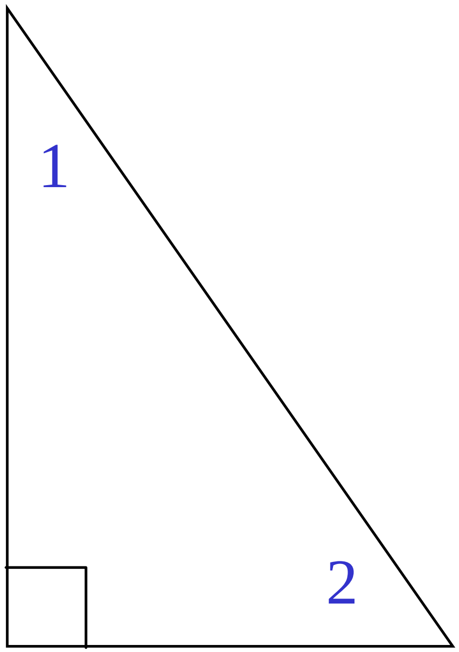
Corollary 3 Theorem 3-11

If a polygon is a triangle, then there can be at most one right or obtuse angle.



★ Corollary 4 Theorem 3-11

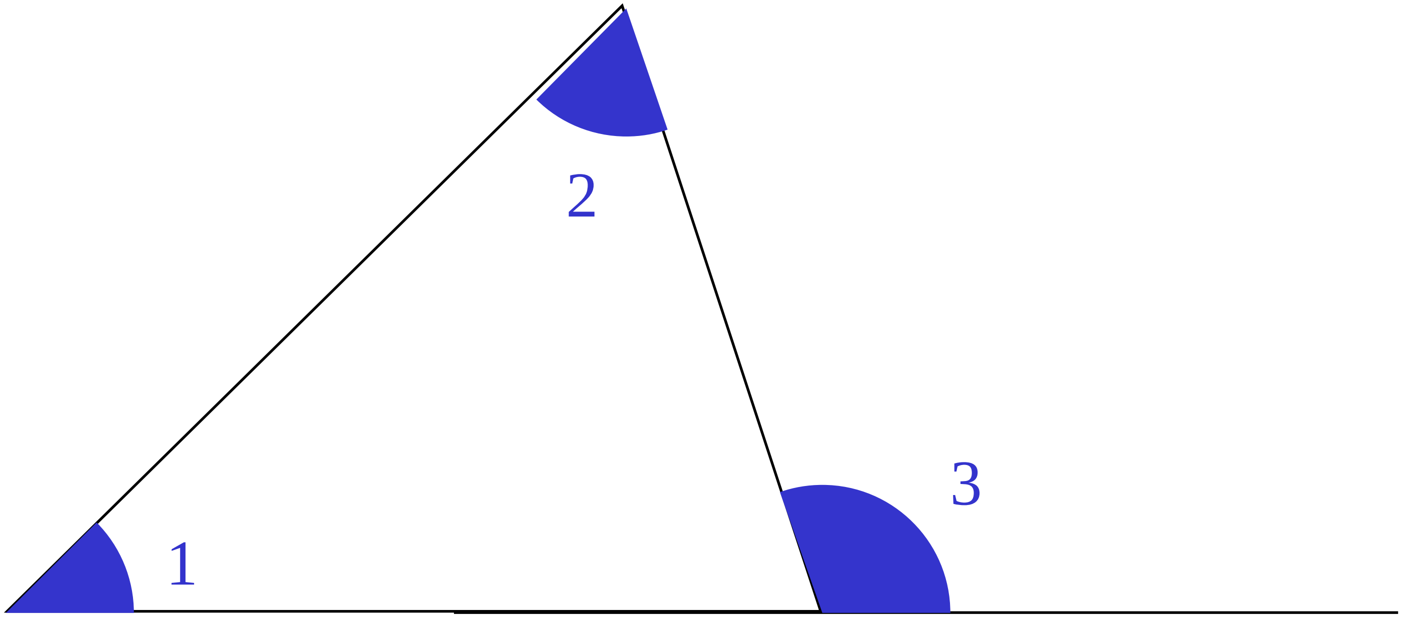
If a polygon is a right triangle, then the acute angles are complementary.



$m\angle 1 + m\angle 2 = 90$

★ Theorem 3-12

If an angle is exterior to a triangle, then its measure equals the sum of the measures of the two remote interior angles.



$$m\angle 1 + m\angle 2 = m\angle 3$$

Sample Problems Section 3-4

Draw a triangle that satisfies the conditions stated. If no triangle can satisfy the conditions, write not possible.

1.a. an acute isosceles triangle

b. a right isosceles triangle

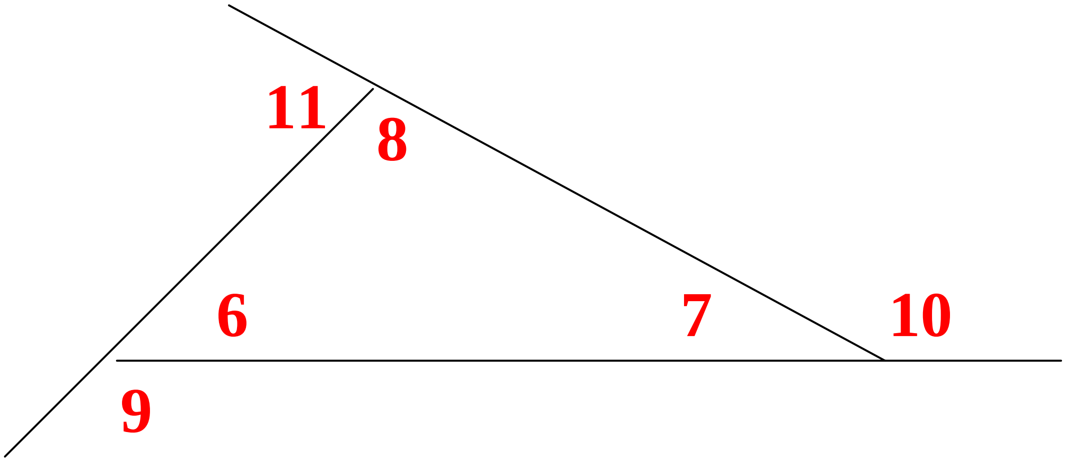
c. an obtuse isosceles triangle.

