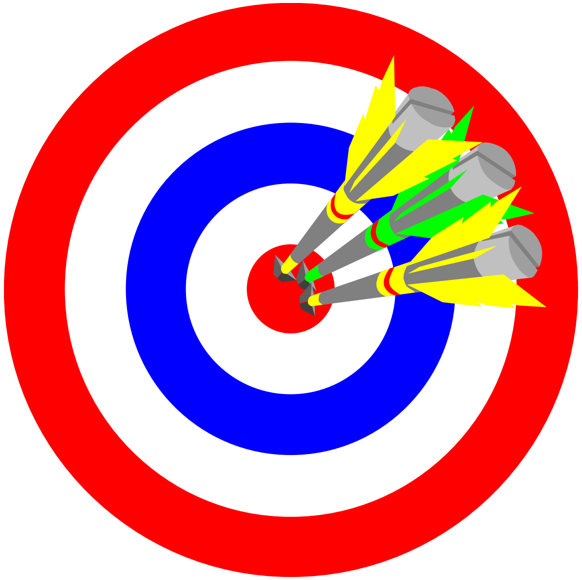
Chapter 5

Quadrilaterals

Objectives



- Use the terms defined in the chapter correctly.
- Properly use and interpret the symbols for the terms and concepts in this chapter.
- Appropriately apply the theorems in this chapter.
- Determine and prove, if necessary, that a quadrilateral is a: parallelogram, rectangle, rhombus, square or trapezoid.

HOMEWORK NOTE!

In homework problems, all proofs MUST be done as two-column proofs. Although various homework problems will ask you to write a ‘paragraph proof’, you are to write all proofs as two-column proofs.

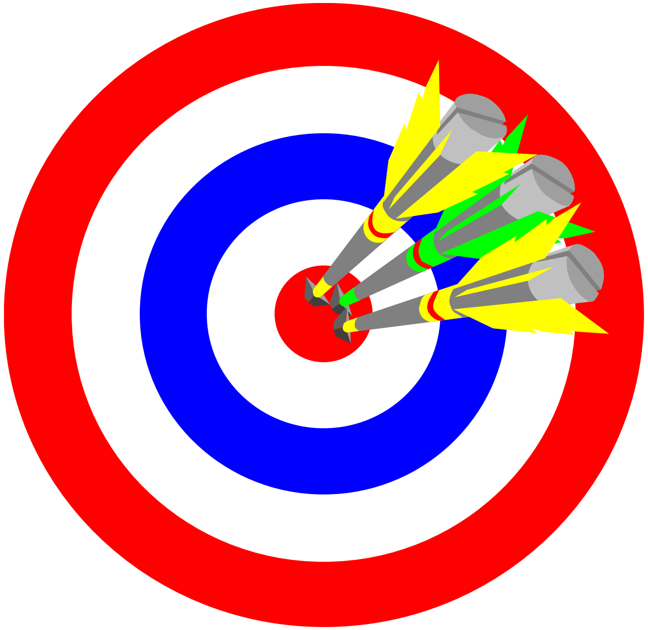
Section 5-1

Properties of Parallelograms

Homework Pages 169-171:

2 – 32 evens, excluding 10, 20, 28

Objectives



A. Use the term parallelogram correctly.

B. Understand and apply the theorems concerning congruent parts of parallelograms.

Definitions

- Polygon: A plane figure formed by coplanar segments (sides) such that:
  - Each segment intersects exactly two other segments, one at each endpoint
  - No two segments with a common endpoint are collinear
- Quadrilateral: A four-sided polygon.
- Parallelogram: a quadrilateral with both pairs of opposite sides parallel

▱ ABCD

Indicates a parallelogram with vertices A, B, C, and D

Reminders

- The slope of a line can be determined by its change in Y over its change in X:
  - Also known as rise over run
  - (Y2 – Y1)/(X2 – X1)

Activity

1. On a piece of graph paper, draw a coordinate plane:
   - With only positive x-axis and positive y-axis
   - With paper in portrait orientation
   - With the y-axis close to the left-hand side of the page
   - With the x-axis slightly above the center of the page
   - Correctly label the coordinate graph
2. Plot and label the following points (remember, the x-coordinate is first)
   - A (12,13), B (32, 13), C (2, 1), D (22,1)
3. Create a quadrilateral by connecting:
   - A to B, A to C, C to D, B to D.
4. On a separate piece of paper, write down any observations of the resulting figure.

Activity - Continued

5. Measure and record the distances between:
   - Points A and B
   - Points A and C
   - Points C and D
   - Points B and D
6. Record any observations based on these measurements.
7. Find the slopes of the lines between (remember slope is change in Y over change in X):
   - Points A and B
   - Points A and C
   - Points C and D
   - Points B and D
8. Record any observations based on these slope calculations

Activity - Continued

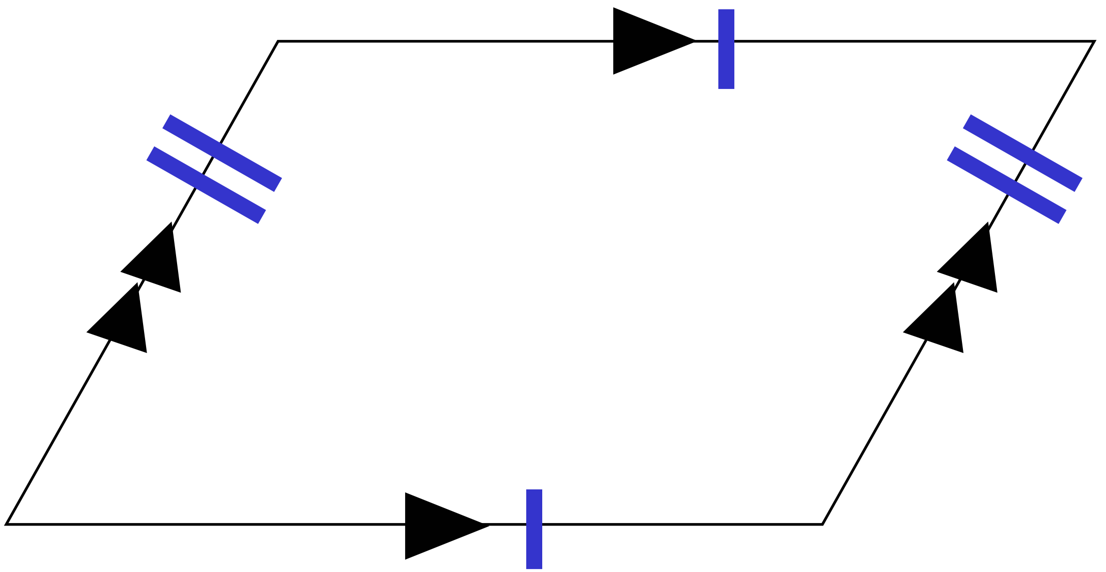
9. Create line segments from:
   - E (14,13) to F (14,1)
   - G (20,13) to H (20,1)
10. Measure and record the lengths of these line segments.
11. Based on these measurements, can you conclude that the line containing A and B and the line containing C and D are parallel? If so, why? If not, why not?
12. What would you conclude if you drew and measured 2 line segments perpendicular to AC and terminating on BD?
13. Based on this information, we conclude the figure is a parallelogram. Define a parallelogram.
14. On the bottom half of the same graph paper, redraw the parallelogram ABDC.

Activity - Continued

15. On the interior of the quadrilateral, label points A, B, C and D.
16. Draw diagonals from A to D and B to C.
17. Carefully cut out the quadrilateral and compare it to the original quadrilateral.
18. Cut the quadrilateral along the diagonal from A to D.
19. Rotate and flip the resulting triangles and write down all observations you can make.
20. Measure and compare the lengths of the segments created by the intersection of the diagonals.
21. Turn in your observations as part of your homework.

★Theorem 5-1

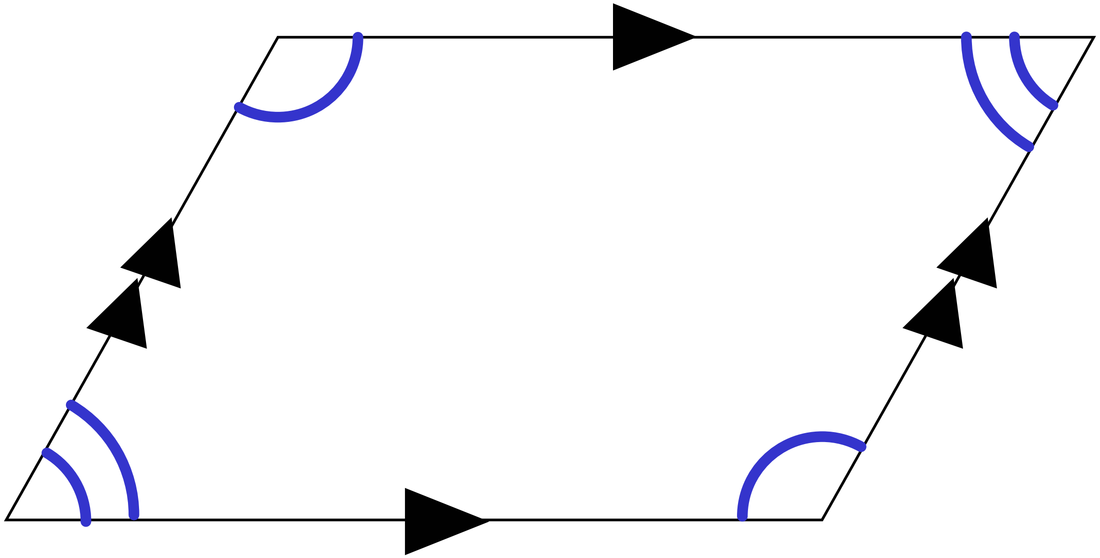
Opposite sides of a parallelogram



are congruent

★ Theorem 5-2

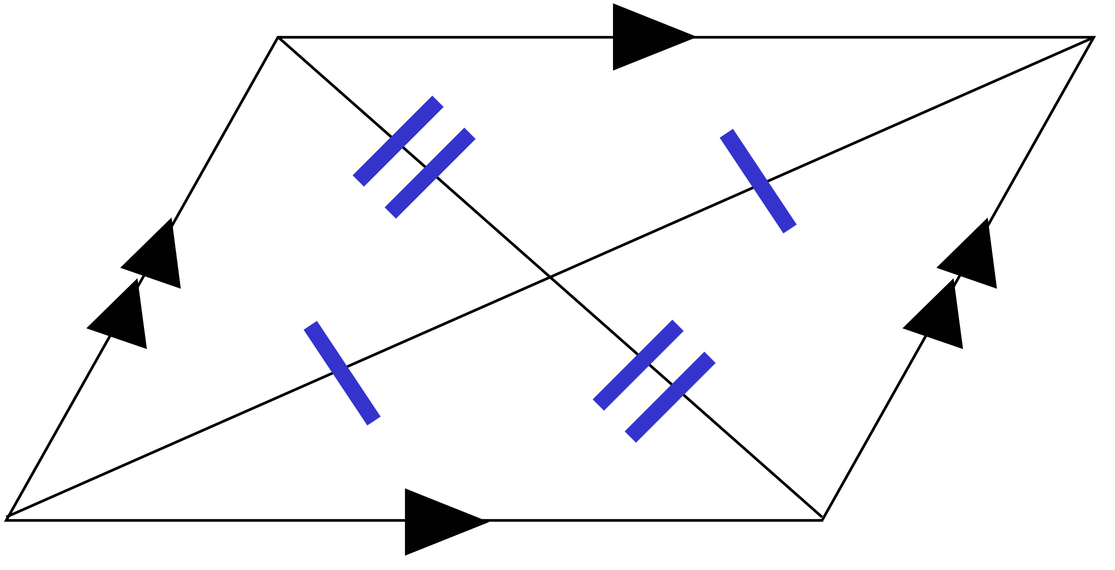
Opposite angles of a parallelogram



are congruent

★ Theorem 5-3

Diagonals of a parallelogram

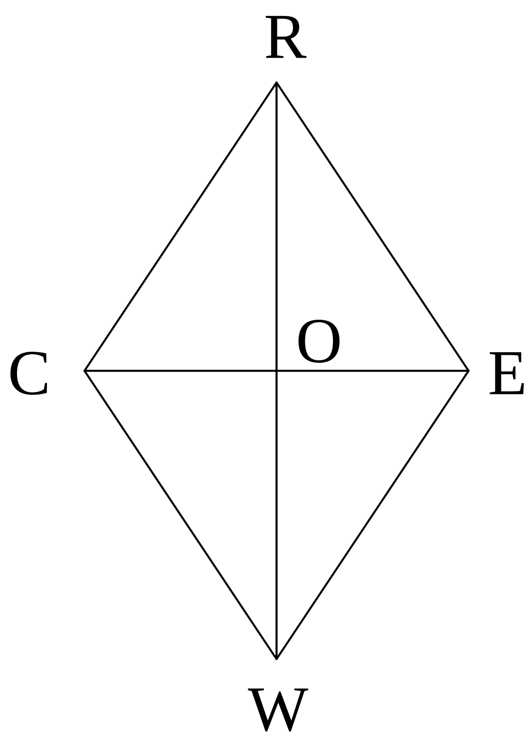


bisect each other.

Sample Problems Section 5-1

Given parallelogram CREW.

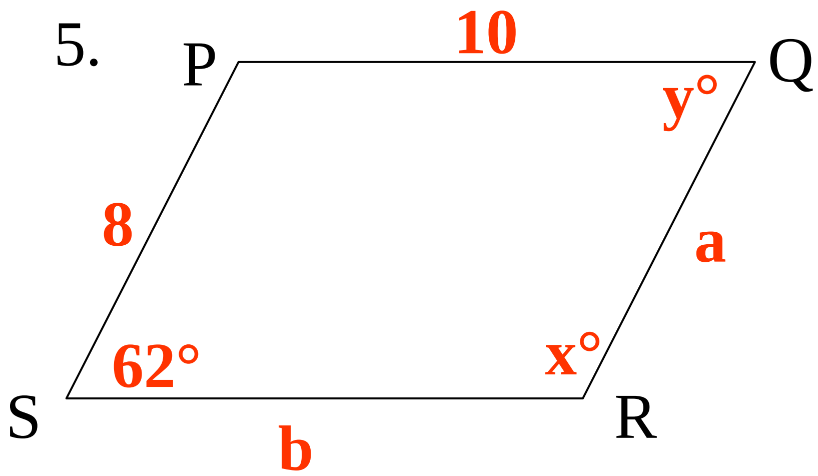
1. If OE = 4 and WE = 8, name two segments congruent to $\overline{WE}$
3. If $\overline{WR} \perp \overline{CE}$ name all segments congruent to $\overline{WE}$



Sample Problems Section 5-1

PQRS is a parallelogram. Find the values of a, b, x and y.

5.



