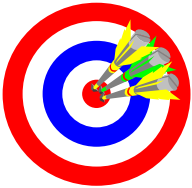


Chapter Four

Congruent Triangles

Objectives



- A. Use the terms defined in the chapter correctly.
- B. Properly use and interpret the symbols for the terms and concepts in this chapter.
- C. Appropriately apply the postulates and theorems in this chapter.

- D. Identify the corresponding parts of congruent figures.
- E. Use pairs of congruent triangles to write proofs about angles and sides.
- F. Use relationships between angles and sides of triangles to prove congruence of triangles.

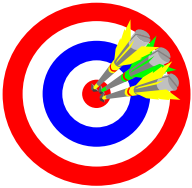
HOMEWORK NOTE!

In homework problems, all proofs MUST be done as two-column proofs. Although various homework problems will ask you to write a 'paragraph proof', you are to write all proofs as two-column proofs.

Section 4-1

Congruent Figures
Homework Pages 120-121:
2-20 evens

Objectives



- A. Use the term 'congruent' correctly when referring to figures.
- B. Recognize the conditions that make two figures congruent.
- C. Rotate figures about several axes to determine congruency.

D. Recognize corresponding sides and corresponding angles.

The Confluence of Congruence

- Congruent → Figures with the same **SIZE** and **SHAPE!**
 - Congruent line segments → two or more line segments with the same length.
 - Congruent angles → two or more angles with the same measure.

Corresponding Parts

- When we talk about 'corresponding parts' of polygons, we are basically matching each part of one polygon to exactly one part of another polygon.
- We can speak of:
 - Corresponding vertices
 - Corresponding sides
 - Corresponding angles
- Corresponding parts do NOT necessarily have to be in the same relative location (upper right, lower left, etc.)
 - However, it may be helpful to ROTATE the figures to place corresponding parts in the same relative locations.

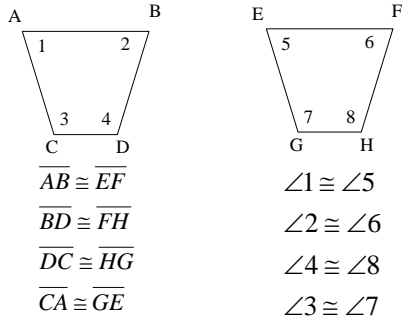
Checking Congruence of Polygons

- Congruent polygons → polygons are congruent if and only if the vertices can be matched up so the corresponding parts (sides and angles) are congruent.
- Congruent triangles → triangles are congruent if and only if the vertices can be matched up so that corresponding parts (sides and angles) are congruent.

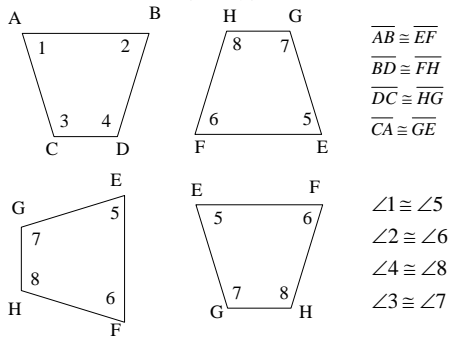
Methods for Checking Congruence of Polygons

- One-to-one matching → show that each side and angle of one polygon is congruent to each corresponding side and angle of the other polygon.
 - NOTE: ORDER MATTERS!
 - A quadrilateral with consecutive sides of length 4, 5, 6, and 7 is NOT congruent to a quadrilateral with consecutive sides of length 4, 6, 5, 7.
- Rotating a polygon → Rotate a polygon clockwise or counterclockwise until each segment and each angle is congruent to the corresponding angles and segments AND in the same relative position of the second polygon.
- 'Flipping' a polygon → Rotate a polygon across an axis until each segment and each angle is congruent to the corresponding angles and segments AND in the same relative position of the second polygon.

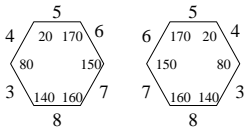
One-to-one Matching



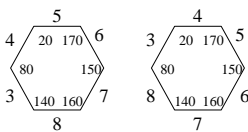
Rotating a Polygon



Corresponding Parts of Congruent Polygons
'Flipping' Polygons

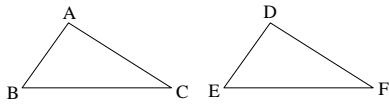


These two polygons are congruent because the left polygon can be flipped over on top of the right polygon and all of the sides and angles will match up.



These two polygons are not congruent because you cannot get both the sides and the angles to match up at the same time. You can get one to work but not the other,

★Corresponding Parts of Congruent Triangles



$\triangle ABC \cong \triangle DEF$ if and only if
 $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$
 $AB = DE, BC = EF, AC = DF$

Sample Problems Section 4-1

Suppose $\triangle BIG \cong \triangle CAT$. Complete.

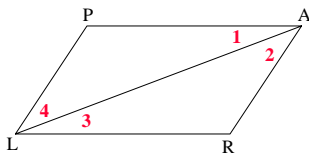
1. $\angle G \cong$ _____
3. $BI =$ _____
5. $\triangle IGB \cong$ _____

7. If $\triangle DEF \cong \triangle RST$, $m\angle D = 100$, and $m\angle F = 40$, name four congruent angles.
9. Suppose $\triangle LXR \cong \triangle FNE$. List six congruencies that can be justified by the following reason: Corresponding Parts of Congruent Triangles.