3. a triangle with two acute exterior angles

Sample Problems Section 3-4

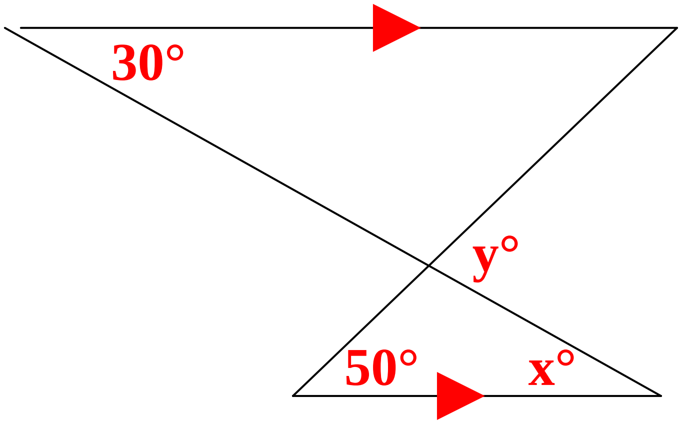
Complete.

5. m ∠ 6 + m ∠ 7 + m ∠ 8 = ?
7. If m ∠ 6 = 55 and m ∠ 10 = 150, then m ∠ 8 = ?
9. If m ∠ 8 = 4x, m ∠ 7 = 30, and m ∠ 9 = 6x - 20, then x = ?



Sample Section 3-4

11.



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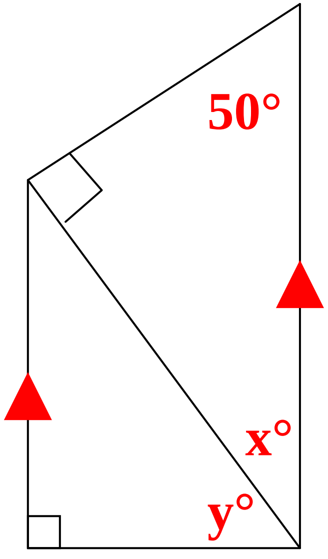

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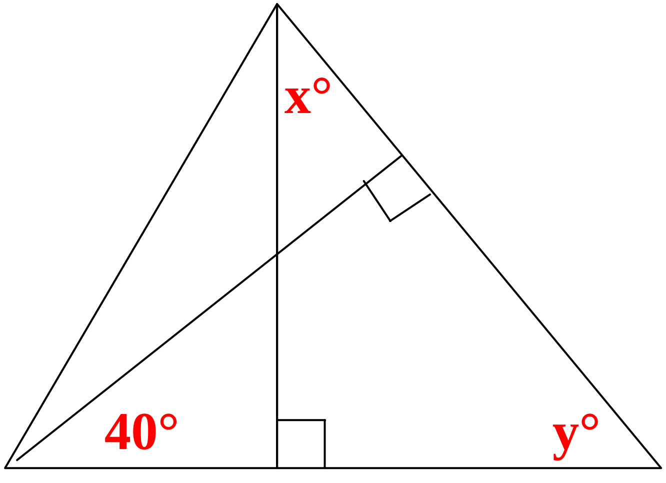


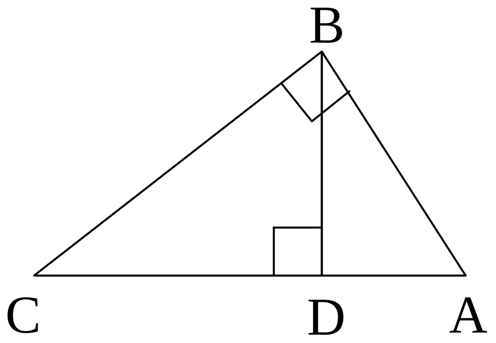
Sample Problems Section 3-4

17. The lengths of the sides of a triangle are 4n, 2n + 10, and 7n – 15. Is there a value of n that make the triangle equilateral?

19. The largest two angles of a triangle are two and three times as large as the smallest angle. Find all three measures.