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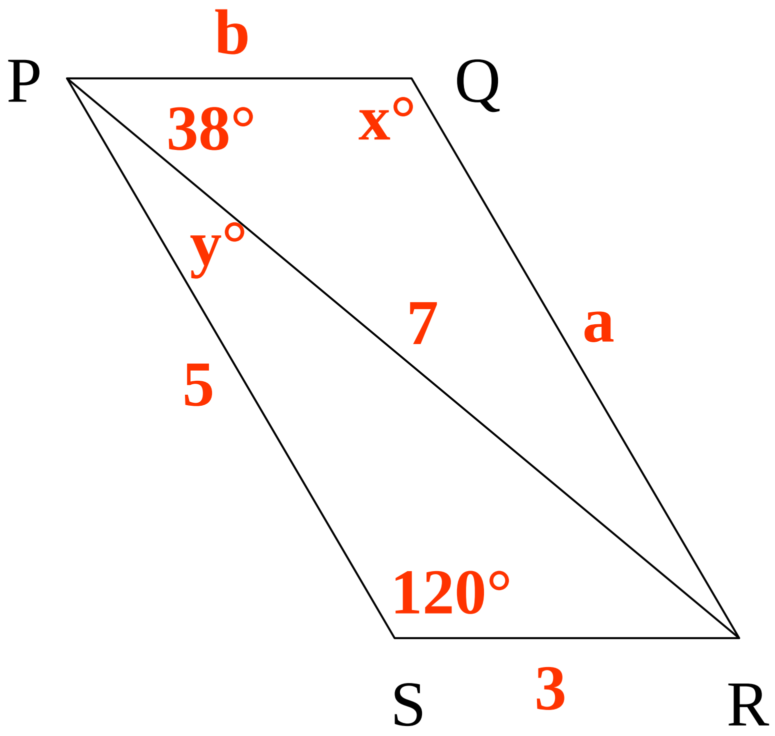

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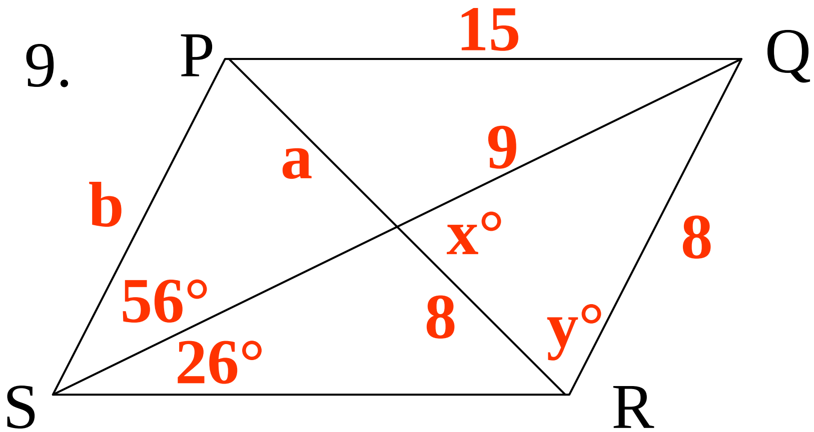


Sample Problems Section 5-1

11. Find the perimeter of parallelogram RISK if RI = 17 and IS = 13.
13. Prove Theorem 5-1
15. Prove Theorem 5-3

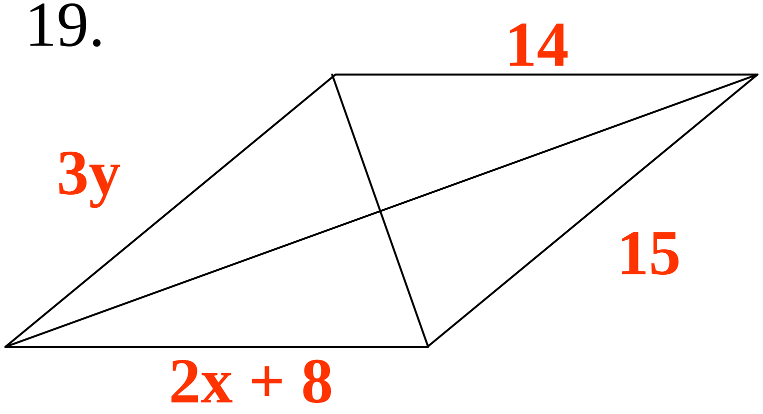
The coordinates of three vertices of parallelogram ABCD are given. Plot the points and find the coordinates of the fourth vertex.

17. A(1, 0) B(5, 0) C(7, 2) D(? , ?)

Sample Problems Section 5-1

Given these parallelograms find x and y.

19.



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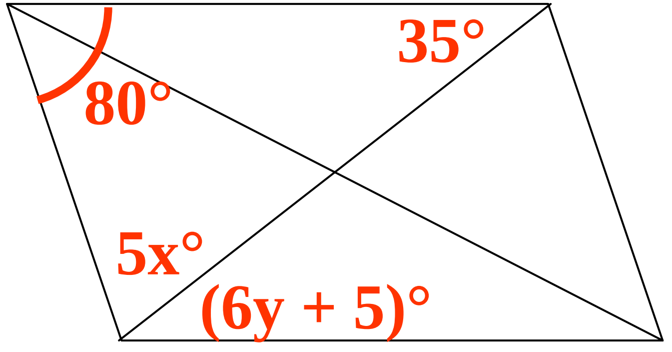

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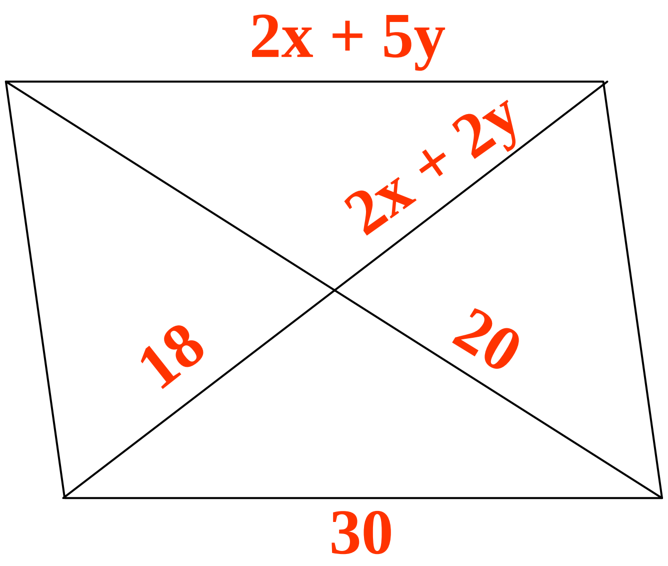
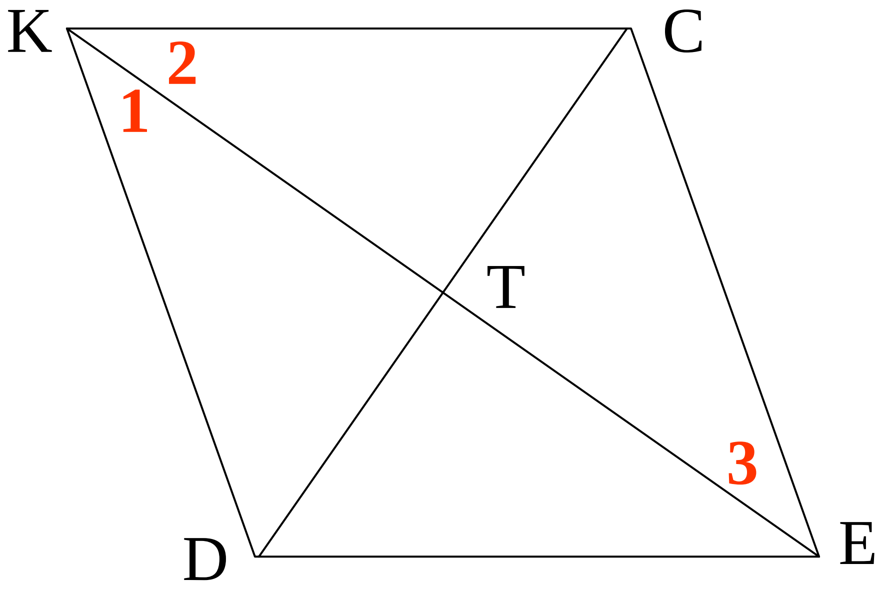


Sample Problems Section 5-1

DECK is a parallelogram.

25. If KT = 2x + y, DT = x + 2y, TE = 12 and TC = 9, then x = ? and y = ?

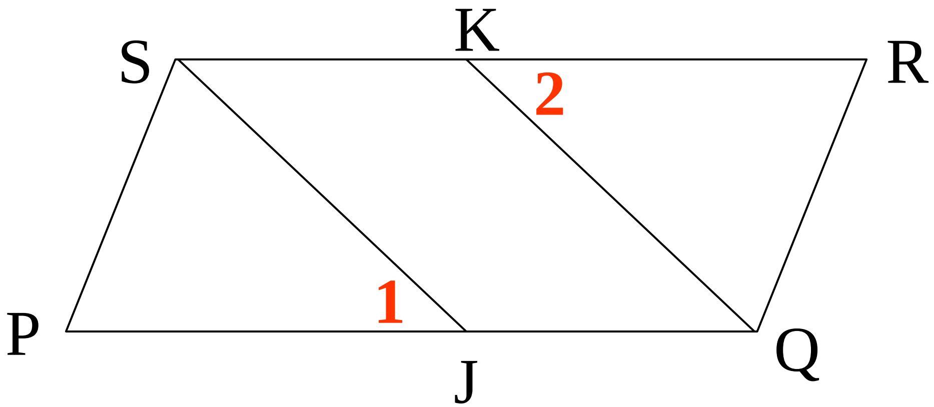
27. If m $\angle$ 1 = 3x, m $\angle$ 2 = 4x and m $\angle$ 3 = x^2 - 70, then x = ? and m $\angle$ CED = ?



Sample Problem Section 5-1

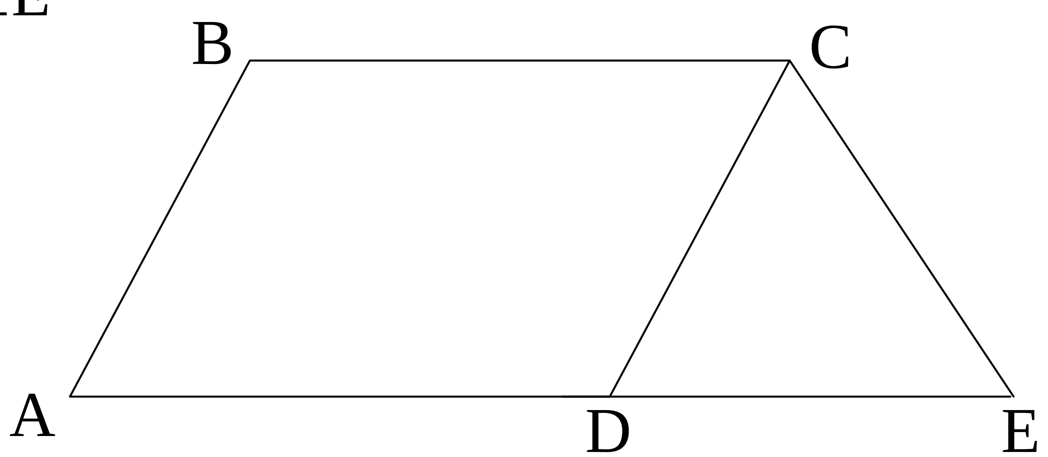
29. Given: parallelogram PQRS; PJ = RK

Prove: SJ = QK



31. Given: parallelogram ABCD; CD = CE

Prove: $\angle A \cong \angle E$



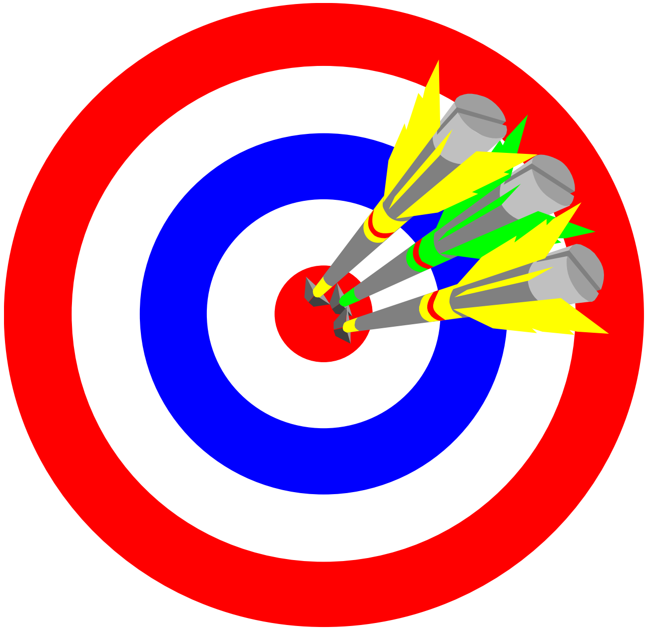
Section 5-2

Ways to Prove that Quadrilaterals are Parallelograms

Homework Pages 174-176:

2-24 evens, excluding 6, 8, 18, 22

Objectives



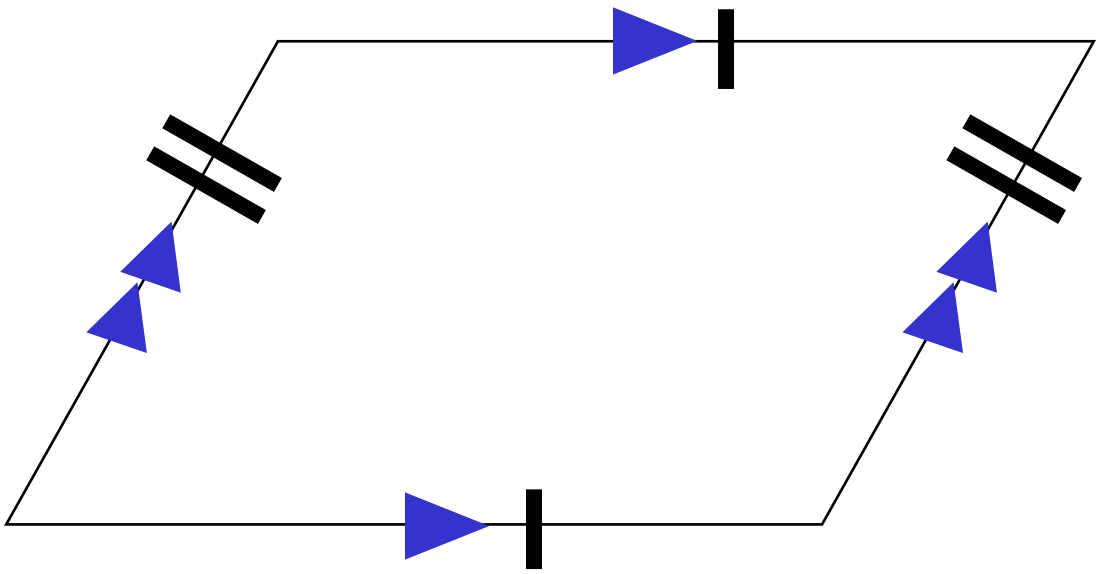
A. Understand and apply Theorems that prove quadrilaterals to be parallelograms.

★ Proving a Quadrilateral is a Parallelogram

- Show that both pairs of opposite sides are parallel (definition).
- Show that both pairs of opposite sides are congruent (theorem 5-4).
- Show that one pair of opposite sides is both congruent and parallel (theorem 5-5).
- Show that both pairs of opposite angles are congruent (theorem 5-6).
- Show that the diagonals bisect each other (theorem 5-7).
- Which of these can you derive from the previous day’s activities?

Theorem 5-4

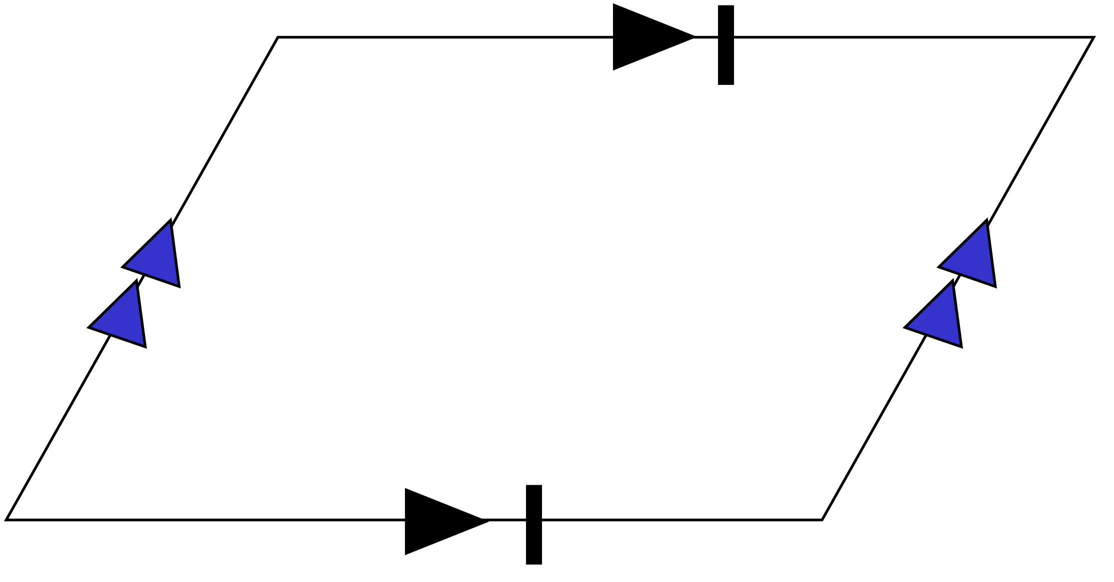
If both pairs of opposite sides are congruent,



then the quadrilateral is a parallelogram.