Sample Problems Section 4-1

11. The two triangles shown are congruent. Complete.

- a. $\triangle PAL \cong$ _____
 - b. $PA =$ _____
 - c. $\angle 1 \cong$ _____ because _____
 - d. $\angle 2 \cong$ _____ because _____
- Then _____ \parallel _____ because _____



Sample Problems Section 4-1

13. Plot the given points on a piece of graph paper. Draw $\triangle FAT$. Locate point C so that $\triangle FAT \cong \triangle CAT$.

F(7, 5) A(-2, 2) T(5, 2)

Plot the given points on graph paper. Draw $\triangle ABC$ and $\triangle DEF$. Copy and complete the statement $\triangle ABC \cong$ _____

15. A(-7, -3) B(-2, -3) C(-2, 0)

D(0, 1) E(5, 1) F(0, -2)

17. A(1, 1) B(8, 1) C(4, 3)

D(3, -7) E(5, -3) F(3, 0)

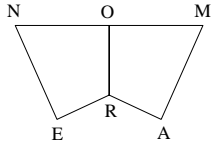
Sample Problems Section 4-1

19. Plot the points on graph paper. Draw $\triangle ABC$ and DE. Find two locations of point F such that $\triangle ABC \cong \triangle DEF$.

A(-1, 0) B(-5, 4) C(-6, 1) D(1, 0) E(5, 4)

21. In your own words explain why each of the following statements must be true.

- a. O is the midpoint of NM.
- b. $\angle NOR \cong \angle MOR$
- c. $RO \perp NM$



Section 4-2

Some Ways to Prove Triangles
Congruent
Homework Pages 124-126:
2-22 evens
Excluding 8

Objectives

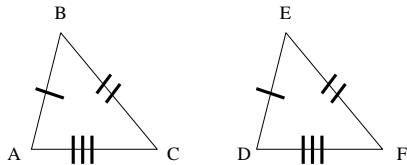


- A. State postulates of congruence of triangles correctly.
- B. Apply postulates of congruence of triangles correctly.
- C. Distinguish between SSS, SAS, and ASA congruence.

D. Correctly interpret and utilize include sides and included angles.

★ Postulate 12 (side-side-side of SSS)

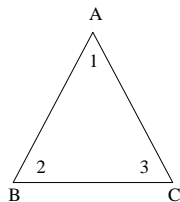
If three sides of one triangle are congruent to three sides of another triangle, **then the triangles are congruent**.



$$\triangle ABC \cong \triangle DEF$$

Included Sides and Angles

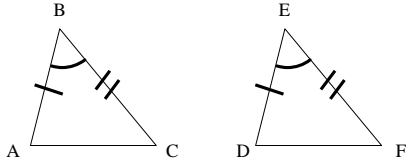
- In a triangle, we say a **side is included** if it is between two referenced angles.
- In a triangle, we say an **angle is included** if it is between two referenced sides.



- Side AC is included between angles 1 and 3.
- Angle 2 is included between sides AB and BC.

★ Postulate 13 (side-angle-side or SAS)

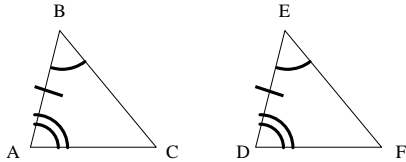
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.



$$\triangle ABC \cong \triangle DEF$$

★ Postulate 14 (angle-side-angle or ASA)

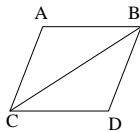
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.



$$\triangle ABC \cong \triangle DEF$$

Proof

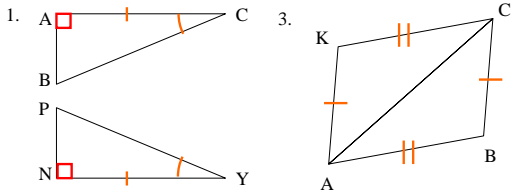
Given $\overline{AB} \cong \overline{CD}$ and $\overline{BD} \cong \overline{AC}$,
prove $\triangle ABC \cong \triangle BDC$.



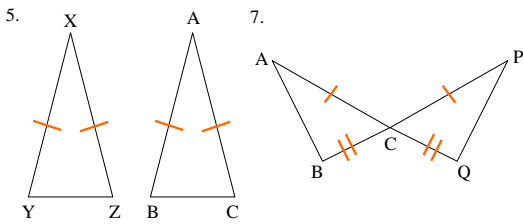
1. $\overline{AB} \cong \overline{CD}$ and $\overline{BD} \cong \overline{AC}$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2. Reflexive property of congruence.
3. $\triangle ABC \cong \triangle BDC$	3. Postulate 12 – If 3 sides of 1 triangle are congruent to 3 sides of another triangle, then the triangles are congruent.

Sample Problems Section 4-2

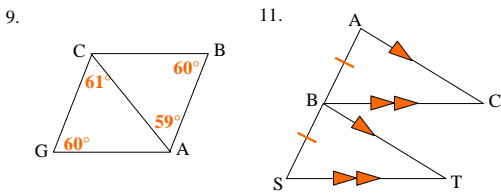
Decide whether you can deduce by SSS, SAS, or ASA that another triangle is congruent to $\triangle ABC$. If so, write the congruence and name the pattern used. If not, write no congruence



Sample Problems Section 4-2

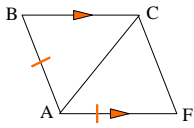


Sample Problems Section 4-2

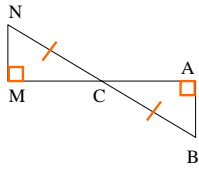


Sample Problems Section 4-2

13.



15.