21. In Δ ABC, m ∠ A = 60 and m ∠ B < 60. What can you say about m ∠ C?

23. Given: AB ⊥ BC; BD ⊥ AC



a. If m ∠ C = 22, find m ∠ ABD

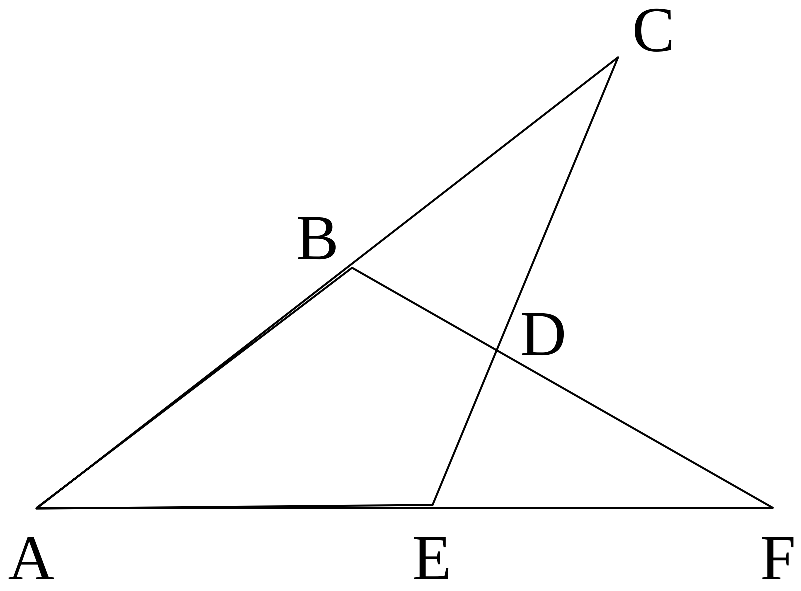
b. If m ∠ C = 23, find m ∠ ABD

c. Explain why m ∠ ABD always equals m ∠ C.

Sample Problems Section 3-4

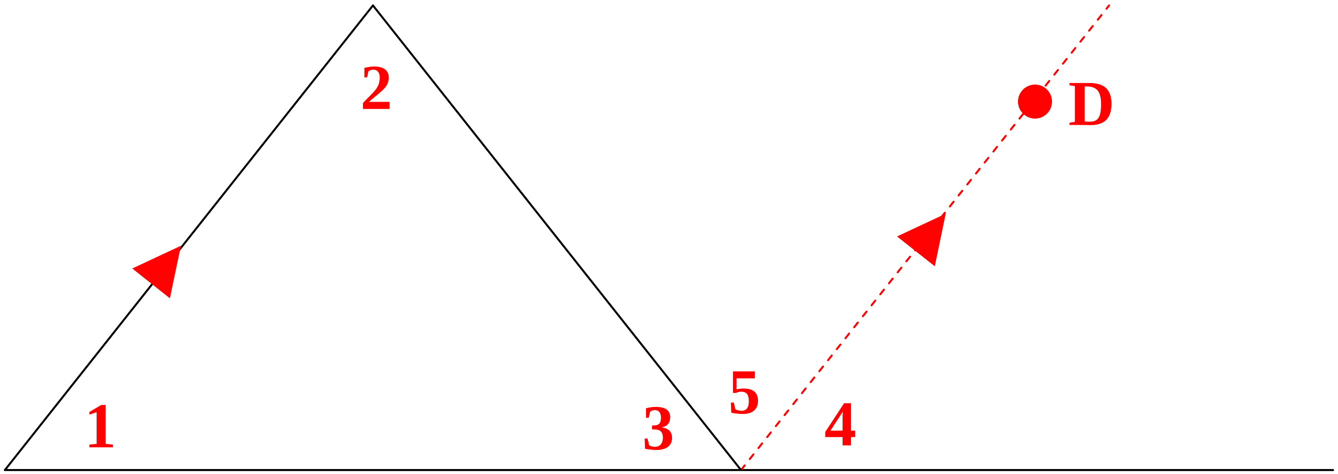
25. Given: $\angle ABD \cong \angle AED$