Theorem 5-5

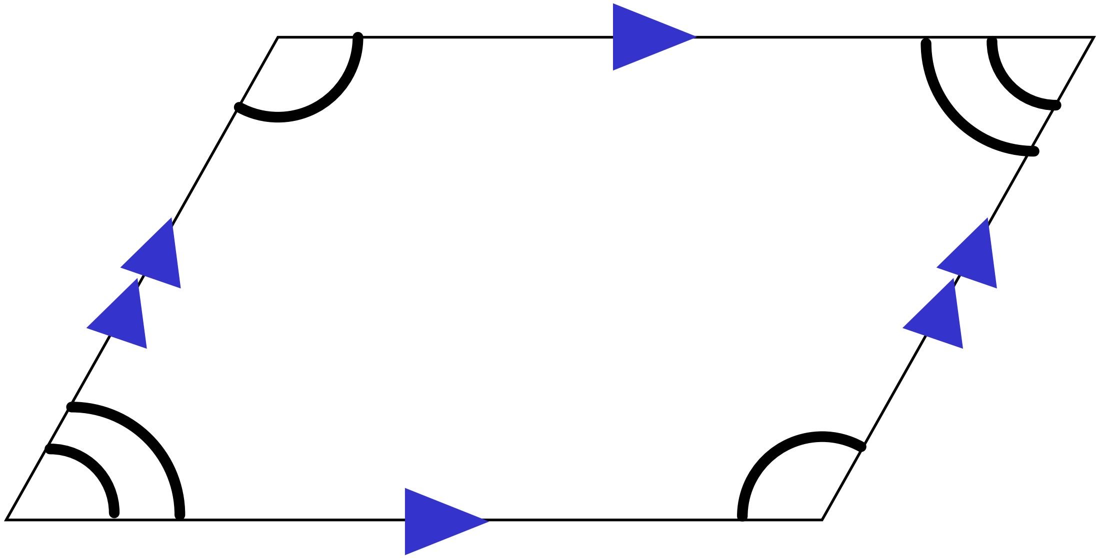
If one pair of opposite sides of a quadrilateral are both congruent and parallel,



then the quadrilateral is a parallelogram.

Theorem 5-6

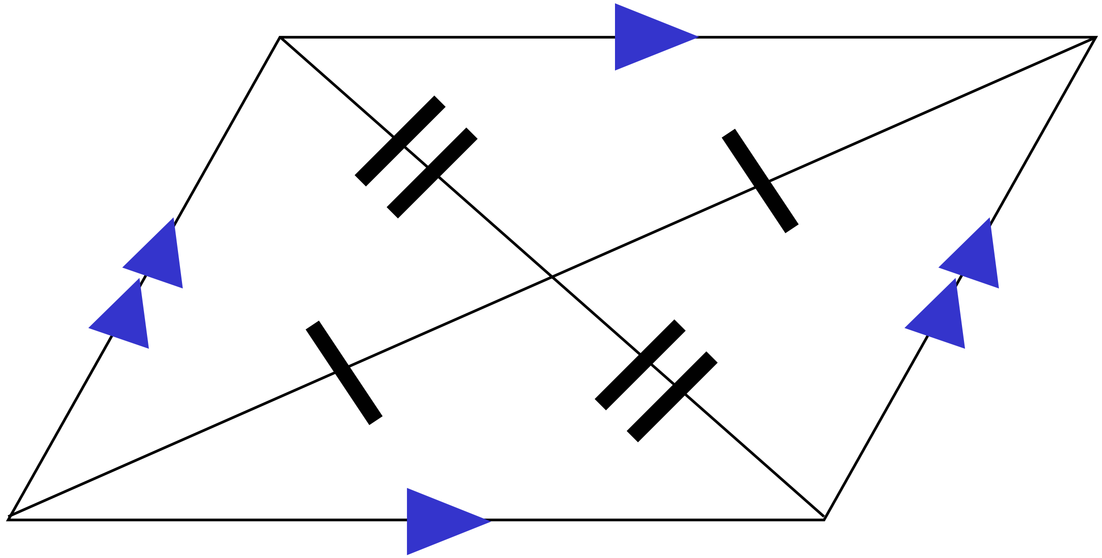
If both pairs of opposite angles of a quadrilateral are congruent,



then the quadrilateral is a parallelogram.

Theorem 5-7

If the diagonals of a quadrilateral bisect each other,

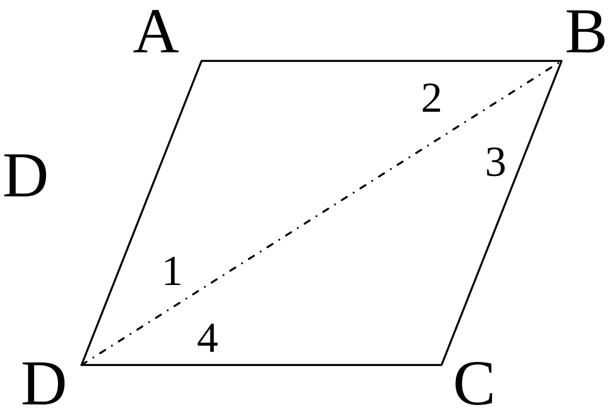


then the quadrilateral is a parallelogram.

Sample Proof – Theorem 5-4 (Exercise 11)

Given: $\overline{AB} \cong \overline{DC}; \overline{AD} \cong \overline{BC}$

Prove: quadrilateral ABCD is a parallelogram.

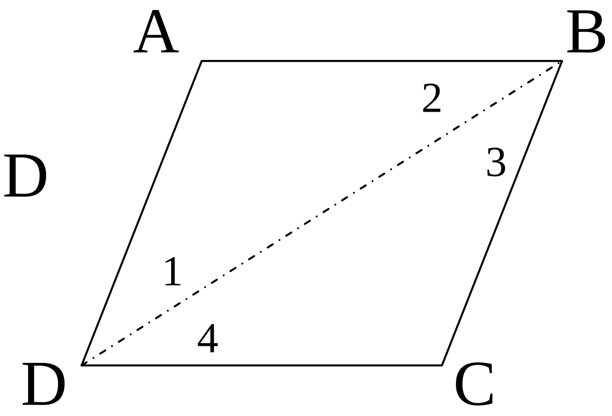


1. $\overline{AB} \cong \overline{DC}; \overline{AD} \cong \overline{BC}$	1. Given
2. $\overline{DB} \cong \overline{DB}$	2. Reflexive Property of Congruence.
3. $\Delta ABD \cong \Delta CDB$	3. SSS Postulate
4. $\angle ABD \cong \angle CDB; \angle ADB \cong \angle CBD$	4. CPCT

Sample Proof – Theorem 5-4 (Exercise 11)

Given: $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$

Prove: quadrilateral ABCD is a parallelogram.

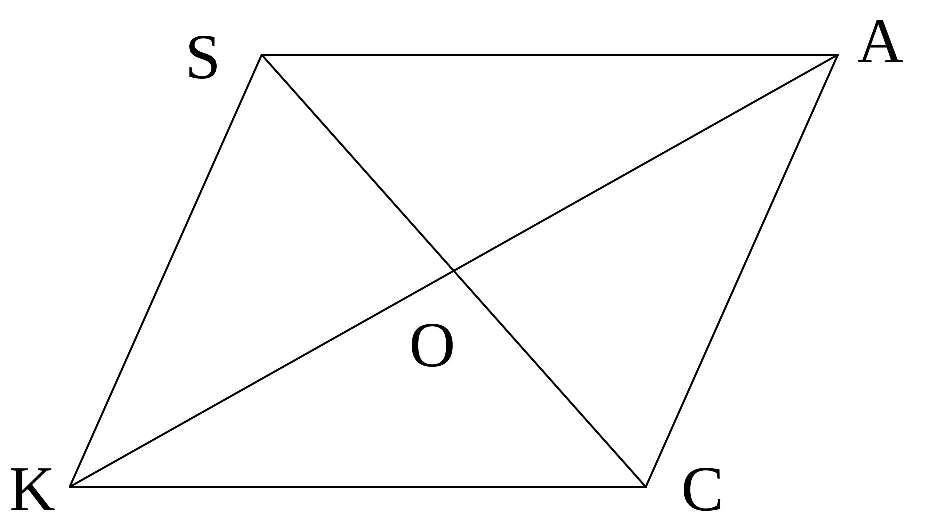


5. $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$	5. If 2 lines are cut by transversal and alt. int. are congruent, lines are parallel.
6. Quadrilateral ABCD is a parallelogram.	6. Definition of parallelogram.

Sample Problems Section 5-2

State the principal definition or theorem that enables you to deduce, from the information given, that SACK is a parallelogram.

1. $\overline{SA} \parallel \overline{KC}$ $\quad \overline{SK} \parallel \overline{AC}$
3. $\overline{SA} \parallel \overline{KC}$ $\quad \overline{SA} \cong \overline{KC}$
5. $\angle SKC \cong \angle CAS$; $\angle KCA \cong \angle ASK$



Sample Problems Section 5-2

9. Which theorem is the converse of:

a. Theorem 5-1

b. Theorem 5-2

c. Theorem 5-3

Draw and label a diagram. List what is given and what is to be proved. Then write a two-column proof.

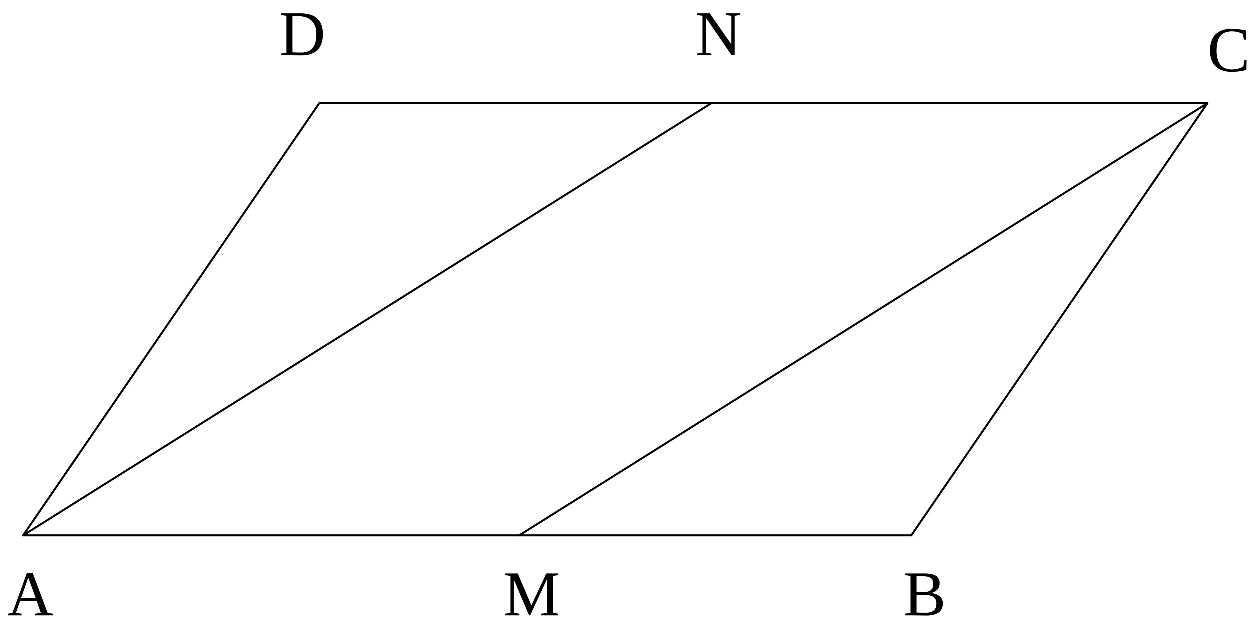
11. Theorem 5-4

13. Theorem 5-7

Sample Problems Section 5-2

15. Given: parallelogram ABCD; AN bisects ∠ DAB;
CM bisects ∠ BCD

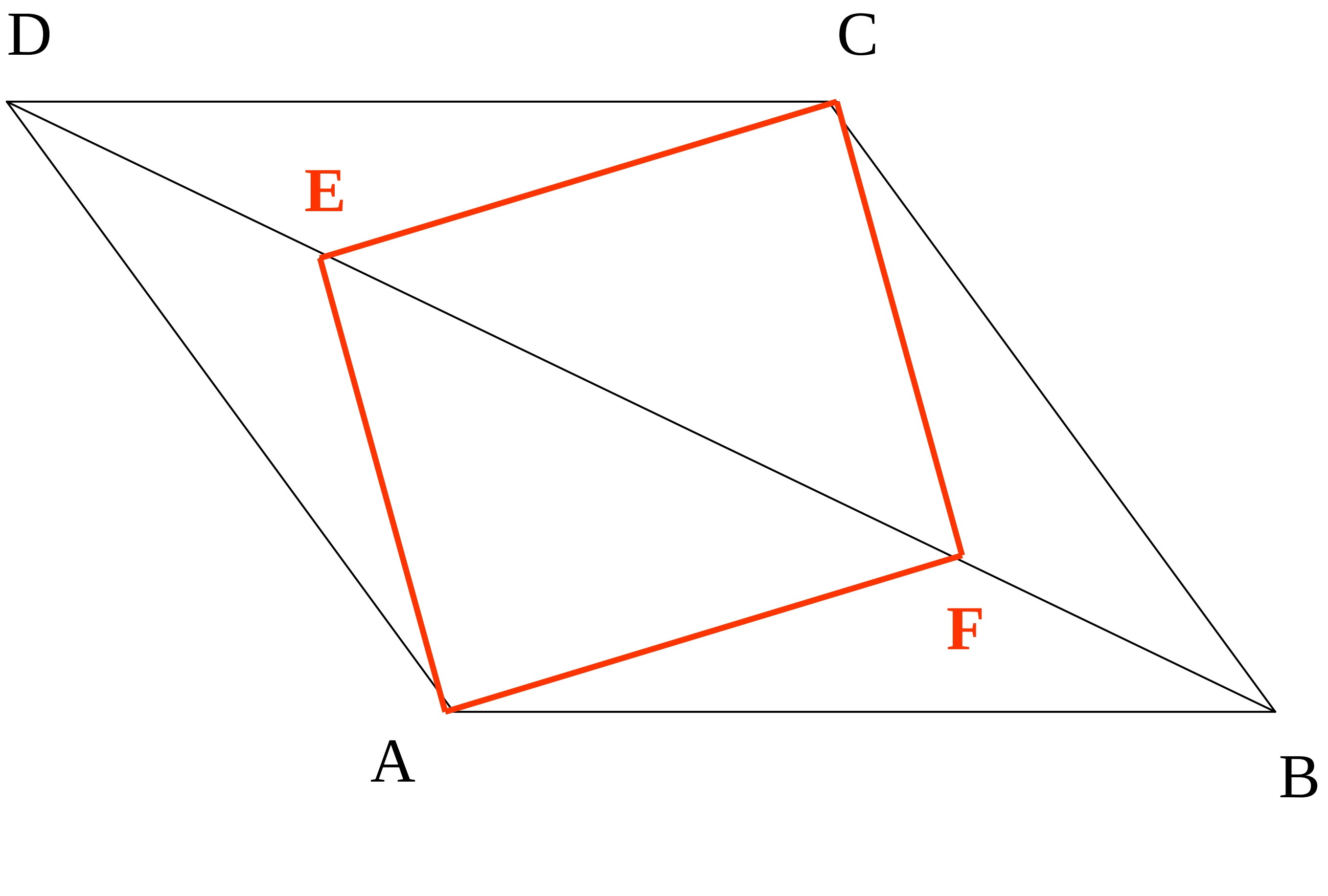
Prove: AMCN is a parallelogram



Sample Problems Section 5-2

17. Given: parallelogram ABCD; DE = BF

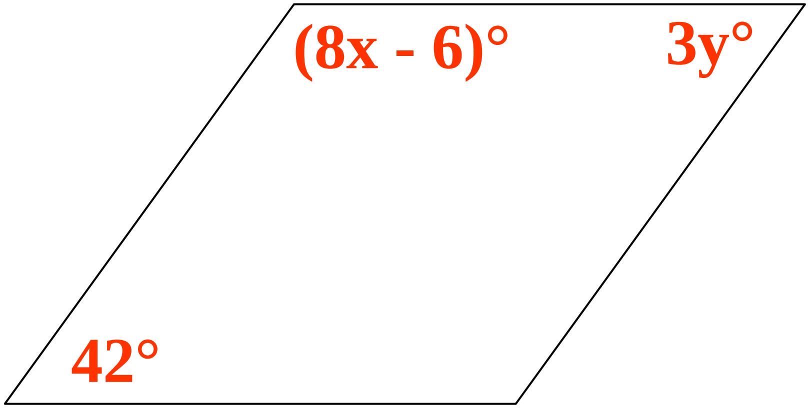
Prove: AFCE is a parallelogram



Sample Problems Section 5-2

What values must x and y have to make the quadrilateral a parallelogram?