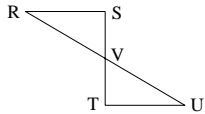


Sample Problems Section 4-2

17. Given: $\overline{RS} \perp \overline{ST}$; $\overline{TU} \perp \overline{ST}$;

V is the midpoint of \overline{ST}

Prove: $\triangle RSV \cong \triangle UTV$

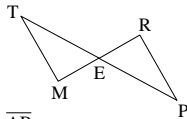


- | | |
|--|----|
| 1. $\overline{RS} \perp \overline{ST}$; $\overline{TU} \perp \overline{ST}$ | 1. |
| 2. $m\angle S = 90$, $m\angle ? = 90$ | 2. |
| 3. $\angle S \cong \angle T$ | 3. |
| 4. V is the midpoint of \overline{ST} | 4. |
| 5. $\overline{SV} \cong ?$ | 5. |
| 6. $\angle RVS \cong \angle ?$ | 6. |
| 7. $\triangle ? \cong \triangle ?$ | 7. |

Sample Problems Section 4-2

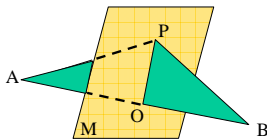
19. Given: E is the midpoint of \overline{TP} & \overline{MR}

Prove: $\triangle TEM \cong \triangle PER$



21. Given: Plane M bisects \overline{AB} ; $\overline{PO} \perp \overline{AB}$

Prove: $\triangle POA \cong \triangle POB$



Sample Problems Section 4-2

Draw and label a diagram. List in terms of the diagram, what is given and what is to be proved. Then write the two column proof.

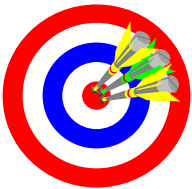
23. In an isosceles triangle, if a segment is drawn from the vertex of the angle between the congruent sides to the midpoint of the opposite side, then congruent triangles are formed.

25. If pentagon ABCDE is equilateral and has right angles at B and E, then diagonals AC and AD form congruent triangles.

Section 4-3

Using Congruent Triangles
Homework Pages 130-131:
2-10 evens

Objectives



- A. Understand and apply the concept of a line perpendicular to a plane.
- B. Apply SSS, SAS, and ASA relationships to prove congruence of angles or segments.

- line perpendicular to a plane: a line is perpendicular to a plane if and only if they intersect and the line is perpendicular to all lines in the plane that pass through the point of intersection.

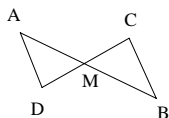
★ Using Congruent Triangles

Ways to Prove Two Segments or Two Angles Congruent:

1. Identify two triangles in which the angles or segments would be corresponding parts.
2. Prove the triangles congruent.
3. State the angles or segments are congruent by Congruent Parts of Congruent Triangles (CPCT).

Proof

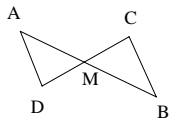
Given \overline{AB} and \overline{CD} bisect each other at M,
prove $\overline{AD} \parallel \overline{BC}$.



1. \overline{AB} and \overline{CD} bisect each other at M.	1. Given
2. M is the midpoint of \overline{AB} and \overline{CD} .	2. Definition of segment bisector
3. $\overline{AM} \cong \overline{MB}; \overline{DM} \cong \overline{MC}$	3. Definition of midpoint.

Proof

Given \overline{AB} and \overline{CD} bisect each other at M,
 prove $\overline{AD} \parallel \overline{BC}$.



4.	$\angle AMD \cong \angle BMC$	4. Vertical angles are congruent.
5.	$\triangle AMD \cong \triangle BMC$	5. Side-angle-side (SAS) postulate.
6.	$\angle A \cong \angle B$	6. Corresponding parts of congruent triangles are congruent.
7.	$\overline{AD} \parallel \overline{BC}$	7. If 2 lines are cut by transversal and alt int angle are congruent, then the lines are parallel.

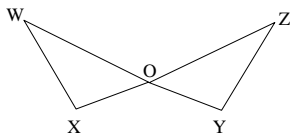
Sample Problems Section 4-3

1. Given: $\angle P \cong \angle S$; O is the midpoint of \overline{PS}
 Prove: O is the midpoint of \overline{RQ}

- | | |
|---|----|
| 1. $\angle P \cong \angle S$ | 1. |
| 2. O is the midpoint of \overline{PS} | 2. |
| 3. $\overline{PO} \cong \overline{SO}$ | 3. |
| 4. $\angle POQ \cong \angle SOR$ | 4. |
| 5. $\triangle POQ \cong \triangle SOR$ | 5. |
| 6. $\overline{QO} \cong \overline{RO}$ | 6. |
| 7. O is the midpoint of \overline{RQ} | 7. |

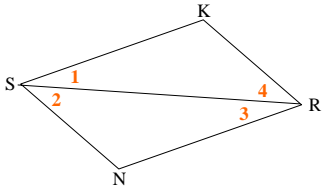
Sample Problems Section 4-3

3. Given: $\overline{WO} \cong \overline{ZO}$; $\overline{XO} \cong \overline{YO}$
 Prove: $\angle W \cong \angle Z$



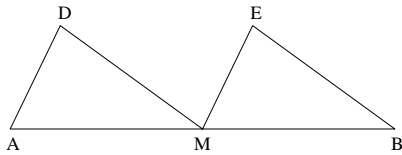
Sample Problems Section 4-3

5. Given: $\overline{SK} \parallel \overline{NR}$ $\overline{SN} \parallel \overline{KR}$
 Prove: $\overline{SK} \cong \overline{NR}$; $\overline{SN} \cong \overline{KR}$