Prove: $\angle C \cong \angle F$



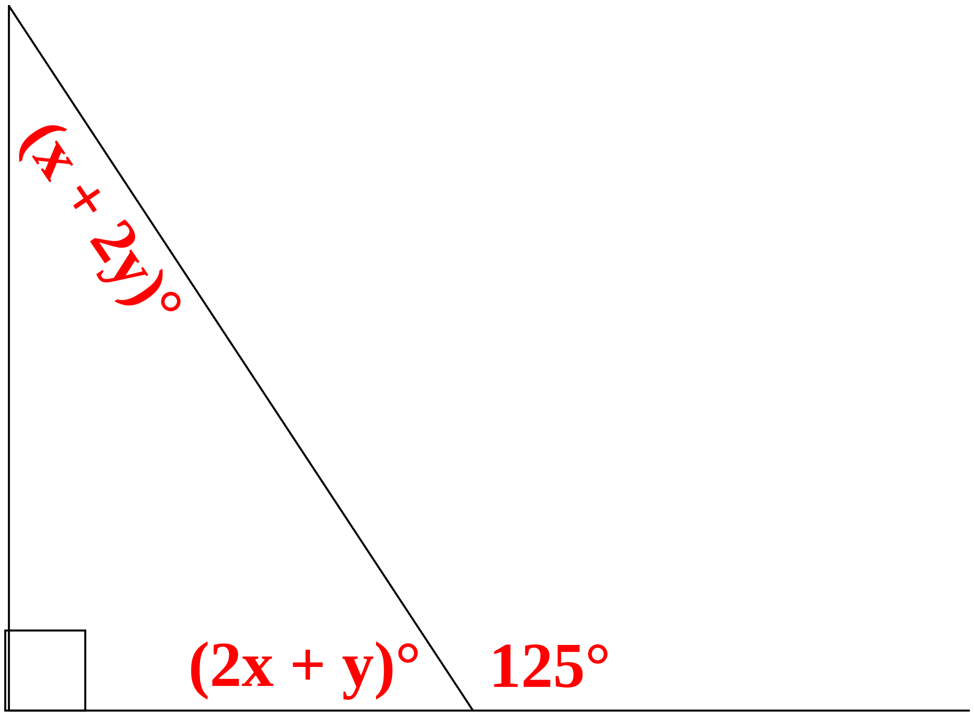
Sample Problems Section 3-4

27. Prove Theorem 3-11 by using the diagram below.



Sample Problems Section 3-4

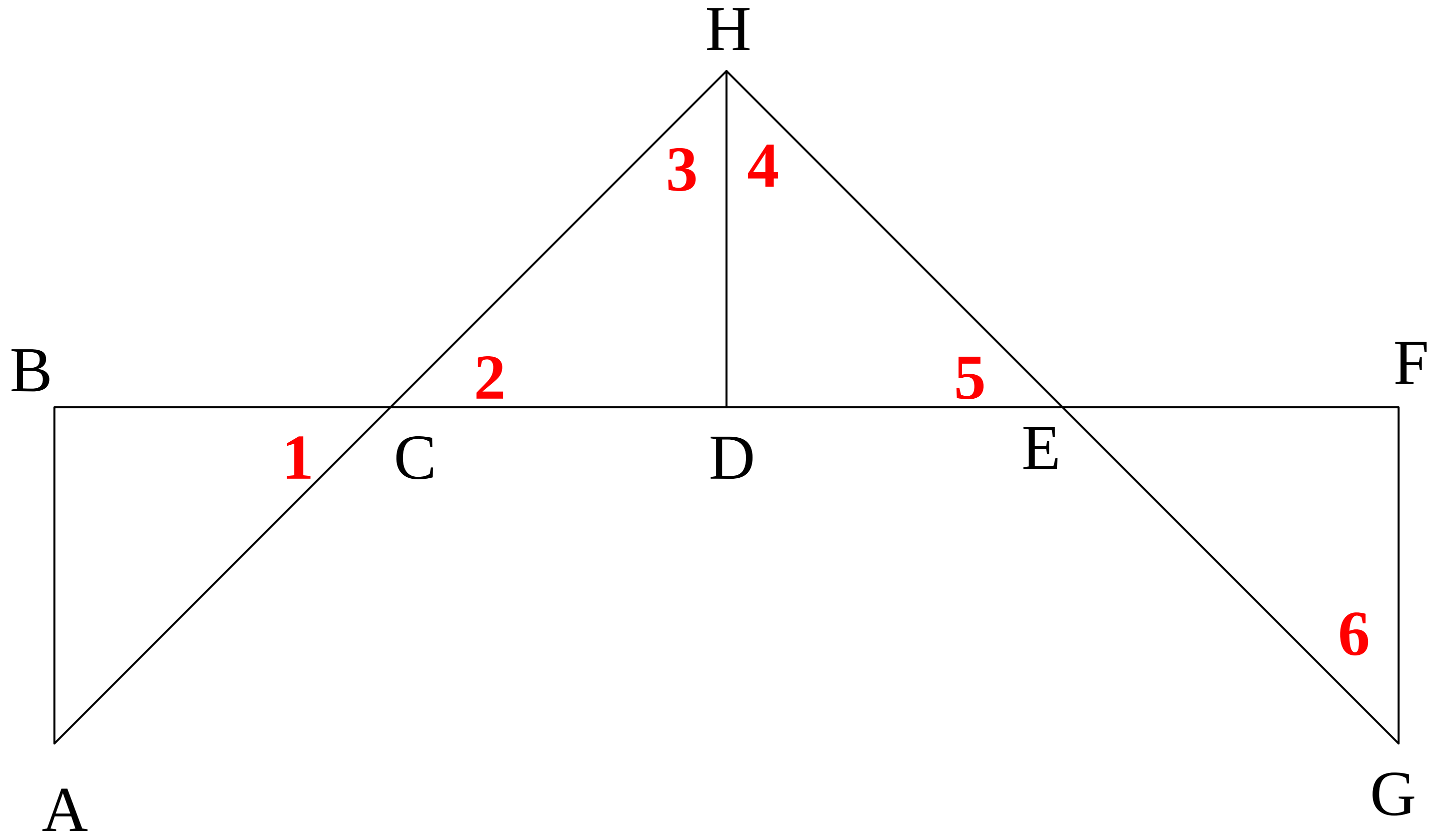
29. Find the values of x and y.



Sample Problems Section 3-4

31. Given $AB \perp BF$; $HD \perp BF$; $GF \perp BF$; $\angle A \cong \angle G$

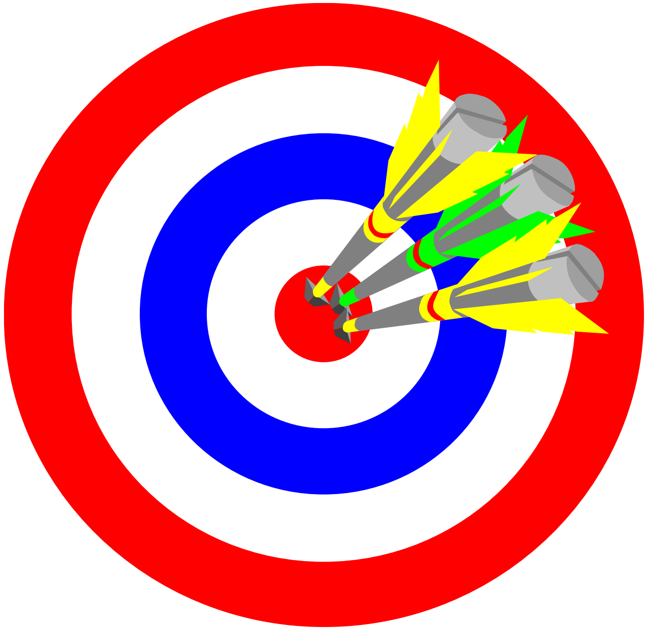
Which numbered angles must be congruent?