19.



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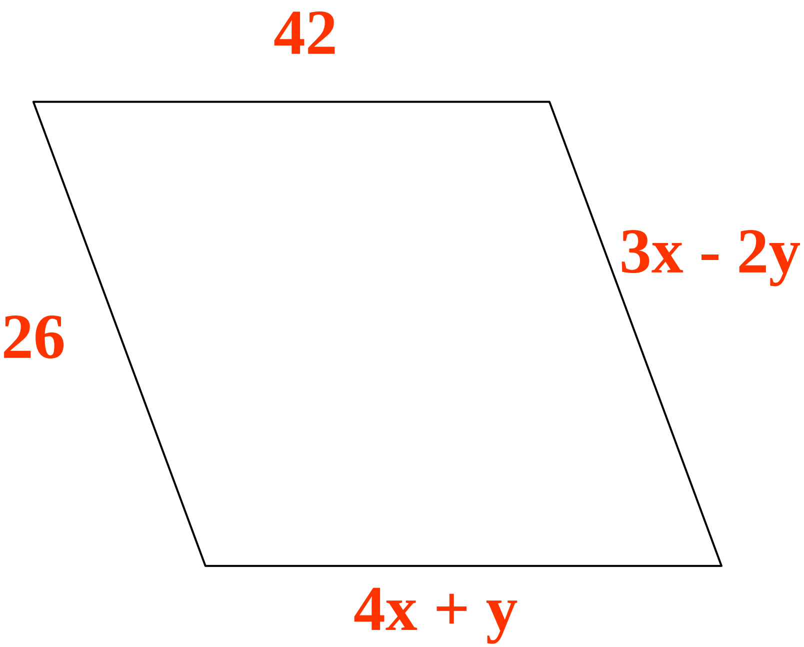
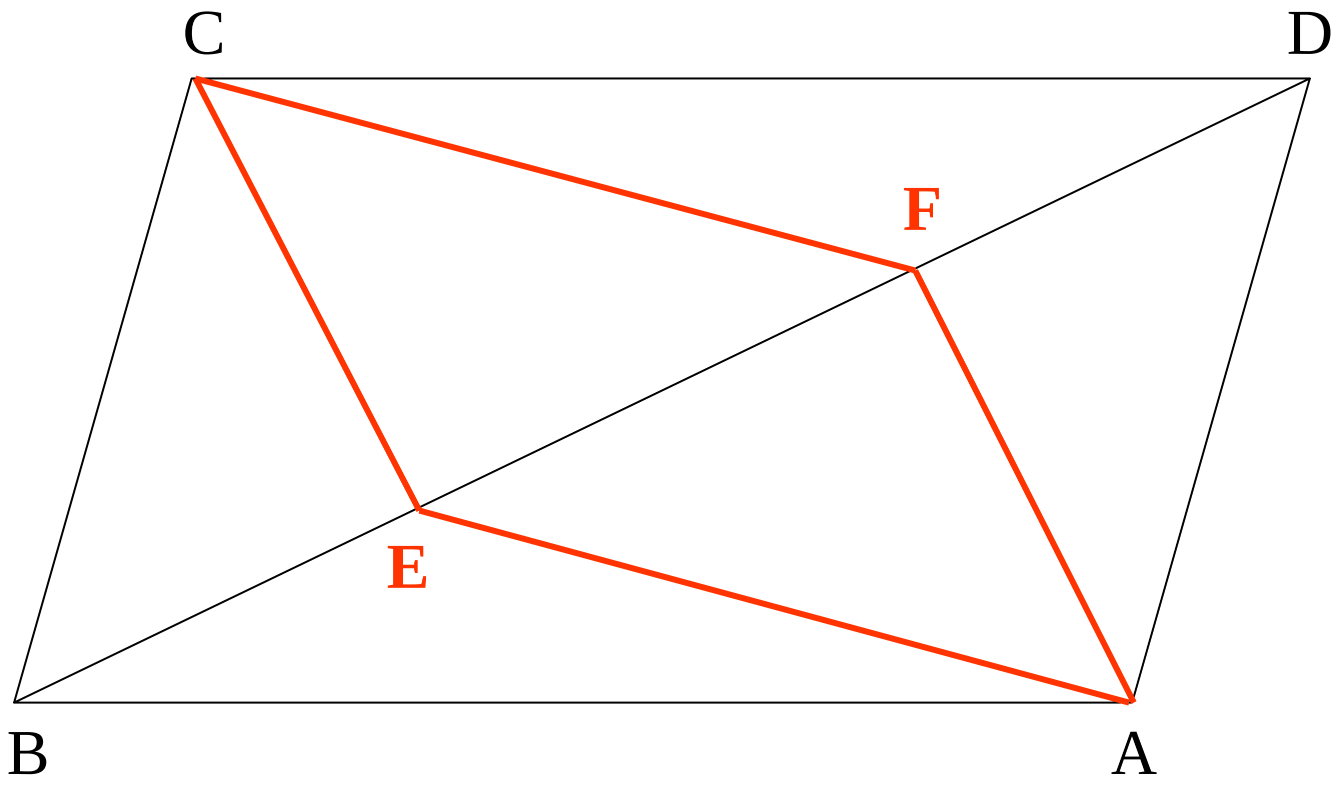


Sample Problems Section 5-2

23. Given: parallelogram ABCD; $\overline{DE} \perp \overline{AC}$; $\overline{BF} \perp \overline{AC}$

Prove: DEBF is a parallelogram



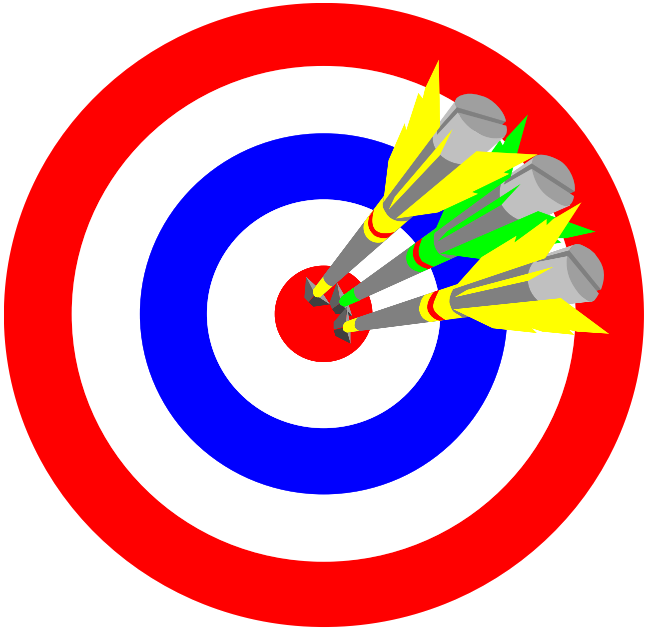
Section 5-3

Theorems Involving Parallel Lines

Homework Pages 180-181:

2-20 evens

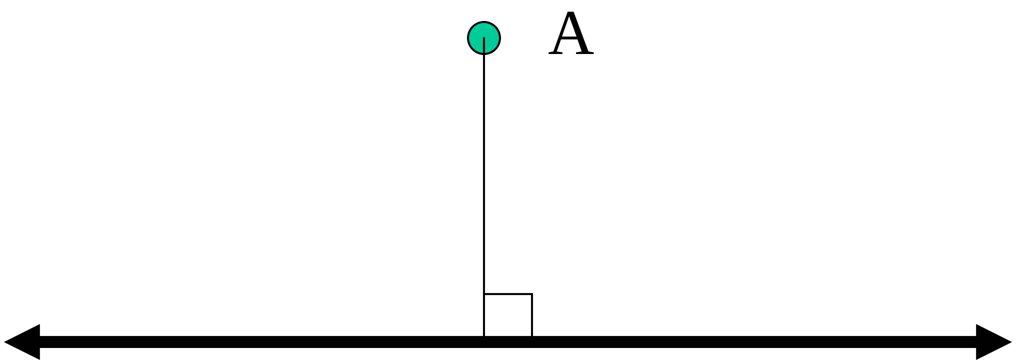
Objectives



A. Understand an apply theorems that relate parallel lines and components of various figures.

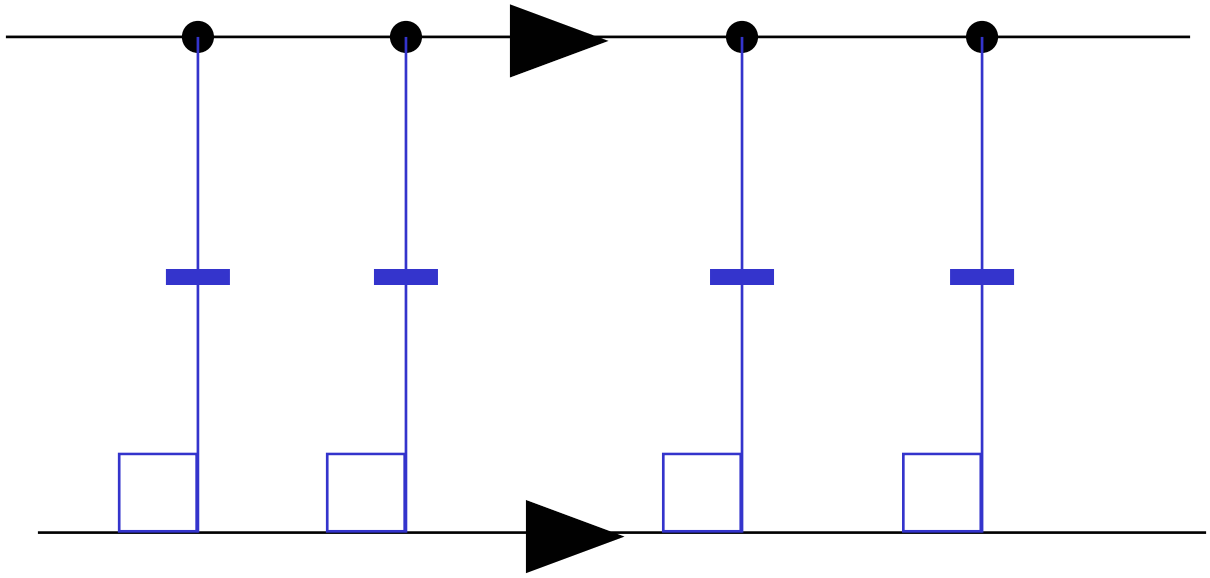
Remember …

The distance between a point A and a line *l* is determined by constructing the line segment perpendicular to line *l* and through point A and measuring the distance from the point to the line along the line segment.



Theorem 5-8

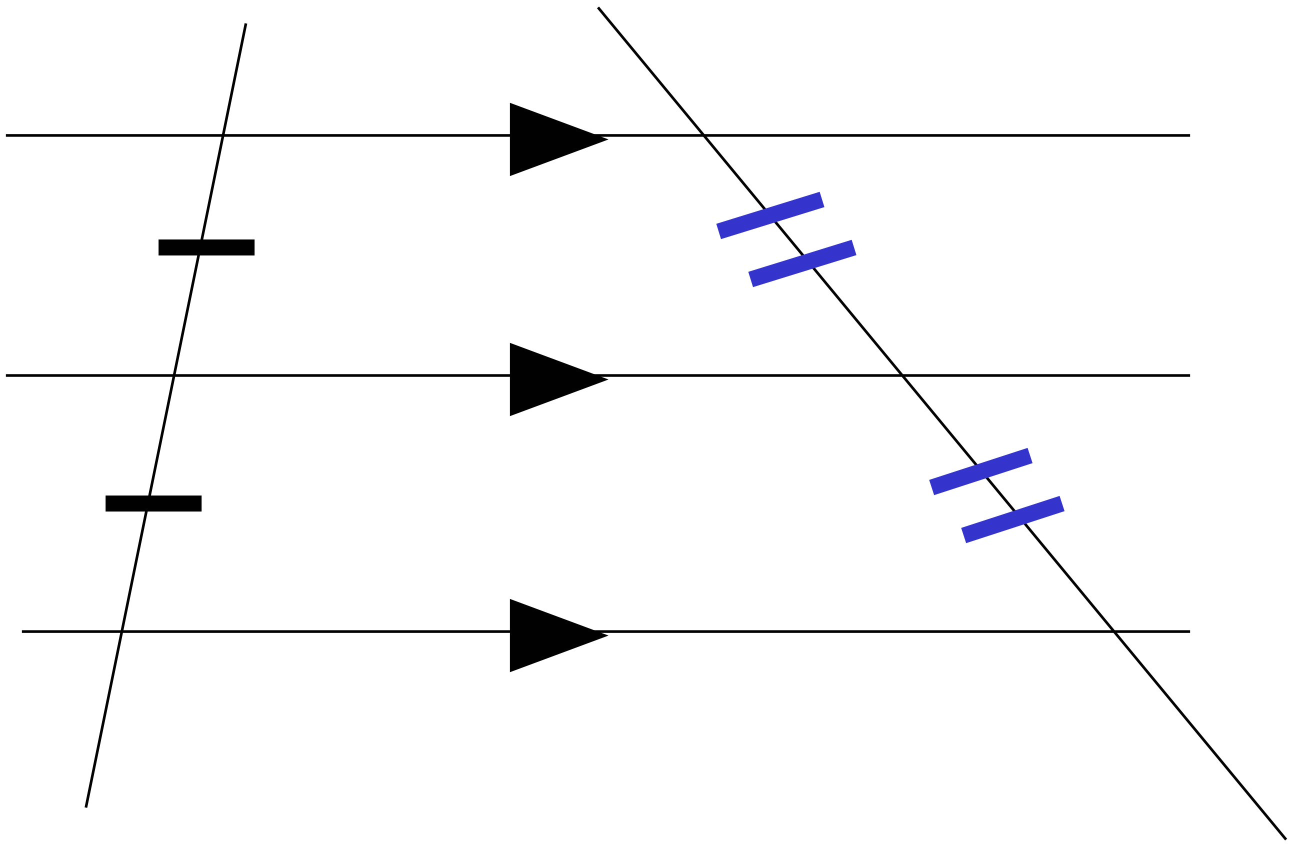
If two lines are parallel,



then all the points on one line are equidistant from the other line.

★ Theorem 5-9

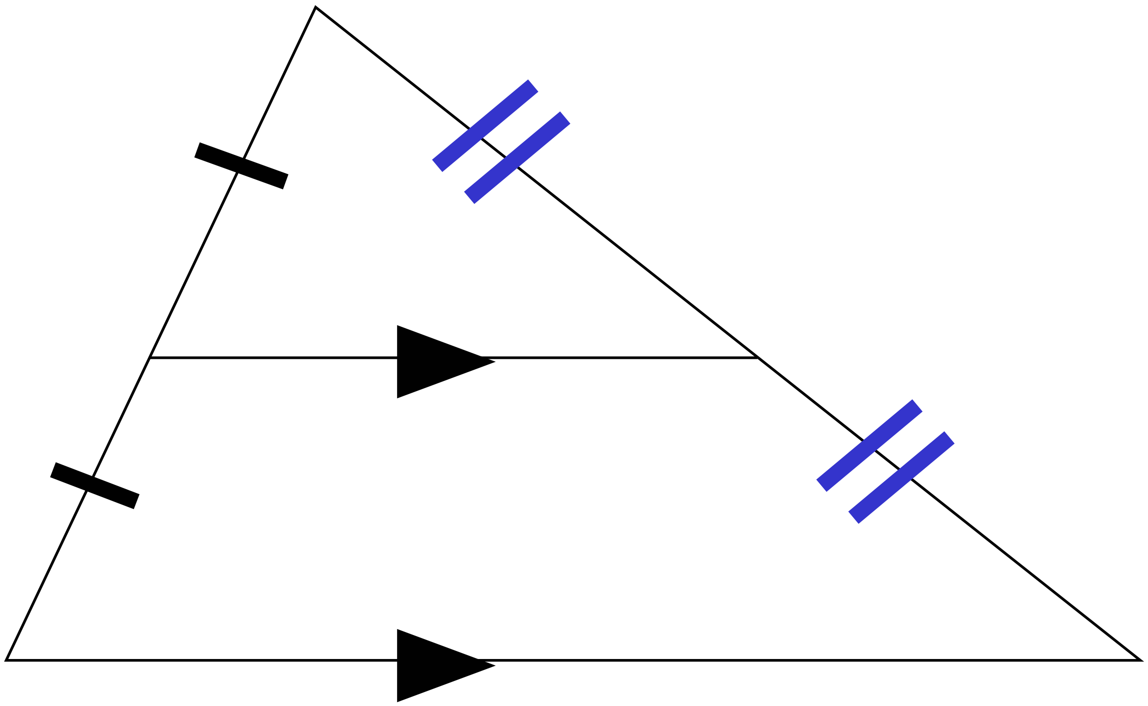
If three parallel lines cut off congruent segments on one transversal,



then they cut off congruent segments on every transversal.