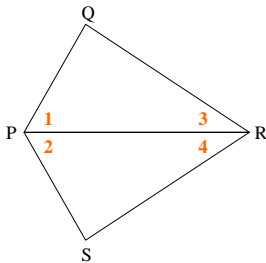
Sample Problems Section 4-3

7. Given: $\overline{AD} \parallel \overline{ME}$ $\overline{MD} \parallel \overline{BE}$
 M is the midpoint of \overline{AB}
 Prove: $\overline{MD} \cong \overline{BE}$



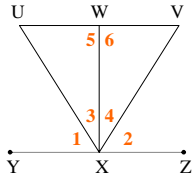
Sample Problems Section 4-3

9. Given: $\overline{PQ} \cong \overline{PS}$; $\overline{QR} \cong \overline{SR}$; $\angle 1 \cong \angle 2$
 Prove: $\angle 3 \cong \angle 4$



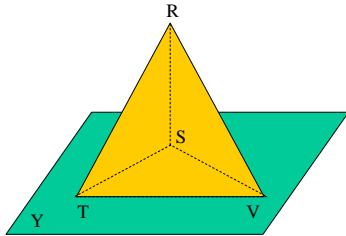
Sample Problems Section 4-3

11. Given: $\overline{WX} \perp \overline{YZ}$; $\angle 1 \cong \angle 2$; $\overline{UX} \cong \overline{VX}$
 Which one(s) of the following statements must be true?
 (1) $\overline{XW} \perp \overline{UV}$ (2) $\overline{UV} \parallel \overline{YZ}$ (3) $\overline{VX} \perp \overline{UX}$



Sample Problems Section 4-3

13. Given: $\overline{RS} \perp$ plane Y ; $\angle TRS \cong \angle VRS$
 Prove: $\triangle RTV$ is isosceles



Section 4-4

The Isosceles Triangle Theorems
 Homework Pages 137-139:
 2-28 evens
 Excluding 16, 20, 22, 26

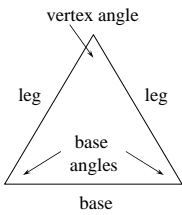
Objectives



- A. Apply the terms 'vertex angle', 'legs', 'base', and 'base angles' in conjunction with isosceles triangles.
- B. Understand and apply the theorems and corollaries associated with isosceles triangles.

C. Apply corollaries that relate isosceles and equilateral triangles.

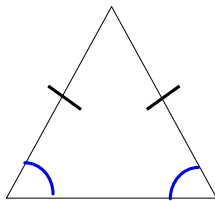
★ Parts of an Isosceles Triangle



- legs: congruent sides of an isosceles triangle
- vertex angle: angle formed by the legs of an isosceles triangle
- base: side opposite the vertex angle of an isosceles triangle
- base angles: angles formed by a leg and the base of an isosceles triangle

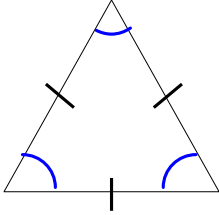
★ Theorem 4-1

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.



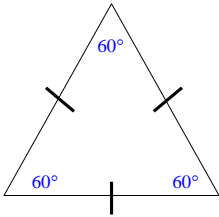
★ Corollary 1 Theorem 4-1

An equilateral triangle is also equiangular.



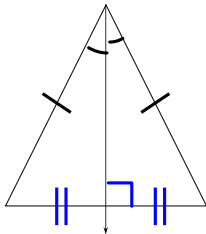
★ Corollary 2 Theorem 4-1

An equilateral triangle has three 60° angles.



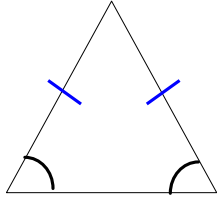
★ Corollary 3 Theorem 4-1

The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.



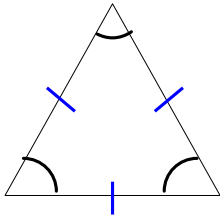
★ Theorem 4-2

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



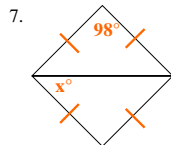
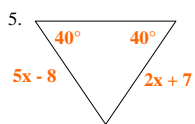
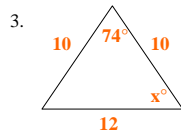
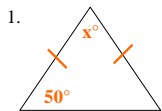
Corollary 1 Theorem 4-2

An equiangular triangle is also equilateral.



Sample Problems Section 4-4

Find the value of x .



Sample Problems Section 4-4

Place the statements in the appropriate order for a proof.

9. Given: $\overline{RS} \cong \overline{RT}$

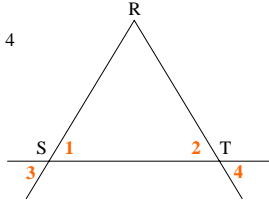
Prove: $\angle 3 \cong \angle 4$

(a) $\angle 3 \cong \angle 4$

(b) $\angle 3 \cong \angle 1$; $\angle 2 \cong \angle 4$

(c) $\overline{RS} \cong \overline{RT}$

(d) $\angle 1 \cong \angle 2$

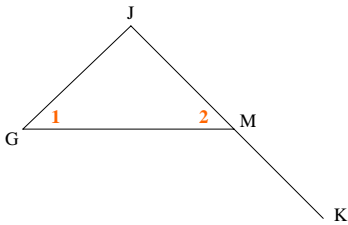


Sample Problems Section 4-4

11. Prove Theorem 4-1.

13. Given: M is the midpoint of \overline{JK} ; $\angle 1 \cong \angle 2$

Prove: $\overline{JG} \cong \overline{MK}$

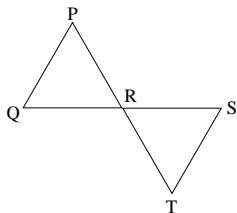


Sample Problems Section 4-4

15. Given: $\overline{PQ} \cong \overline{PR}$; $\overline{TR} \cong \overline{TS}$

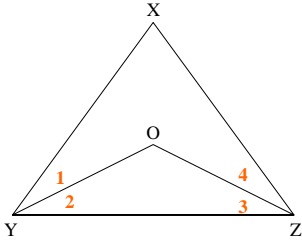
Which one(s) of the following must be true?