Section 3-5

Angles of a Polygon
Homework Pages 104-105:
2-22 evens
Excluding 14

Objectives



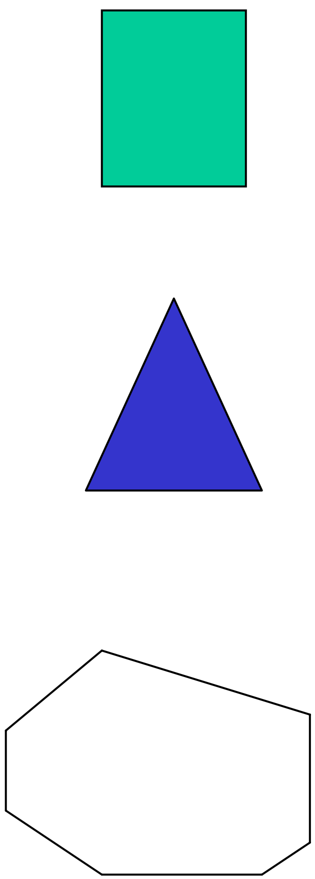
A. Use the term ‘polygon’ correctly.
B. Recognize and name convex, concave, and regular polygons.
C. Find the measures of interior and exterior angles of convex polygons.
D. Understand and use the theorems relating to measures of interior and exterior angles of polygons correctly.
E. Use the term ‘diagonal’ correctly and apply it to problems and diagrams.

Poly Wants A …

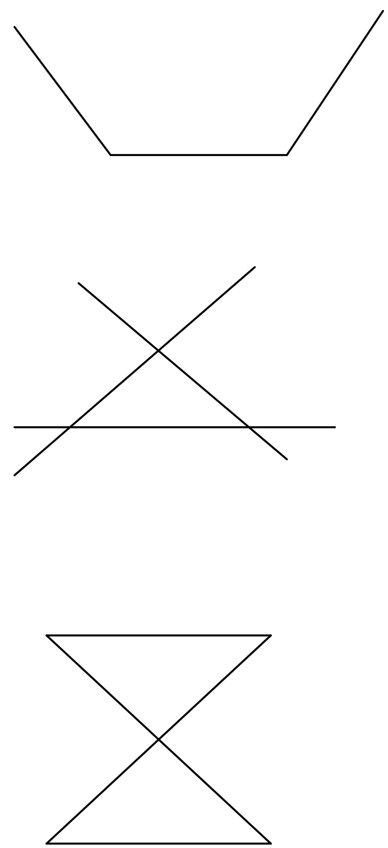
- Polygon →
  - ‘poly-’ from Latin ‘polys’ meaning many
  - ‘-gon’ from Latin ‘gonum’ meaning figure of angles.
  - Polygon is a figure with many angles
- polygon: a figure formed by coplanar segments such that:
  - each segment intersects exactly two other segments, one at each endpoint
  - no two segments with a common endpoint are collinear
- Diagonal → A diagonal of a polygon is a segment that joins two non-consecutive vertices.