★ Theorem 5-10

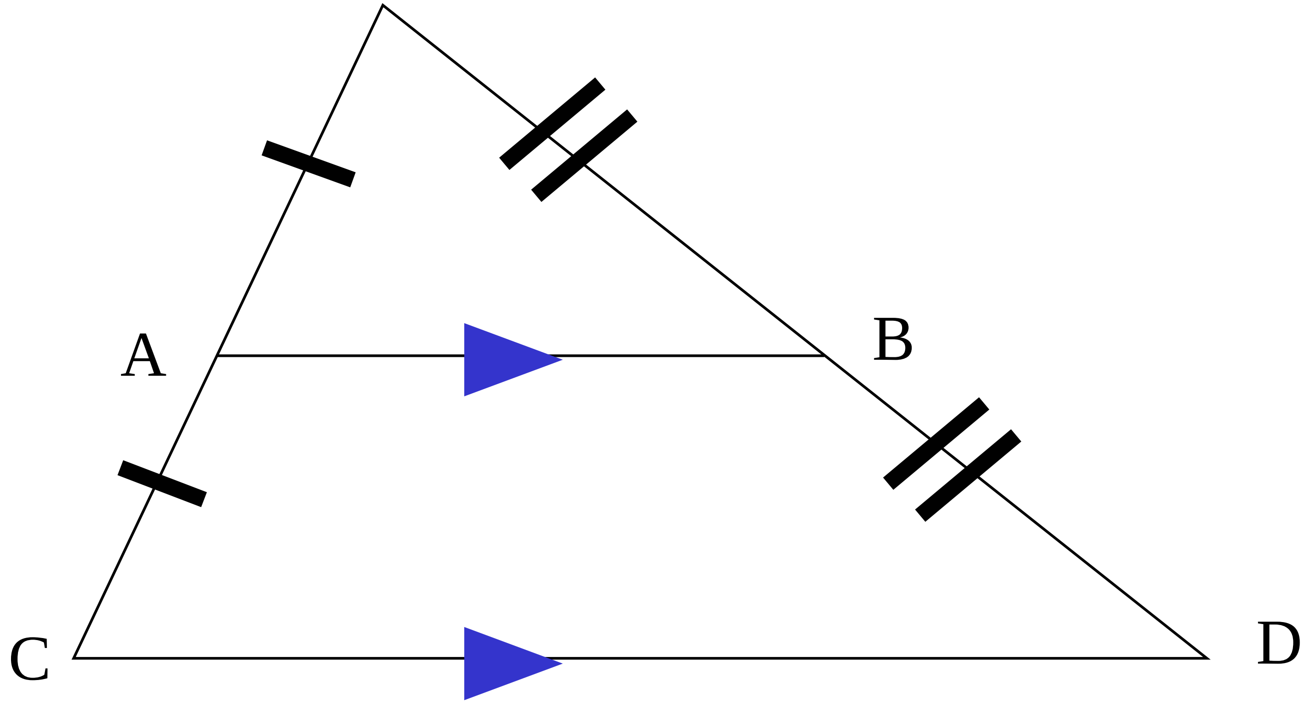
A line that passes through the midpoint of one side of a triangle and is parallel to another side



passes through the midpoint of the third side of the triangle.

★ Theorem 5-11

The segment that joins the midpoints of two sides of a triangle

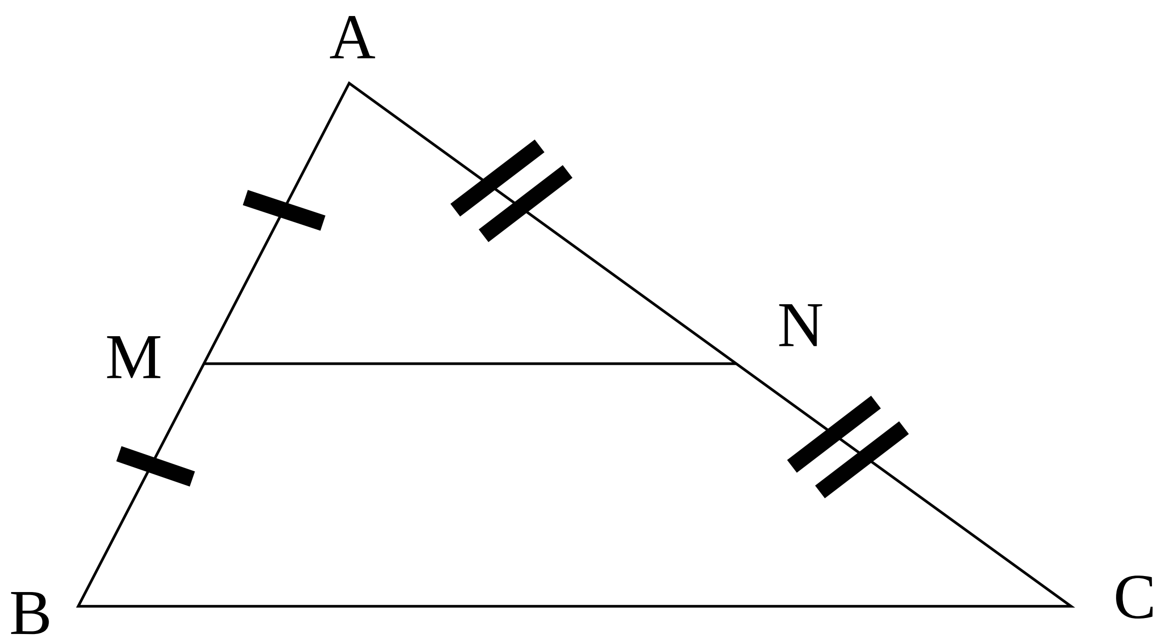


is parallel to the third side.

is half as long as the third side. $AB = \frac{1}{2} CD$

★ Proving Theorem 5-11

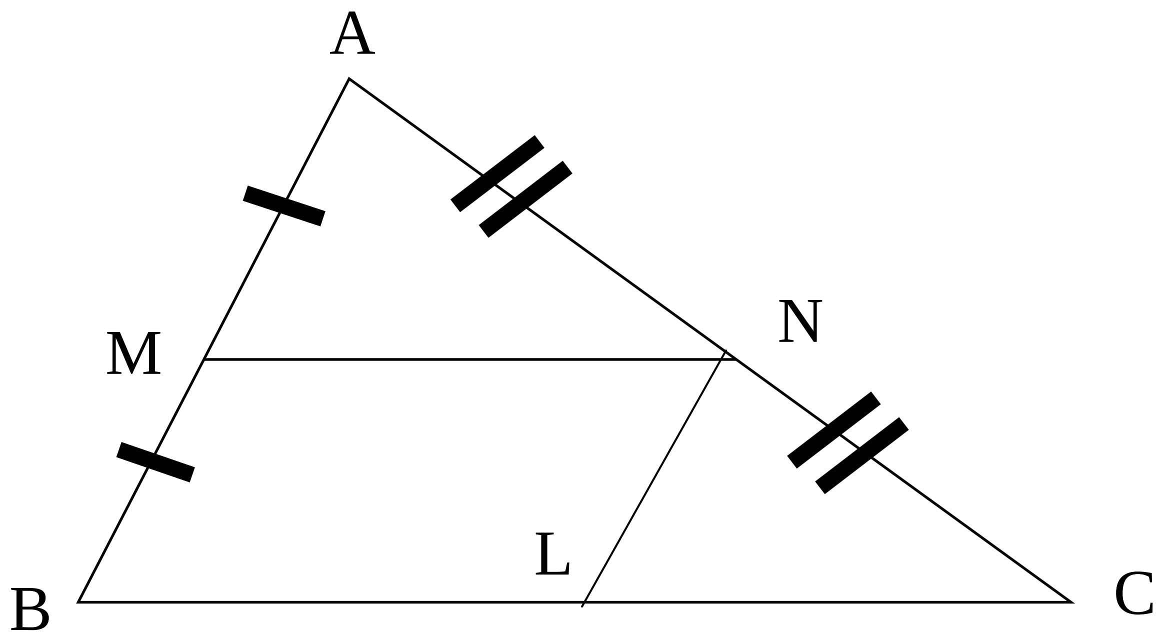
The segment that joins the midpoints of two sides of a triangle is parallel to the third side.



There is exactly one line through M parallel to line segment BC. It must pass through the midpoint N of line segment AC by Theorem 5-10. Therefore $\overline{MN} \parallel \overline{BC}$

★ Proving Theorem 5-11

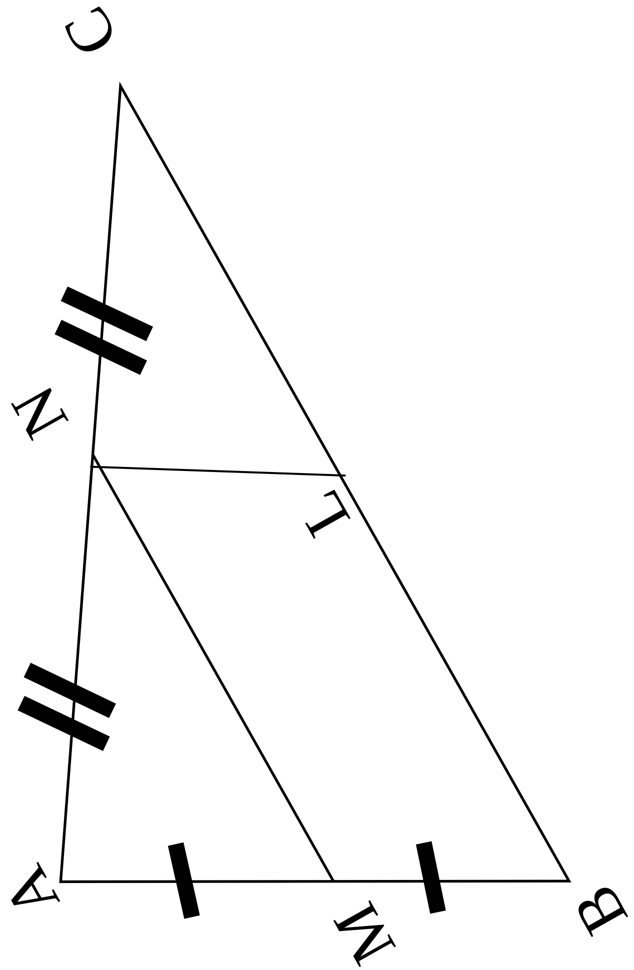
The segment that joins the midpoints of two sides of a triangle is half as long as the third side.



The trick is to construct line segment LN such that L is the midpoint of BC. Any ideas from here?

★ Proving Theorem 5-11

Does rotating the figure give you any ideas?
Apply Theorem 5-10 again and then look at parallelogram.

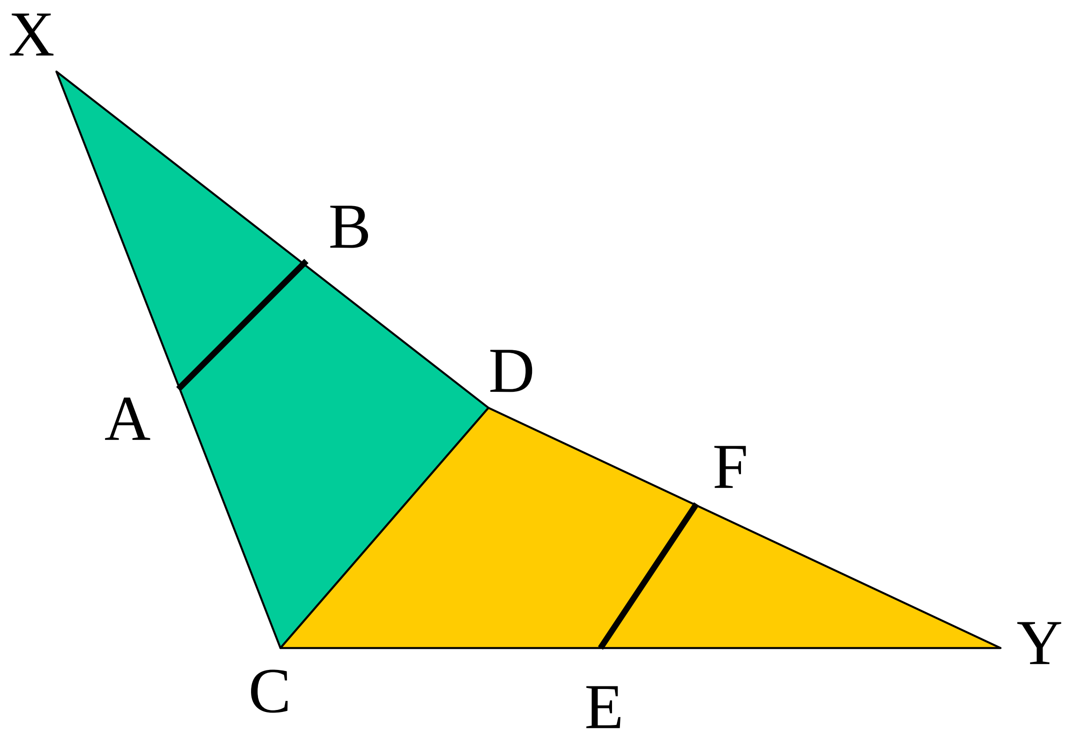


Sample Problems Section 5-3

Points A, B, E and F are the midpoints of $\overline{XC}$, $\overline{XD}$, $\overline{YC}$ and $\overline{YD}$.

1. If CD = 24 then AB = ? and EF = ?

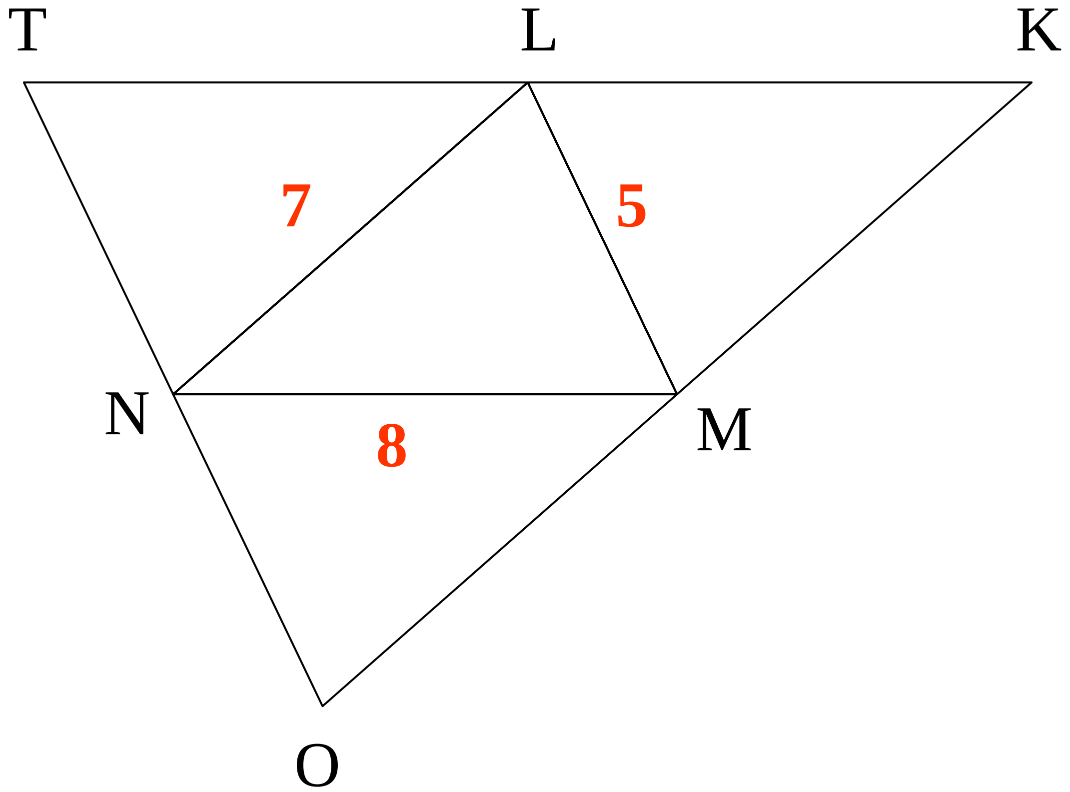
3. If AB = 5x - 8 and EF = 3x, then x = ?