(1) $\overline{ST} \parallel \overline{QP}$ (2) $\overline{ST} \cong \overline{QP}$ (3) $\angle T \cong \angle P$



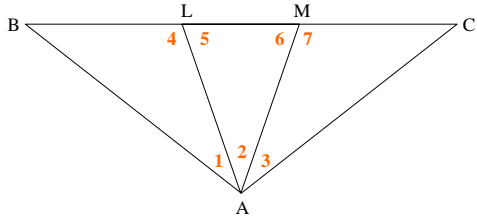
Sample Problems Section 4-4

17. Given: $\overline{XY} \cong \overline{XZ}$; $\overline{OY} \cong \overline{OZ}$
 Prove: $m\angle 1 = m\angle 4$



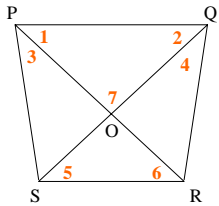
Sample Problems Section 4-4

19. Given: $\overline{AB} \cong \overline{AC}$; \overline{AL} and \overline{AM} trisect $\angle BAC$
 Prove: $\overline{AL} \cong \overline{AM}$



Sample Problems Section 4-4

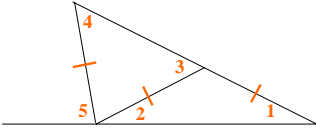
21. Given: $\angle 3 \cong \angle 4$; $\overline{OP} \cong \overline{OQ}$
 Prove: $\angle 5 \cong \angle 6$



Sample Problems Section 4-4

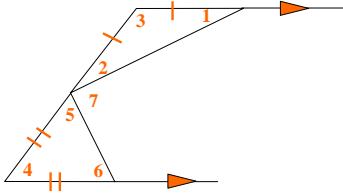
23. Complete.

- a. If $m \angle 1 = 20$ then $m \angle 3 = ?$
 $m \angle 4 = ?$ and $m \angle 5 = ?$
 b. If $m \angle 1 = x$ then $m \angle 3 = ?$



Sample Problems Section 4-4

- 25.a. If $m \angle 1 = 23$, find $m \angle 7$.
 b. If $m \angle 1 = k$, find $m \angle 7$.



Sample Problems Section 4-4

Find the values of x and y .

27. In equiangular $\triangle ABC$, $AB = 4x - y$, $BC = 2x + 3y$, and $AC = 7$.
 29. In $\triangle JKL$, $JK = JL$, $m \angle J = 2x - y$, $m \angle K = 2x + 2y$, and $m \angle L = x + 2y$.

Section 4-5

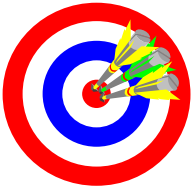
Other Methods of Proving Triangles
Congruent

Homework Pages 143-145:

2-18 evens

Excluding 8, 10, 12

Objectives

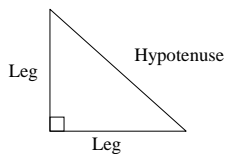


- A. Understand and apply the angle-angle-side (AAS) theorem.
- B. Understand and apply the hypotenuse-leg (HL) theorem.
- C. Properly choose between the various ways to prove two triangles congruent.

D. Understand and apply the terms 'hypotenuse' and 'leg' in reference to right triangles.

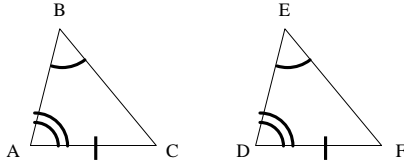
Right Triangle Definitions

- Remember a right triangle is a triangle that contains exactly one right angle.
- Hypotenuse → side opposite the right angle of a right triangle.
- Leg → sides forming the right angle of a right triangle.



★ Theorem 4-3 (angle-angle-side)

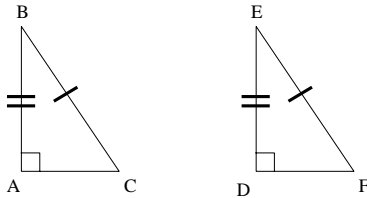
If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent.



$$\triangle ABC \cong \triangle DEF$$

★ Theorem 4-4 (hypotenuse-leg)

If the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of another right triangle, then the triangles are congruent.



$$\triangle ABC \cong \triangle DEF$$

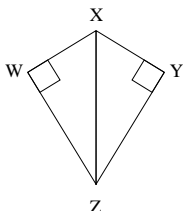
Ways to Prove Two Triangles Congruent

- You have postulates, corollaries, definitions, and theorems that allow you to CONCLUDE two triangles are congruent:
 - SSS → Side-side-side
 - SAS → Side-angle-side
 - ASA → Angle-side-angle
 - AAS → Angle-angle-side
 - HL → Hypotenuse-leg (right triangles ONLY!)
 - CPCT → Corresponding Parts of Congruent Triangles
- These are major theorems that will be used extensively for the rest of your mathematical lives!
- These are generally referred to by their name, but you need to know what they say, when to apply them, and how to apply them!