Polygons and Diagonals

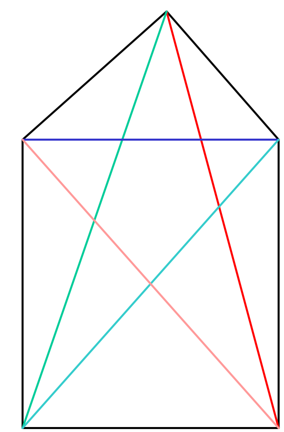
Polygons



Not Polygons



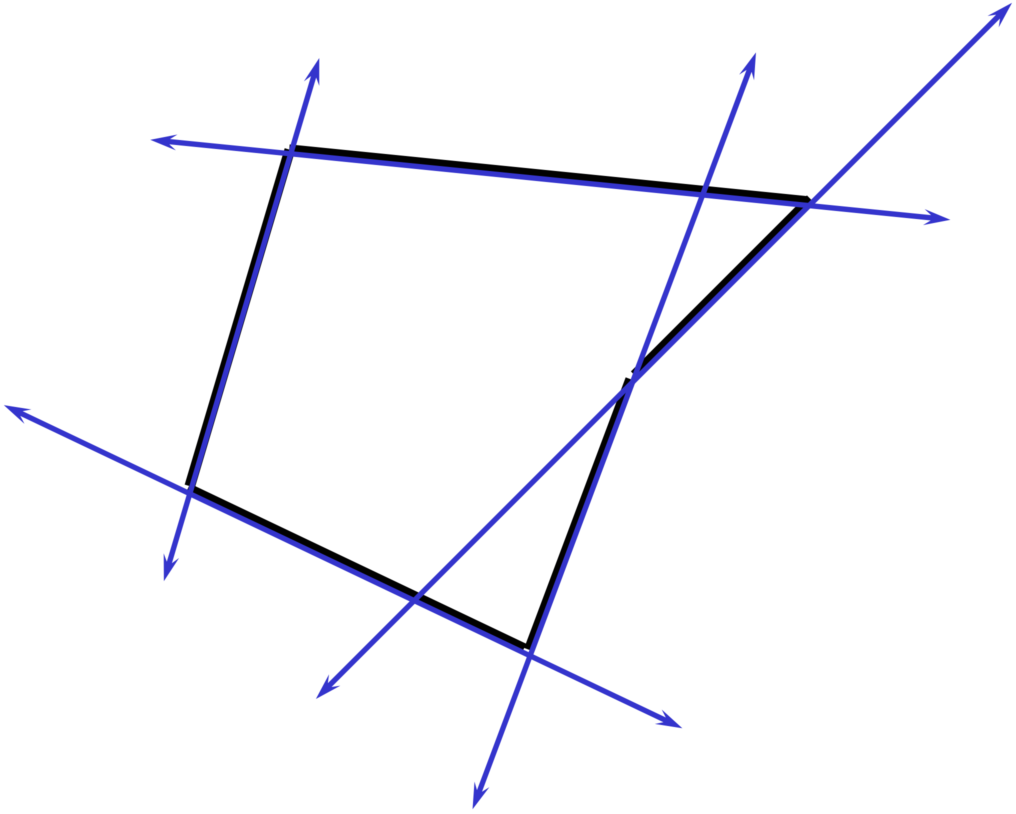
Diagonals



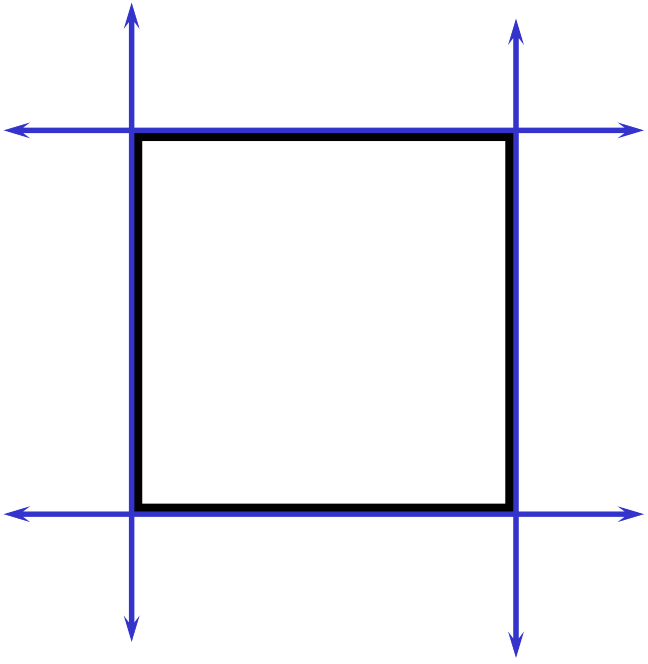
Classifications of Polygons

- Concave Polygon (non-convex) → A polygon such that at least one line containing the side of a polygon contains a point in the interior of the polygon.
- Convex Polygon → A polygon such that no line containing a side of the polygon contains a point in the interior of the polygon.
- Regular polygon → Any polygon that is both equilateral and equiangular.