Sample Problems Section 5-3

5. Given L, M, and N are midpoints of the sides of Δ TKO. Find the perimeter of each figure.

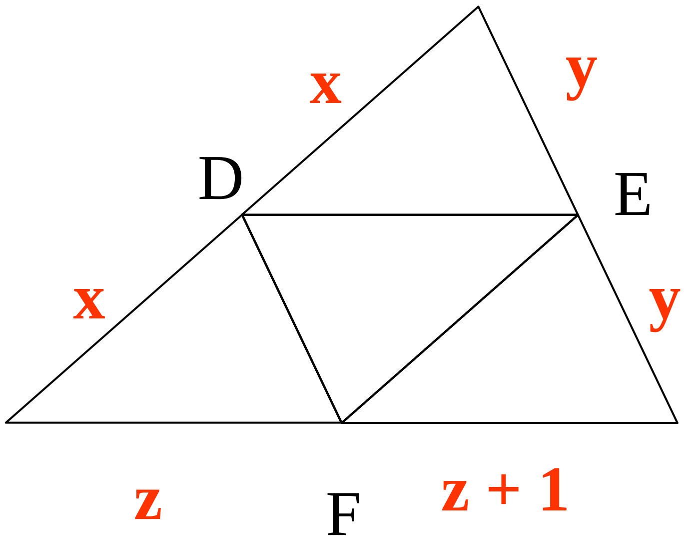
   a. Δ TKO b. Δ LMK c. TNML d. LNOK



Sample Problems Section 5-3

Name all of the points shown that must be midpoints of the sides of the large triangle.

7.



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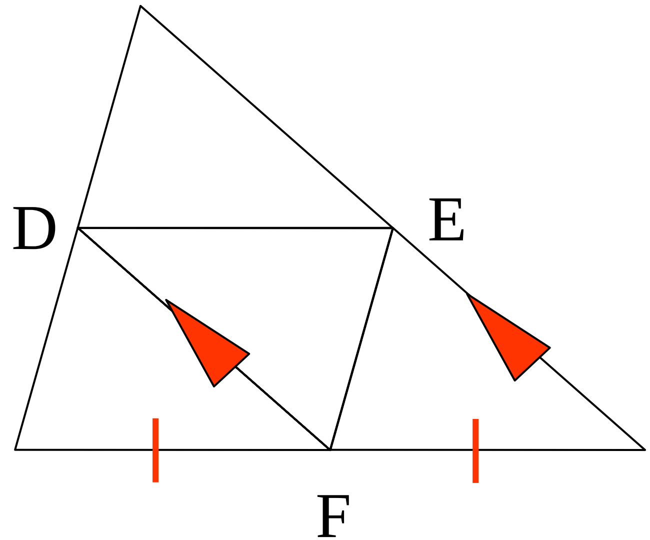
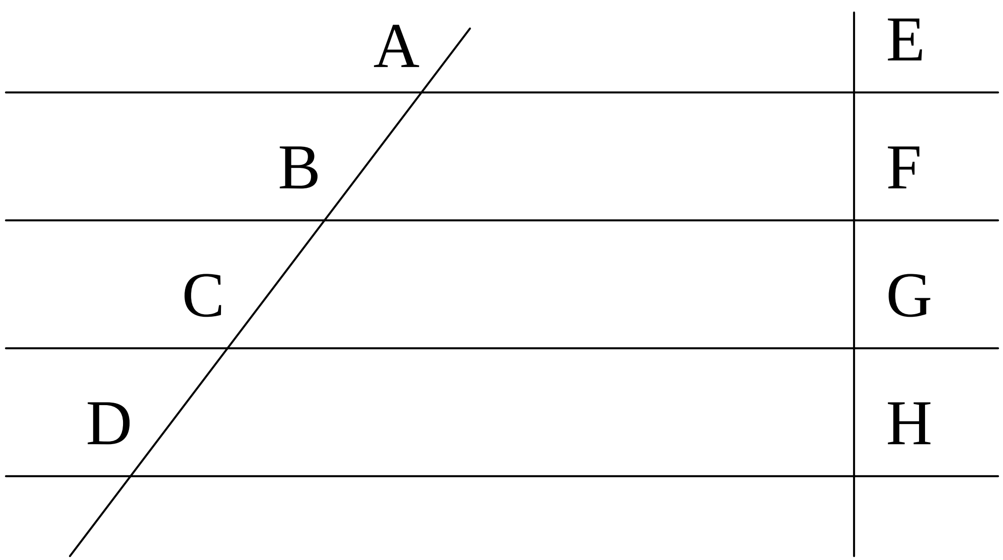


Sample Problems Section 5-3

$\overleftrightarrow{AE}$, $\overleftrightarrow{BF}$, $\overleftrightarrow{CG}$, and $\overleftrightarrow{DH}$ are parallel, with EF = FG = GH. Complete.

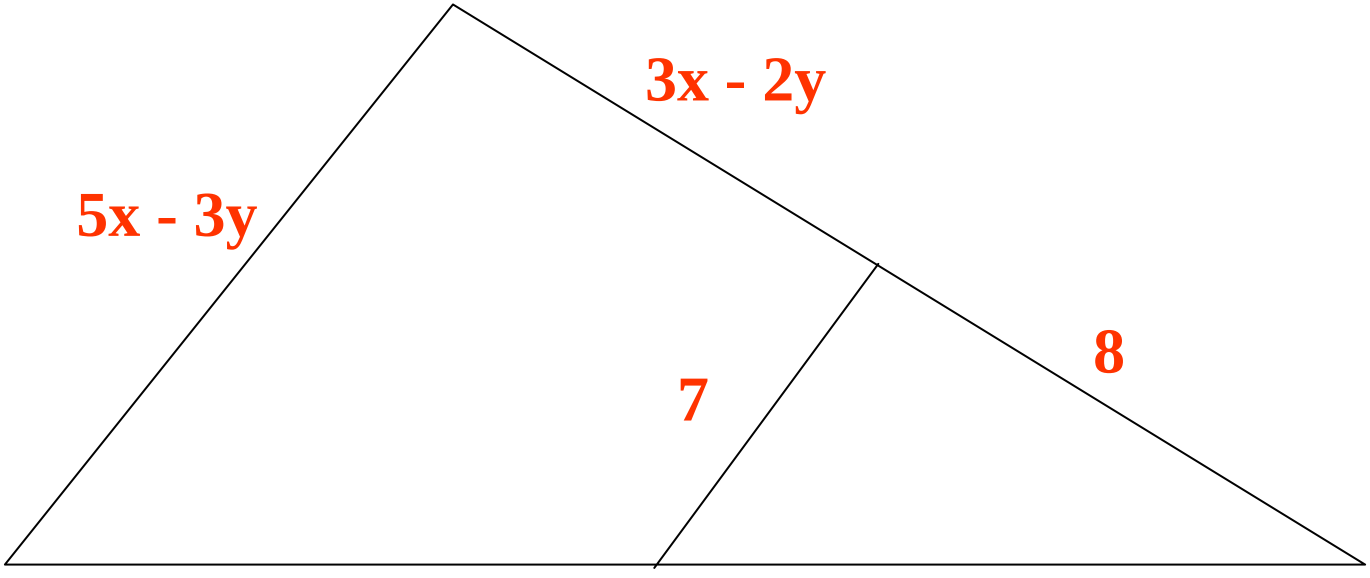
11. If AC = 12, then CD = ?
13. If AC = 22 - x and BD = 3x - 22, then x = ?
15. If AB = 12, BC = 2x + 3y, and BD = 8x, then x = ? and y = ?



Sample Problems Section 5-3

The segment joins the midpoints of the sides of the triangle.
Find x and y.

17.

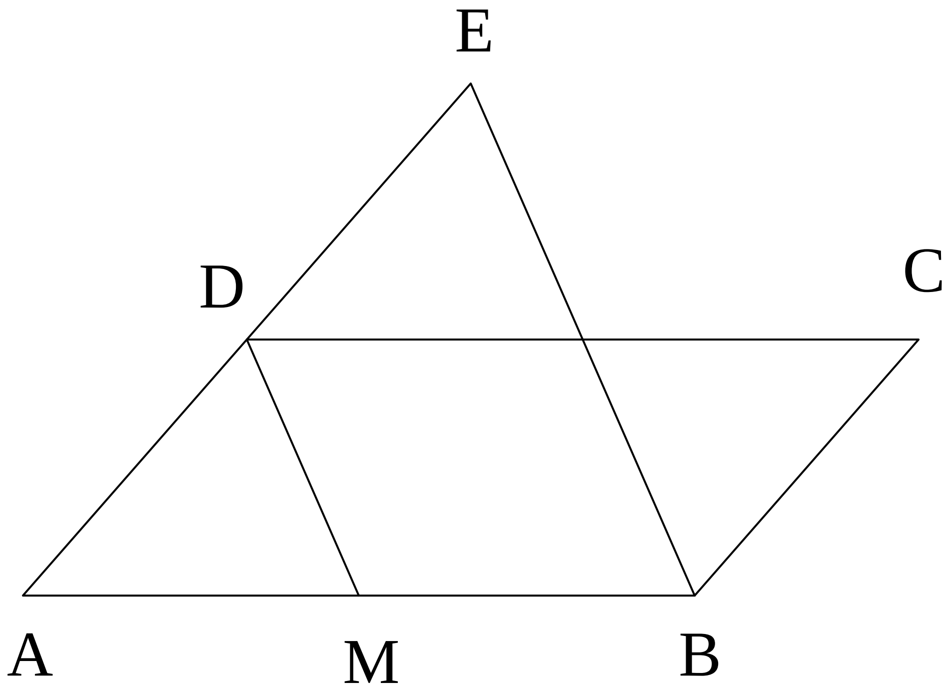


Sample Problems Section 5-3

19. Given: parallelogram ABCD; M is the midpoint of $\overline{AB}$

$\overline{BE} \parallel \overline{MD}$

Prove: DE = BC



Sample Problems Section 5-3

21. EFGH is a parallelogram whose diagonals intersect at P. M is the midpoint of segment FG. Prove MP = ½EF.
23. Draw Δ ABC and let D be the midpoint of AB. Let E be the midpoint of CD. Let F be the intersection of ray AE and BC Draw DG parallel to EF meeting BC at G. Prove BG = GF = FC.

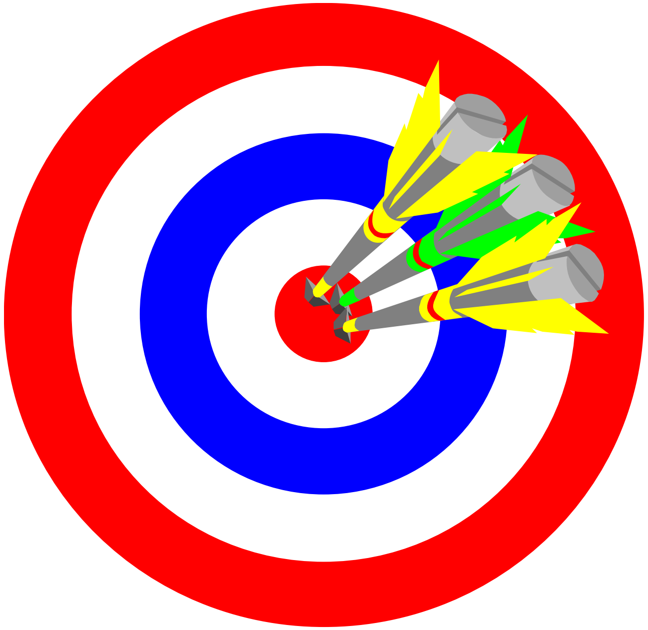
Section 5-4

Special Parallelograms

Homework Pages 187-188:

2-32 evens, excluding 22

Objectives



A. Apply the definitions of rectangle, rhombus, and square.

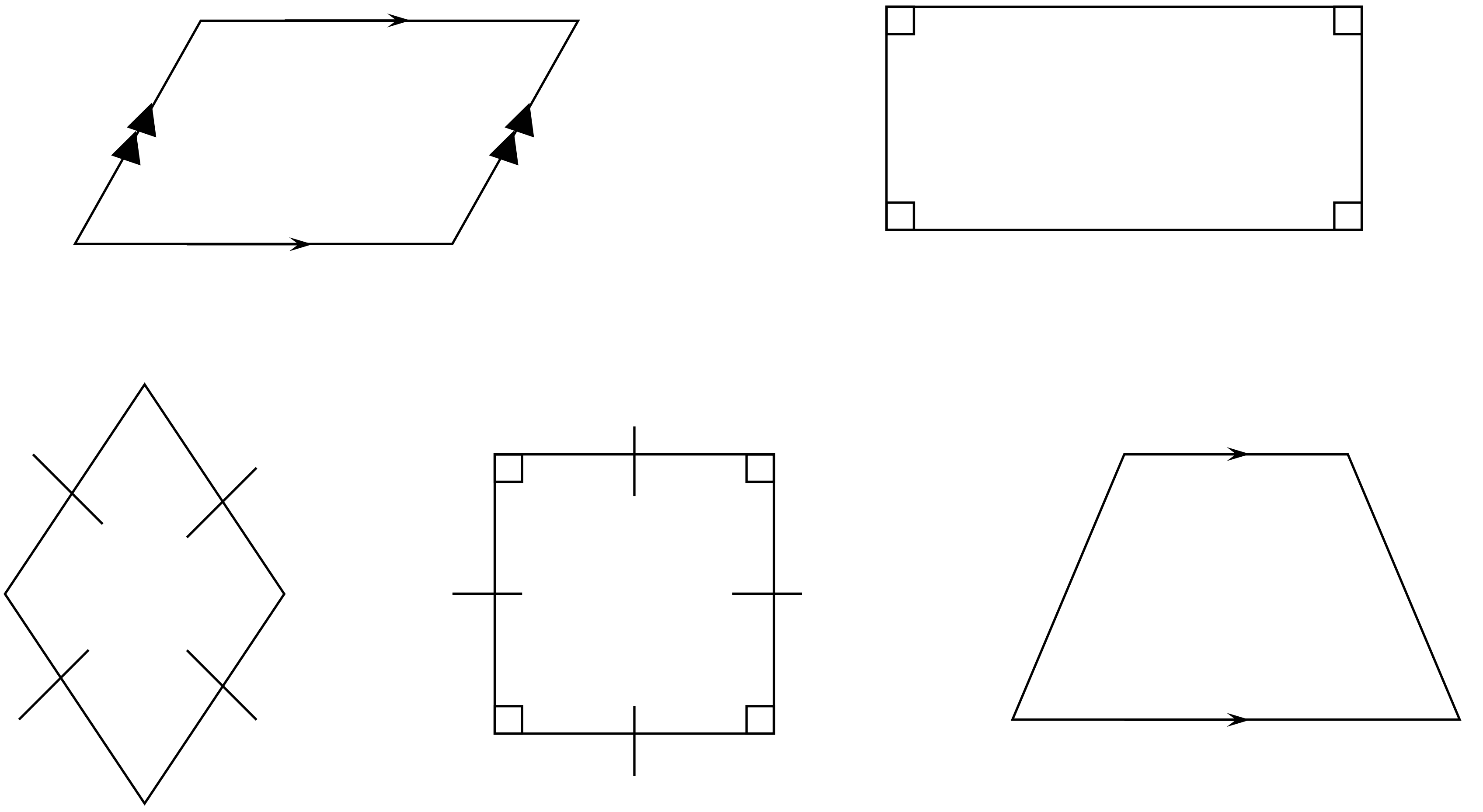
B. Identify the special properties of rectangles, rhombi, and squares.