Sample Homework Problems Section 4-5

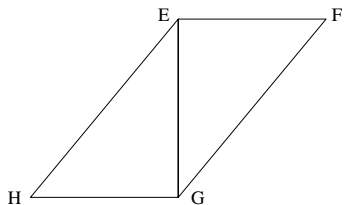
1. Given: $\angle W$ and $\angle Y$ are rt. \angle 's; $\overline{WX} \cong \overline{YX}$
 Prove: $\overline{WZ} \cong \overline{YZ}$

1. $\angle W$ and $\angle Y$ are rt. \angle 's
 2. $\triangle XWZ$ & $\triangle XYZ$ are rt \triangle 's
 3. $\overline{WX} \cong \overline{YX}$
 4. ?
 5. $\triangle XWZ \cong \triangle ?$
 6. ?



Sample Homework Problems Section 4-5

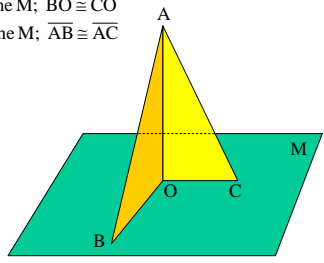
3. Given: $\overline{EF} \perp \overline{EG}$; $\overline{HG} \perp \overline{EG}$; $\overline{EH} \cong \overline{GF}$
 Prove: $\angle H \cong \angle F$



Sample Homework Problems Section 4-5

Name the method you would use to prove $\triangle AOB \cong \triangle AOC$.
 Do not write the proof.

5. Given: $\overline{AO} \perp$ plane M; $\overline{BO} \cong \overline{CO}$
 7. Given: $\overline{AO} \perp$ plane M; $\overline{AB} \cong \overline{AC}$



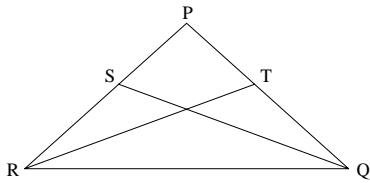
Sample Homework Problems Section 4-5

9. Given: $\overline{PR} \cong \overline{PQ}$; $\overline{SR} \cong \overline{TQ}$

Prove: $\overline{QS} \cong \overline{RT}$

9a. Write a two column proof that uses $\triangle RQS \cong \triangle QRT$ by SAS

9b. Write a two column proof that uses $\triangle PQS \cong \triangle PRT$ by SAS



Sample Homework Problems Section 4-5

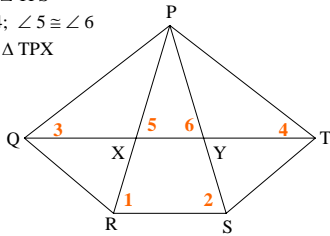
Tell which pair of congruent parts and what method you would use to prove the triangles congruent.

11. Given: $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$; $\overline{QR} \cong \overline{TS}$

Prove: $\triangle QPR \cong \triangle TPS$

13. Given: $\angle 3 \cong \angle 4$; $\angle 5 \cong \angle 6$

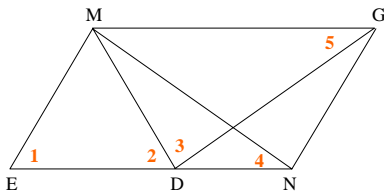
Prove: $\triangle QPY \cong \triangle TPX$



Sample Homework Problems Section 4-5

15. Given: $\angle 1 \cong \angle 2 \cong \angle 3$; $\overline{EN} \cong \overline{DG}$

Prove: $\angle 4 \cong \angle 5$



Sample Homework Problems Section 4-5

Draw and label a diagram. List in terms of the diagram what is given and what is to be proved. Then write a two column proof.

17. If segments are drawn from the endpoints of the base of an isosceles triangle perpendicular to the opposite legs, then those segments are congruent.

19. If segments are drawn from the midpoints of the legs of an isosceles triangle perpendicular to the base, then those segments are congruent.

Section 4-6

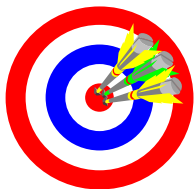
Using More Than One Pair of Congruent Triangles

Homework Pages 148-150:

2-14 evens

Excluding 8

Objectives



- A. Recognize and apply the transitive property of congruence.
- B. Recognize and apply the reflexive property of congruence.
- C. Prove 2 triangles congruent by first proving 2 other triangles congruent.

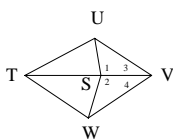
Using Congruent Triangles

Ways to Prove Two Segments or Two angles Congruent:

- Identify two triangles in which the angles or segments would be corresponding parts.
- Prove the triangles congruent.
- State the angles or segments are congruent by C.P.C.T.

Proof

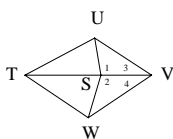
Given $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$,
prove $\overline{TU} \cong \overline{TW}$.



1. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	1. Vertical angles are congruent.
2. $\overline{SV} \cong \overline{SV}$	2. Reflexive property of congruence.
3. $\Delta USV \cong \Delta WSV$	3. Angle-side-angle postulate.
4. $\overline{UV} \cong \overline{WV}$	4. Corresponding parts of corresponding triangles are congruent.

Proof

Given $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$,
prove $\overline{TU} \cong \overline{TW}$.

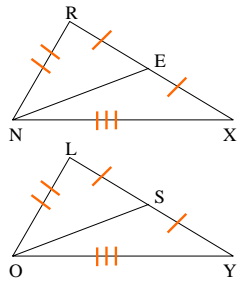


5. $\overline{TV} \cong \overline{TV}$	5. Reflexive property of congruence.
6. $\Delta TUV \cong \Delta TWV$	6. Side-angle-side postulate.
7. $\overline{TU} \cong \overline{TW}$	7. Corresponding parts of congruent triangles are congruent.