Concave & Convex Polygons

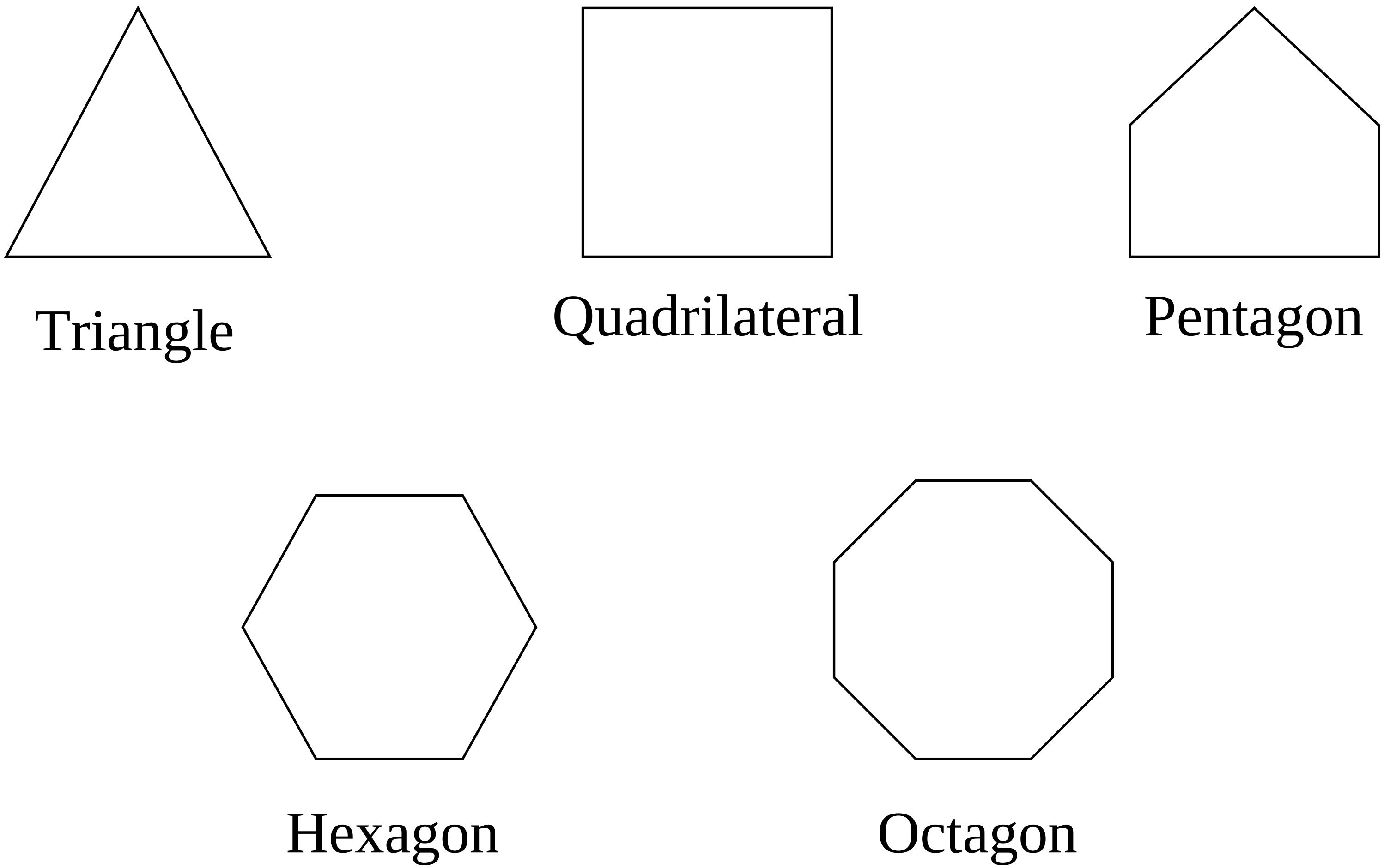


Concave



Convex

Polygons



★ Names for Polygons

triangle: 3 sides

quadrilateral: 4 sides

pentagon: 5 sides

hexagon: 6 sides

octagon: 8 sides

nonagon: 9 sides

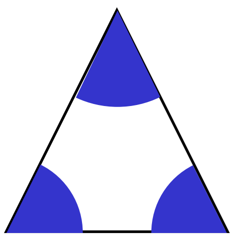
decagon: 10 sides

dodecagon: 12 sides

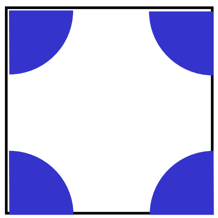
n-gon: n sides

★ Theorem 3-13

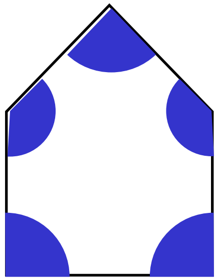
If a polygon is convex with n sides, then the sum of the measures of the interior angles is equal to (n - 2)180.