C. Determine when a parallelogram is a rhombus, square or rectangle.

D. Understand and apply the theorems associated with rectangles, rhombi, and squares.

- ★ rectangle: is a quadrilateral with four right angles
- ★ rhombus: is a quadrilateral with four congruent sides
- ★ square: is a quadrilateral with four right angles and four congruent sides

Types of Quadrilaterals

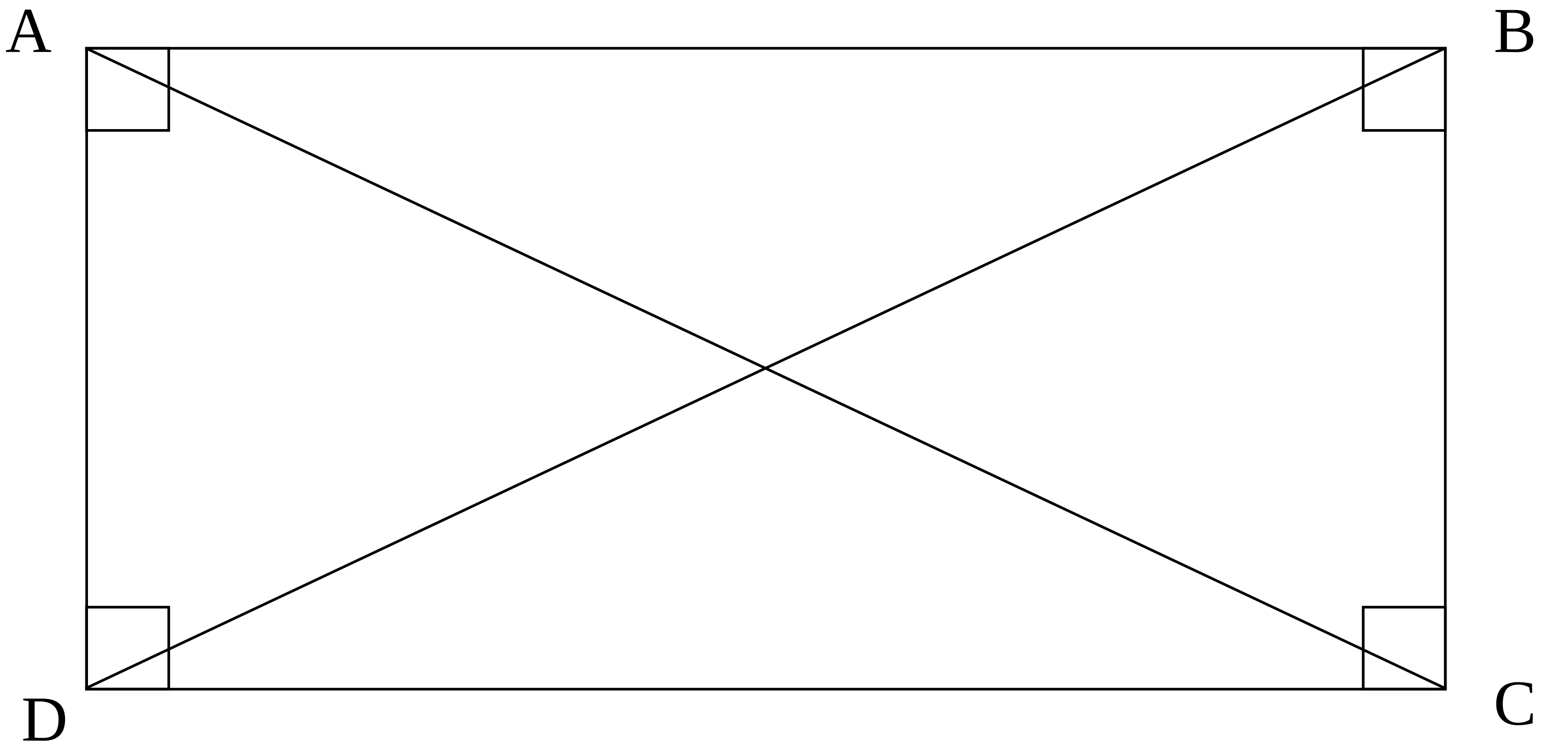


Questions you should be able to answer

- Is every square a rectangle?
- Is every rectangle a square?
- Is every rhombus a square?
- Is every square a rhombus?
- Is every rectangle a rhombus?
- Is every rhombus a rectangle?
- Is every rhombus/square/rectangle a parallelogram?
- Is every parallelogram a square or rectangle or rhombus?

★ Theorem 5-12

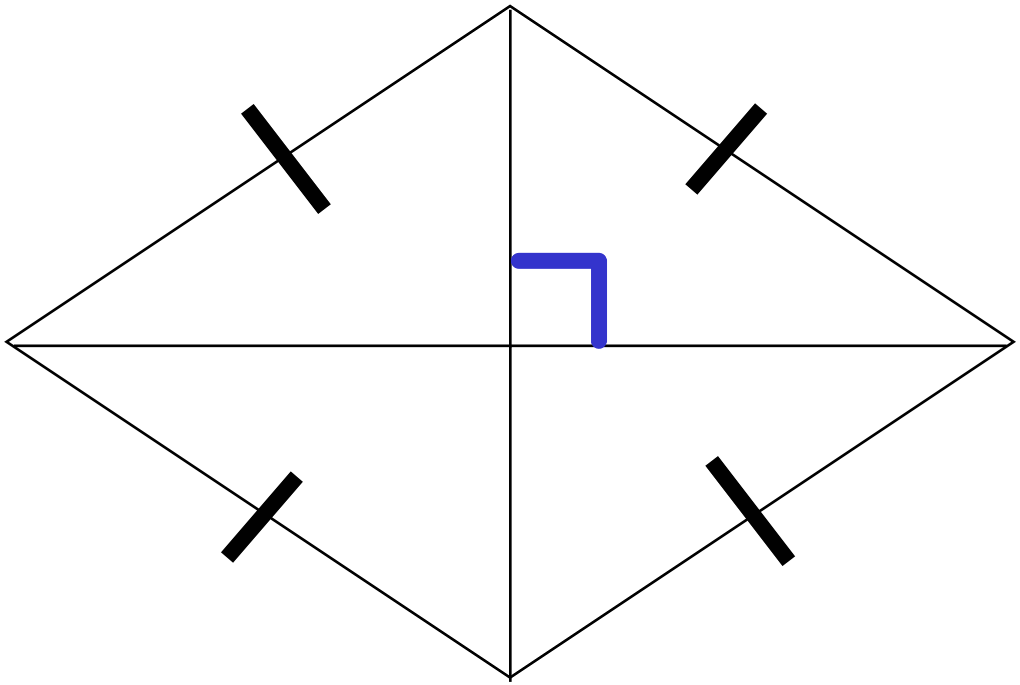
The diagonals of a rectangle



are congruent. AC = BD

★ Theorem 5-13

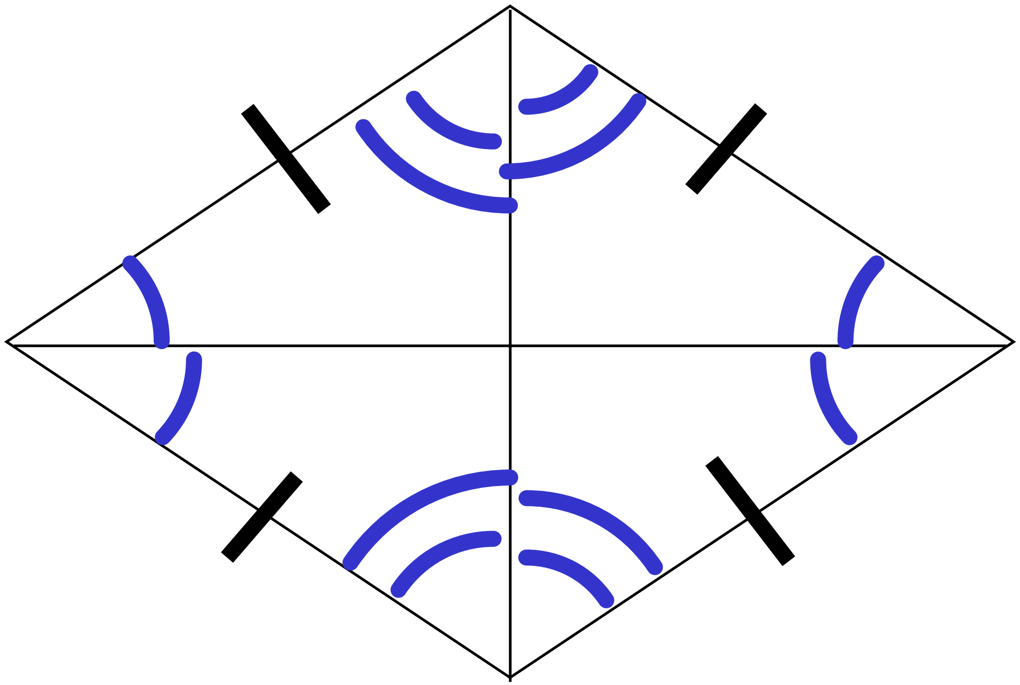
The diagonals of a rhombus



are perpendicular.

★ Theorem 5-14

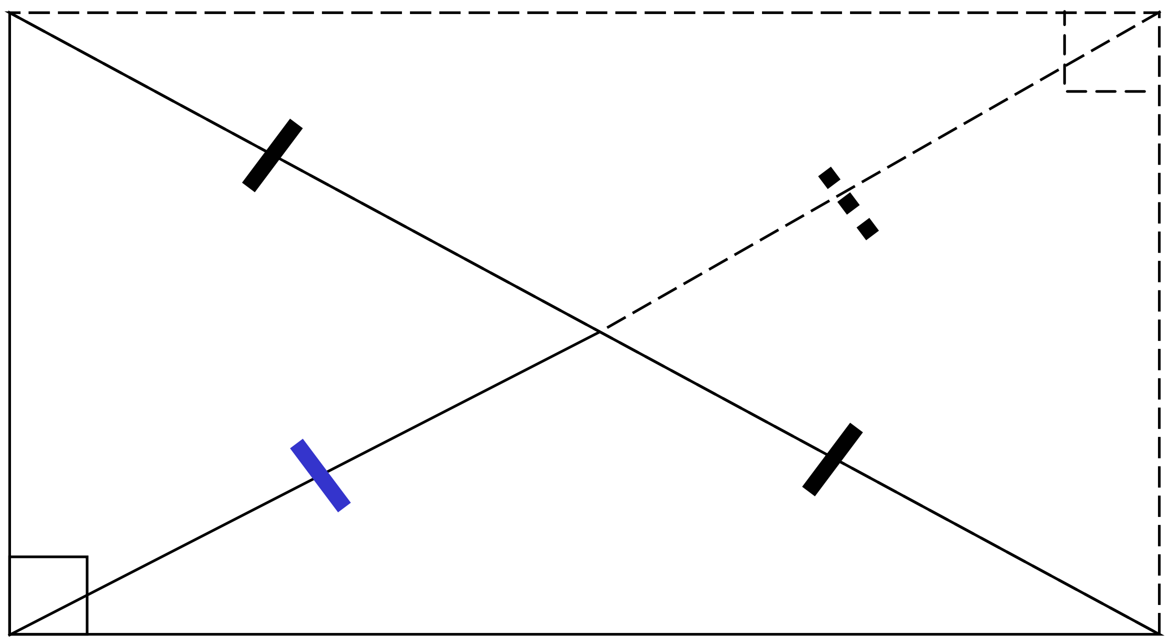
The diagonals of a rhombus



bisect two angles of the rhombus.

★ Theorem 5-15

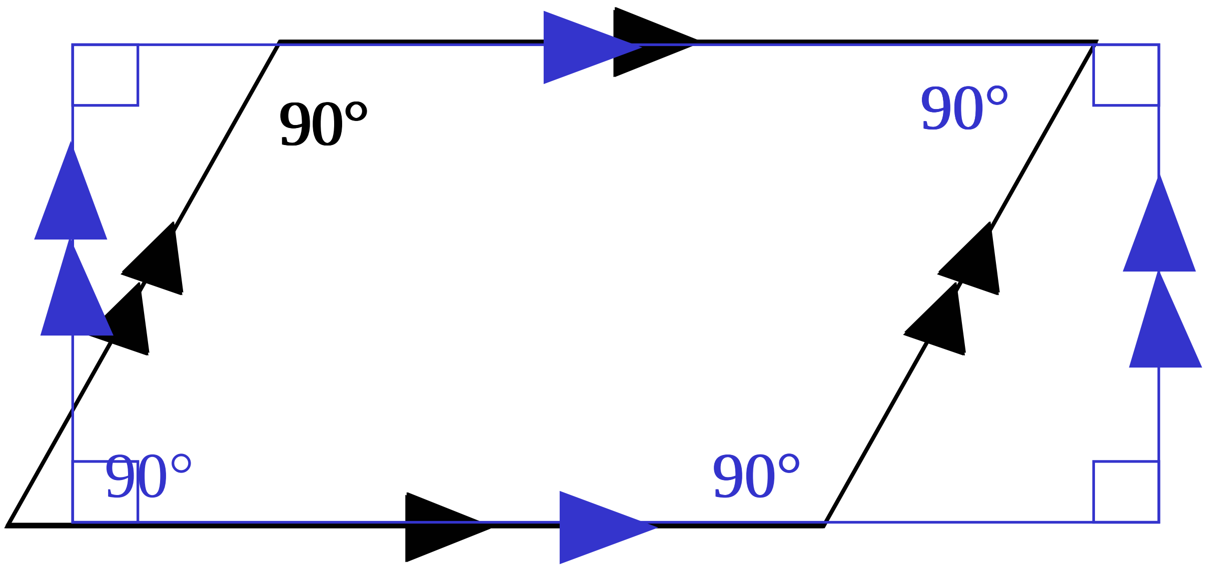
The midpoint of the hypotenuse of a right triangle



is equidistant from the three vertices.

Theorem 5-16

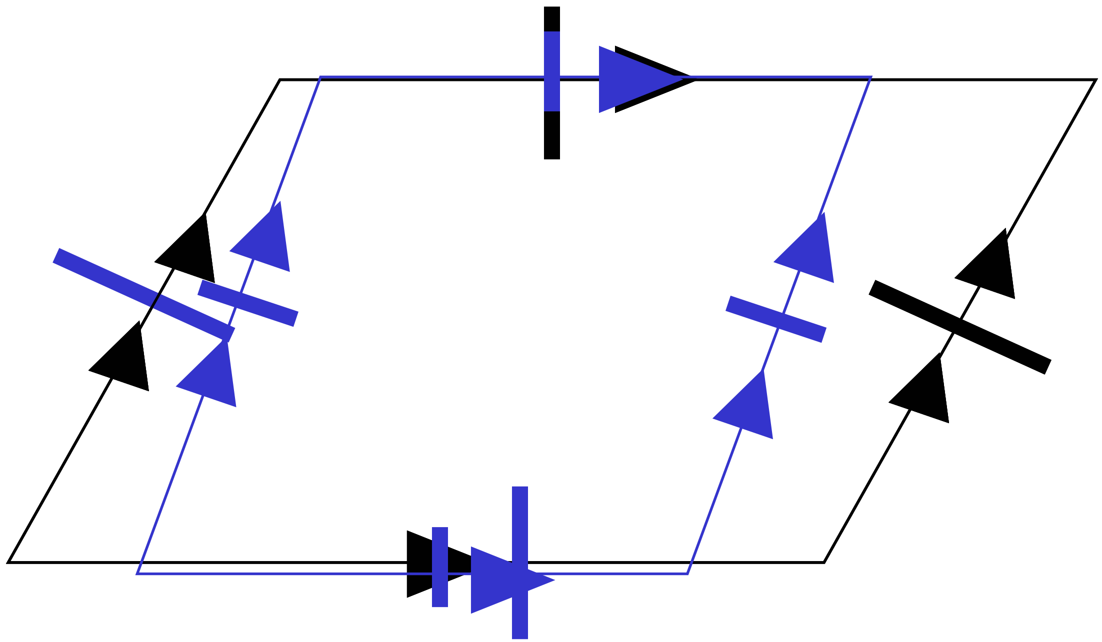
If an angle of a parallelogram is a right angle,



then the parallelogram is a rectangle.