Sample Problems Section 4-6

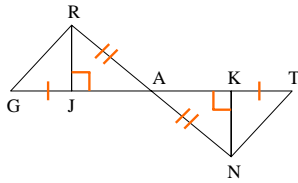
Give the reasons for the steps from the proof.

1. Prove: $\overline{NE} \cong \overline{OS}$
- a. $\triangle RNX \cong \triangle LOY$
- b. $\angle X \cong \angle Y$
- c. $\triangle NEX \cong \triangle OSY$
- d. $\overline{NE} \cong \overline{OS}$



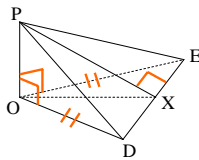
Sample Problems Section 4-6

3. Prove: $\angle G \cong \angle T$
- a. $\triangle RAJ \cong \triangle NAK$
- b. $\overline{RJ} \cong \overline{NK}$
- c. $\triangle GRJ \cong \triangle TNK$
- d. $\angle G \cong \angle T$



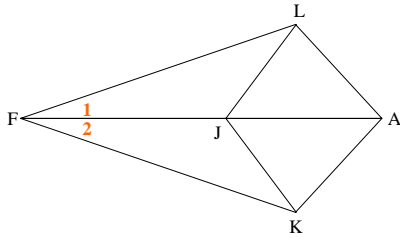
Sample Problems Section 4-6

5. Prove: $\overline{DX} \cong \overline{EX}$
- a. $\triangle POD \cong \triangle POE$
- b. $\overline{PD} \cong \overline{PE}$
- c. $\triangle PDX \cong \triangle PEX$
- d. $\overline{DX} \cong \overline{EX}$



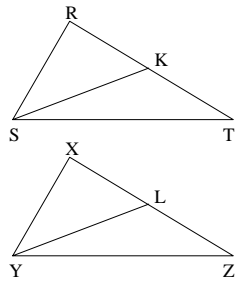
Sample Problems Section 4-6

7. Given: $\overline{LF} \cong \overline{KF}$; $\overline{LA} \cong \overline{KA}$
 Prove: $\overline{LJ} \cong \overline{KJ}$



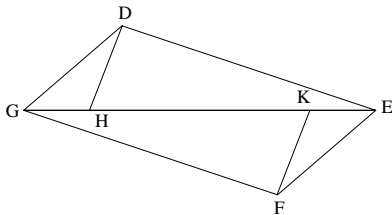
Sample Problems Section 4-6

9. Given: $\triangle RST \cong \triangle XYZ$
 \rightarrow SK bisects $\angle RST$;
 \rightarrow YL bisects $\angle XYZ$
 Prove: $\overline{SK} \cong \overline{YL}$



Sample Problems Section 4-6

11. Given: $\angle HDE$ and $\angle KFG$ are rt. \angle 's
 $\overline{DE} \cong \overline{FG}$; $\overline{GD} \cong \overline{EF}$
 Prove: $\overline{DH} \cong \overline{FK}$



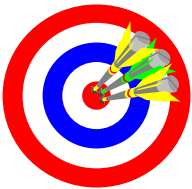
Sample Problems Section 4-6

13. Draw two line segments KL and MN, that bisect each other at O. Mark a point P on segment KN and let Q be the point where \vec{PO} intersects ML. Prove that O is the midpoint of PQ.

Section 4-7

Medians, Altitudes and Perpendicular Bisectors
Homework Pages 156-157:
8-20 evens

Objectives



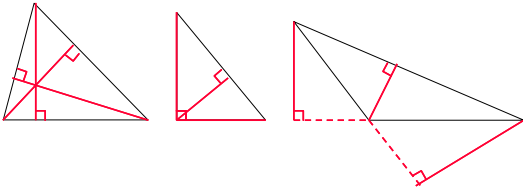
- A. Understand and apply the term 'median'.
- B. Understand and apply the term 'altitude'.
- C. Understand and apply the term 'perpendicular bisector'.

D. Properly apply the theorems relating to medians, altitudes and perpendicular bisectors.

- ★ altitude: segment from a vertex perpendicular to the line containing the opposite side
- ★ median: segment from a vertex to the midpoint of the opposite side
- ★ perpendicular bisector: line, segment or ray perpendicular to a segment at its midpoint

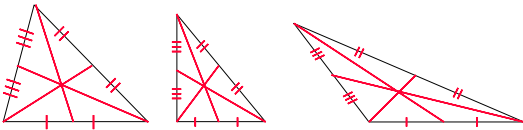
★ Altitudes

Every triangle has three altitudes.



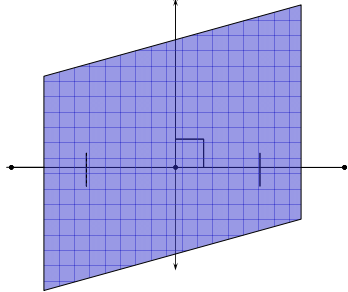
★ Medians

Every triangle has three medians.



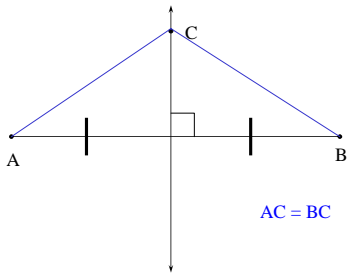
★ Perpendicular Bisectors

You can only have perpendicular bisectors for segments.
However, that segment may live in a plane or a ray.