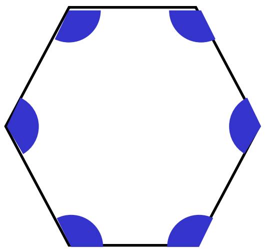
(3 - 2)180



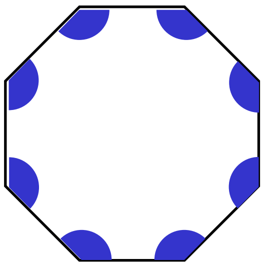
(4 - 2)180



(5 - 2)180



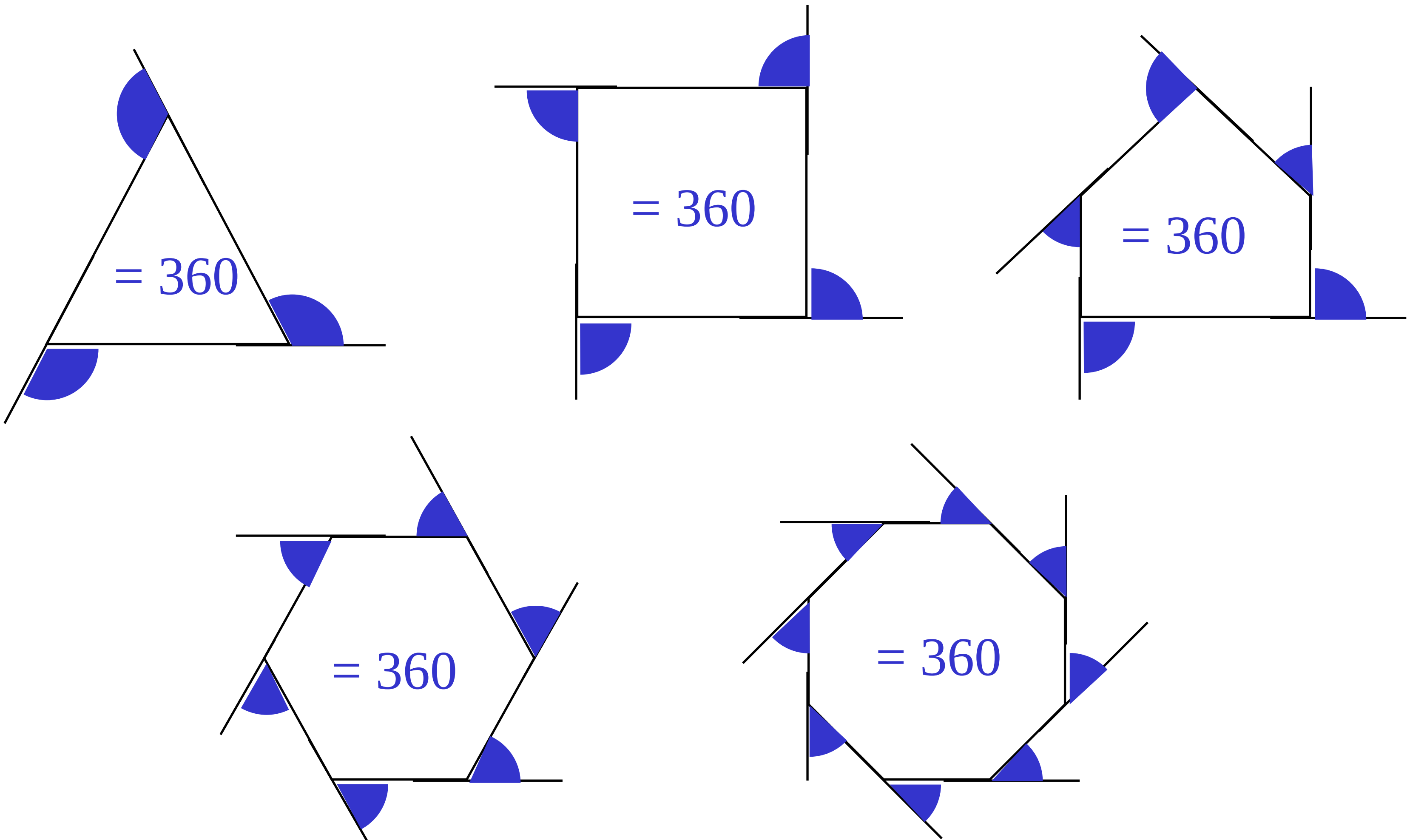
(6 - 2)180



(8 - 2)180

★ Theorem 3-14

If a polygon is convex with n sides, then the sum of the measures of the exterior angles, one angle at each vertex, is 360.



Sample Problems Section 3-5

For each polygon find (a) the interior angle sum and (b) the exterior angle sum.

1. quadrilateral
3. hexagon
5. decagon

9. A baseball diamond’s home plate has three right angles. The other two angles are congruent. Find their measure.
11. The face of a honeycomb consists of interlocking regular hexagons. What is the measure of each angle of these hexagons?

Sample Problems Section 3-5

Sketch the polygon described. If no such polygon exists, write not possible.

13. A quadrilateral that is equilateral but not equiangular.
15. A regular polygon, one of whose angles has a measure of 130.

17. The measure of each interior angle of a regular polygon is eleven times that of an exterior angle. How many sides does the polygon have?
19. Make a sketch showing how to tile a floor using both squares and regular octagons.
21. In quadrilateral ABCD, $m\angle A = x$, $m\angle B = 2x$, $m\angle C = 3x$ and $m\angle D = 4x$. Find the value of x and then state which pair of sides of ABCD must be parallel.

Sample Problems Section 3-5

23. ABCDEFGHIJ is a regular decagon. If sides AB and CD are extended to meet at K, find the measure of ∠ K.

25. The sum of the measures of the interior angles of a polygon is known to be between 2100 and 2200. How many sides does the polygon have?

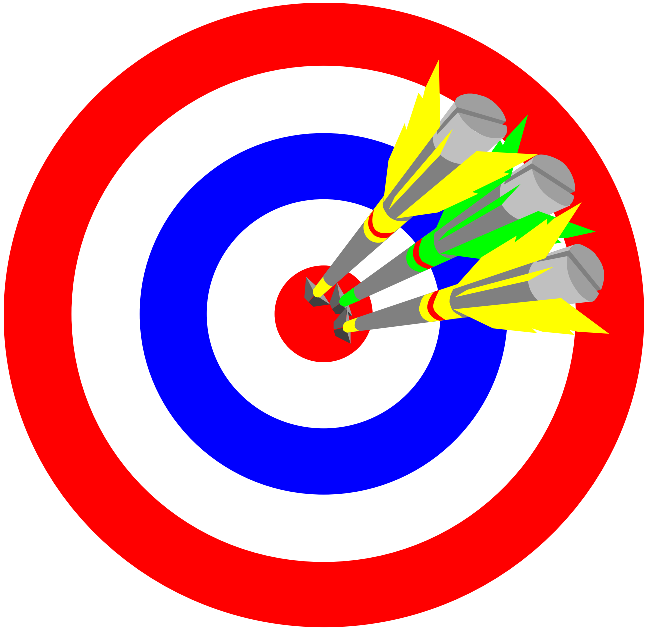
Section 3-6

Inductive Reasoning

Homework Pages 107-108:

2-22 evens

Objectives



A. Explain the difference between inductive and deductive reasoning.
B. Properly apply deductive and inductive reasoning to problems.
C. Understand the importance of inductive reasoning.

★ Deductive vs. Inductive Reasoning

Deductive Reasoning

– conclusion based on accepted statements such as definitions, postulates, properties, theorems, corollaries & given information
– conclusion must be true if the hypothesis is true

Inductive Reasoning

– conclusion based on several past observations such as those observed through experimentation
– conclusion is probably true, but not necessarily true

Sample Problems Section 3-6

Look for a pattern and predict the next two terms in the sequence.

1. 1, 4, 16, 64, …
3. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, ...$
5. 2, 3, 5, 8, 12,…
7. 40, 39, 36, 31, 24, ...
9. 2, 20, 10, 100,.50, ...

Sample Problems Section 3-6

Accept the two statements as given information. State a conclusion based on deductive reasoning. If no conclusion can be reached, then write none.

11. Valerie is older than Greg.
    Dan is older than Greg.
13. Polygon G has more than six sides.
    Polygon K has more than six sides.

Sample Problems Section 3-6

For each exercise write the equation that you think should come next.

15. $1 \times 9 + 2 = 11$

    $12 \times 9 + 3 = 111$

    $123 \times 9 + 4 = 1111$

17. $9^2 = 81$

    $99^2 = 9801$

    $999^2 = 998001$

Sample Problems Section 3-6

Decide whether each statement is true or false. If it is false show a counterexample. If it is true, draw and label a diagram you could use in a proof.

19. If a triangle has two congruent angles, then the sides opposite those angles are congruent.
21. All diagonals of a regular pentagon are congruent.
23. If the diagonals of a quadrilateral are congruent and also perpendicular, then the quadrilateral is a regular quadrilateral.
25. The diagonals of an equilateral quadrilateral are perpendicular.

Chapter Three

Parallel Lines and Planes

Review

Homework pages 112-113:

2-14 evens