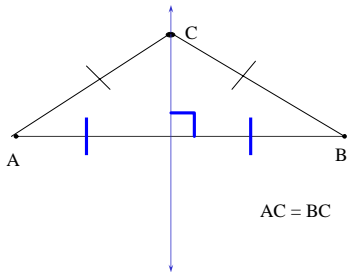
★ Theorem 4-5

If a point lies on the perpendicular bisector of a segment, then that point is equidistant from the endpoints of the segment.



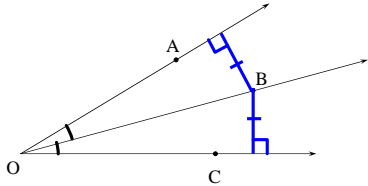
Theorem 4-6

If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.



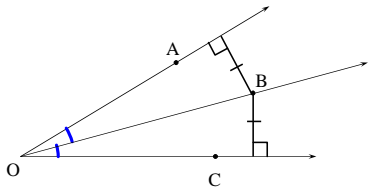
★ Theorem 4-7

If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.



Theorem 4-8

If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

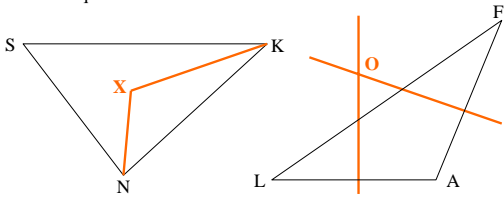


Sample Problems Section 4-7

- 1a. Draw a large scalene triangle $\triangle ABC$. Carefully draw the bisector of $\angle A$, the altitude from A, and the median from A. These three should all be different.
- 1b. Draw a large isosceles triangle $\triangle ABC$ with vertex angle A. Carefully draw the bisector of A, the altitude from A, and the median from A. Are these three different?
3. Draw a right triangle. Then draw its three altitudes using a different color.
5. Draw a large scalene right triangle. Then draw the perpendicular bisectors of its three sides and tell whether they appear to meet in a point. If so where is the point.

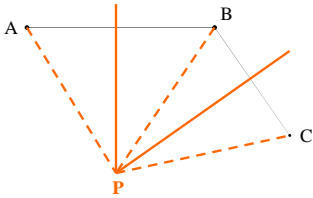
Sample Problems Section 4-7

7. If X is on the bisector of $\angle SKN$, then X is equidistant from ? and ?
9. If X is equidistant from segments SK and SN , then X lies on the ?
11. If O is on the perpendicular bisector of segment AF , then O is equidistant from ? and ?



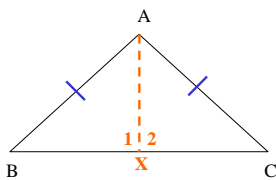
Sample Problems Section 4-7

13. Given: P is on the perpendicular bisector of \overline{AB}
 P is on the perpendicular bisector of \overline{BC}
 Prove: $PA = PC$



Sample Problems Section 4-7

15. Prove theorem 4-6
 Given: $AB = AC$
 Prove: A is on the perpendicular bisector of \overline{BC}

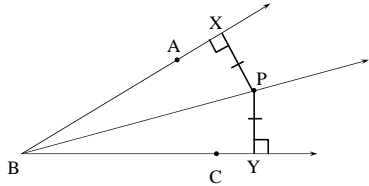


Sample Problems Section 4-7

17. Prove theorem 4-8

Given: $PX \perp BA$; $PY \perp BC$; $PX = PY$

Prove: \vec{BP} bisects $\angle ABC$



Sample Problems Section 4-7

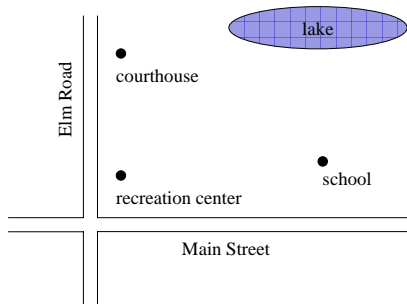
19a. A town wants to build a beach house on the lakefront equidistant from the recreation center and the school.

Copy the diagram and show the point B where the beach house should be located.

19b. The town also wants to build a boat-launching site that is equidistant from Elm Road and Main Street. Find the point L where it should be.

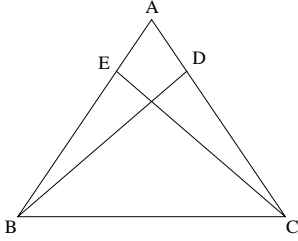
19c. On your diagram, locate the point F for a flagpole that is to be the same distance from the recreation center, the school, and the courthouse.

Sample Problems Section 4-7



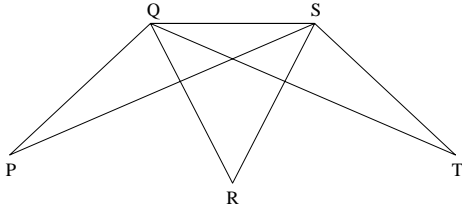
Sample Problems Section 4-7

21. Given: $\overline{AB} \cong \overline{AC}$; $\overline{BD} \perp \overline{AC}$; $\overline{CE} \perp \overline{AB}$
 Prove: $\overline{BD} \cong \overline{CE}$



Sample Problems Section 4-7

23. Given: \overline{SR} is the perpendicular bisector of \overline{QT}
 \overline{QR} is the perpendicular bisector of \overline{SP}
 Prove: $\overline{PQ} \cong \overline{TS}$



Sample Problems Section 4-7

25. Given: Plane M is the perpendicular bisector of \overline{AB} .
 Prove: $\overline{AD} \cong \overline{BD}$; $\overline{AC} \cong \overline{BC}$; $\angle CAD \cong \angle CBD$

Chapter Four

Congruent Triangles
Review
Homework Page 162:
2-18 evens