Theorem 5-17

If two consecutive sides of a parallelogram are congruent,



then the parallelogram is a rhombus.

Sample Problems Section 5-4

Place a check in the appropriate spaces

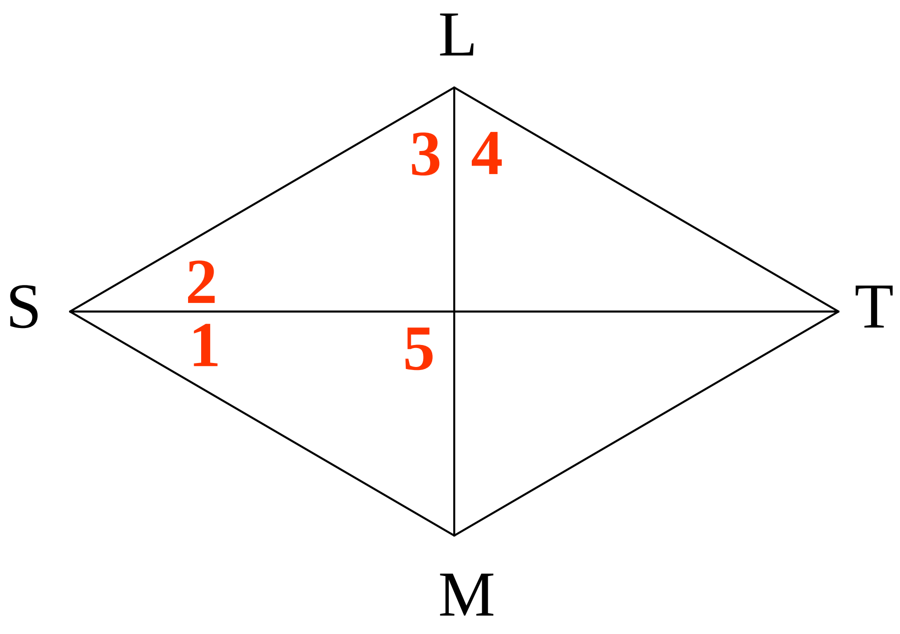
Property	Parallelogram	Rectangle	Rhombus	Square
1. Opposite sides are parallel.				
3. Opposite angles are congruent.				
5. Diagonals bisect each other.				
7. Diagonals are perpendicular				
9. All angles are right angles.				

Sample Problems Section 5-4

Quad SLTM is a rhombus.

11. If m $\angle 1 = 25$, find the measures of $\angle 2$, $\angle 3$, $\angle 4$, and $\angle 5$

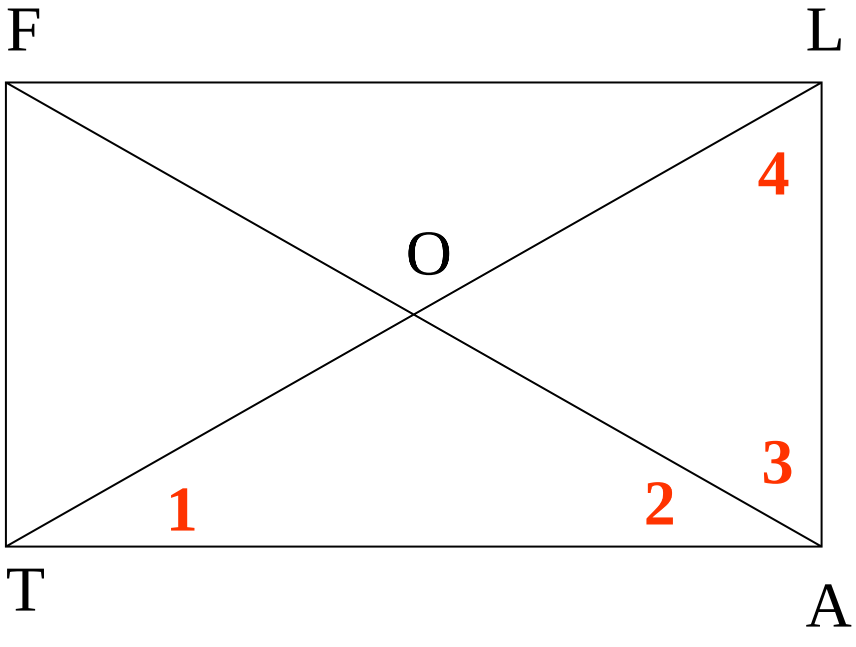
13. If m $\angle 1 = 3x + 1$ and m $\angle 3 = 7x - 11$, find the value of x.



Sample Problems Section 5-4

Quad FLAT is a rectangle.

15. If FA = 27, find LO.

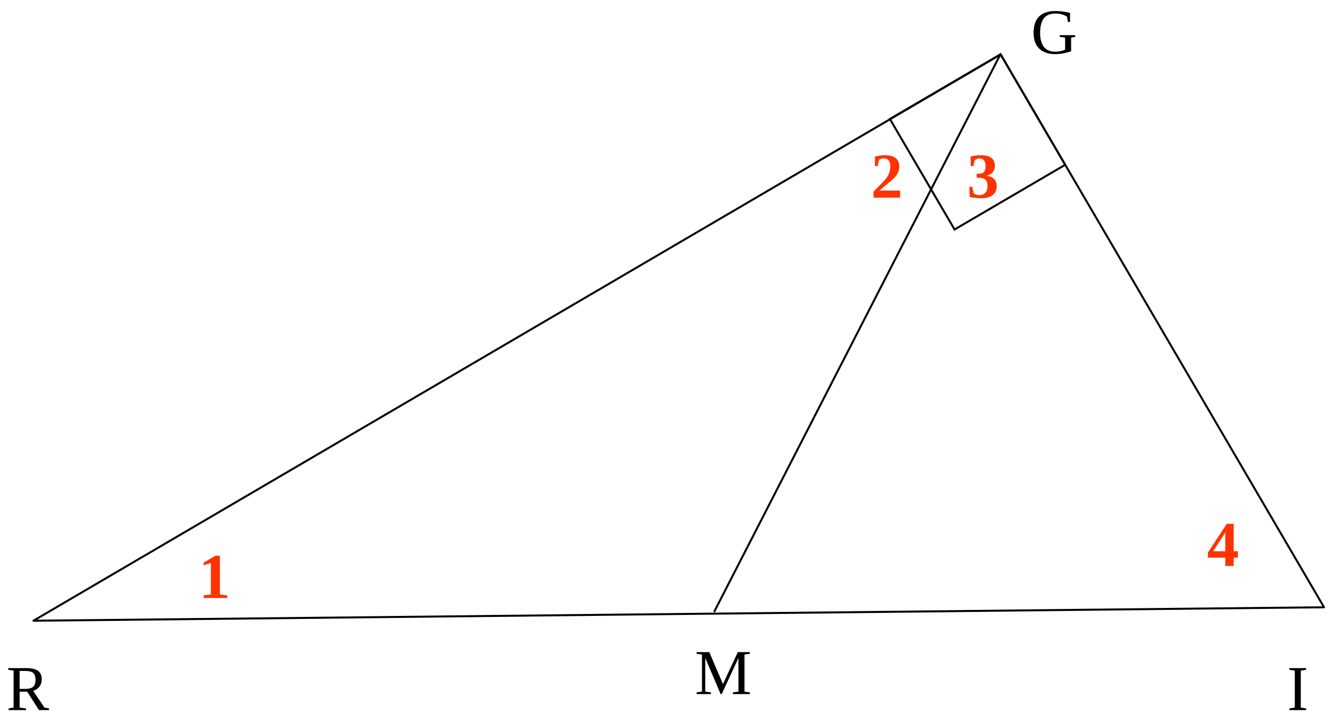


Sample Problems Section 5-4

GM is the median of right Δ IRG.

17. If m ∠ 1 = 32, find the measures of ∠ 2, ∠ 3, and ∠ 4.

19. If GM = 2y + 3 and RI = 12 - 8y, find the value of y.



Sample Problems Section 5-4

The coordinates of three vertices of a rectangle are given. Plot the points and find the coordinates of the fourth vertex. Is the rectangle a square?

21. A(2, 1) B(4, 1) C(4, 5) D(? , ?)

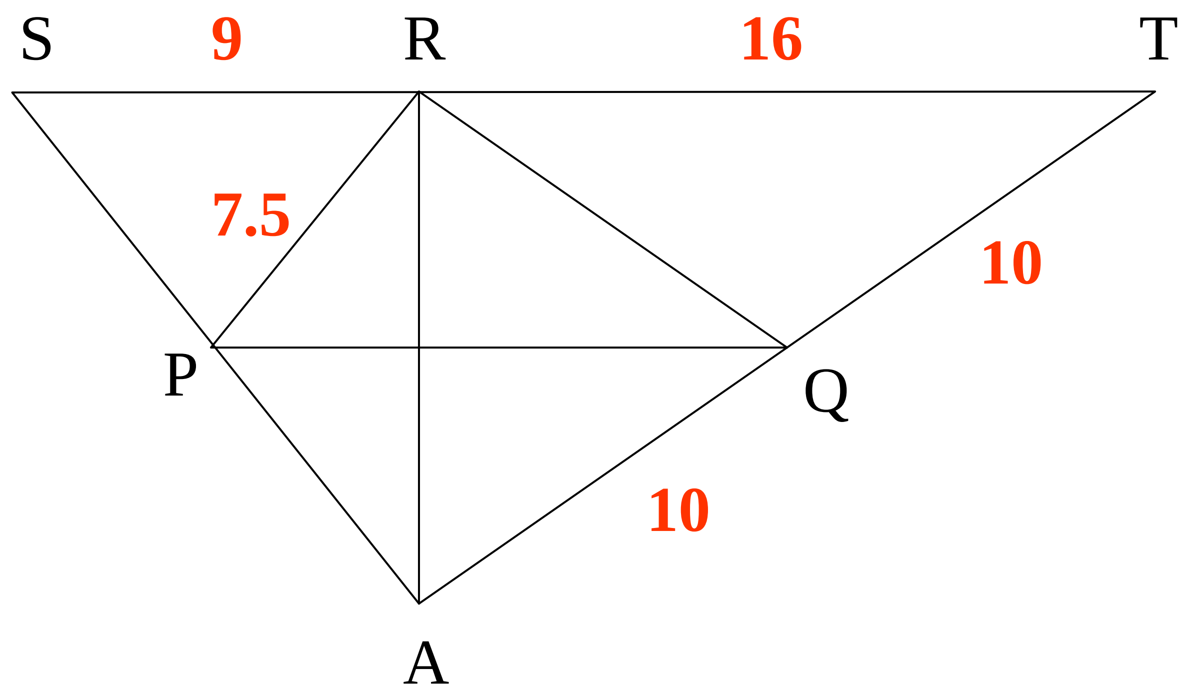
23. H(1, 3) I(4, 3) J(? , ?) K(1, 6)

Sample Problems Section 5-4

RA is an altitude of Δ SAT. P and Q are midpoints of SA and TA. SR = 9, RT = 16, QT = 10, and PR = 7.5

25. Find SA.

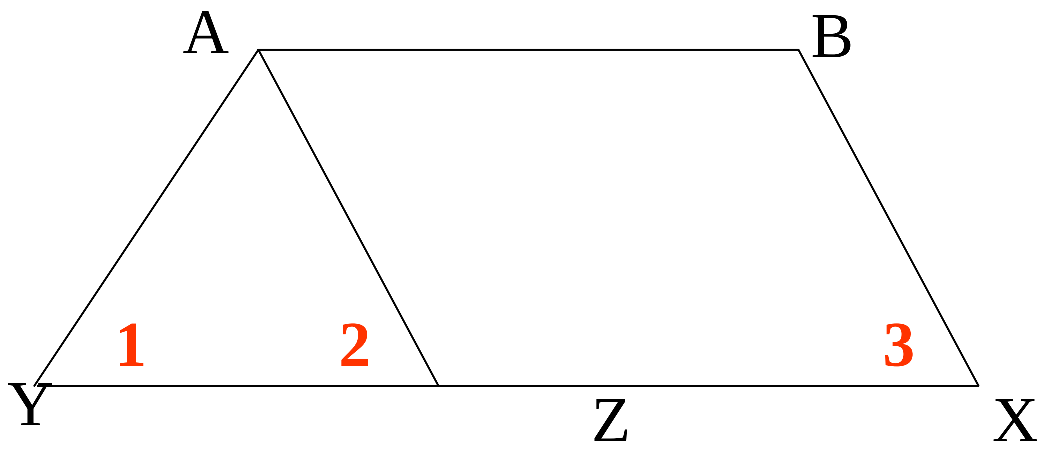
27. Find the perimeter of Δ SAT.



Sample Problems Section 5-4

29. Given: parallelogram ABZY; AY = BX

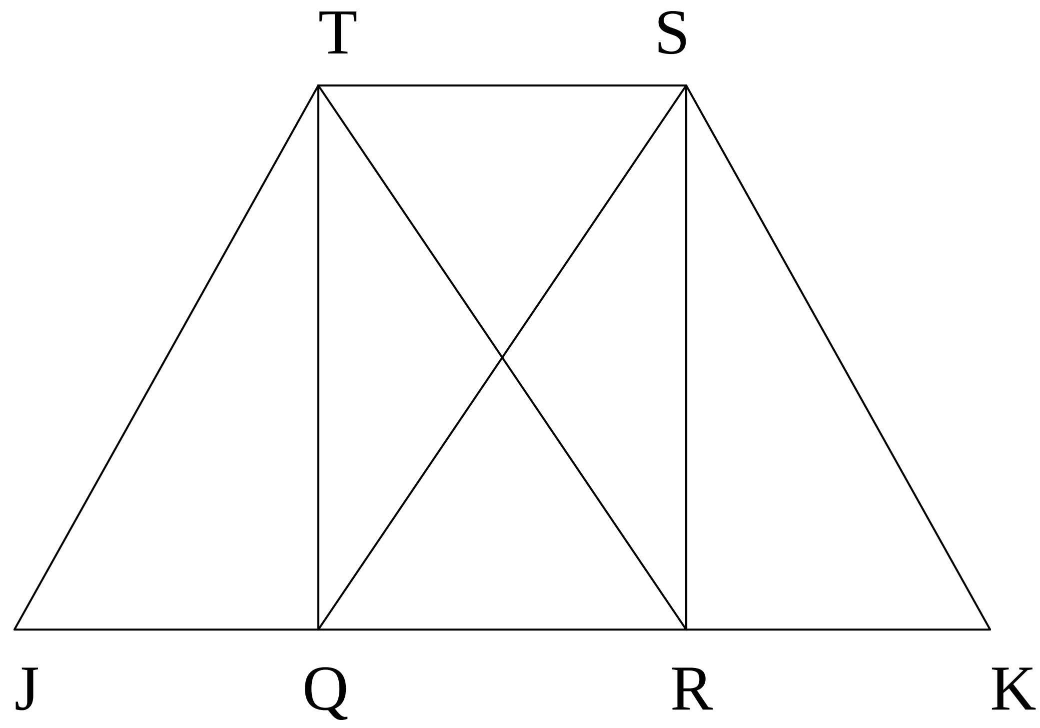
Prove: m ∠ 1 = m ∠ 2 and m ∠ 1 = m ∠ 3



Sample Problems Section 5-4

31. Given: rectangle QRST; parallelograms RKST & JQST

Prove: JT = KS



Sample Problems Section 5-4

33. Prove theorem 5-14 for one diagonal of the rhombus.
35. Prove: If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
37. Draw a rectangle and bisect its angles. The bisectors intersect to form what kind of quadrilateral?

The coordinates of three vertices of a rhombus are given, not necessarily in order. Plot the points and find the coordinates of the fourth vertex. Measure the sides to check your answer.

39. O(0, 0) S(0, 10) E(6, 18) W(? , ?)

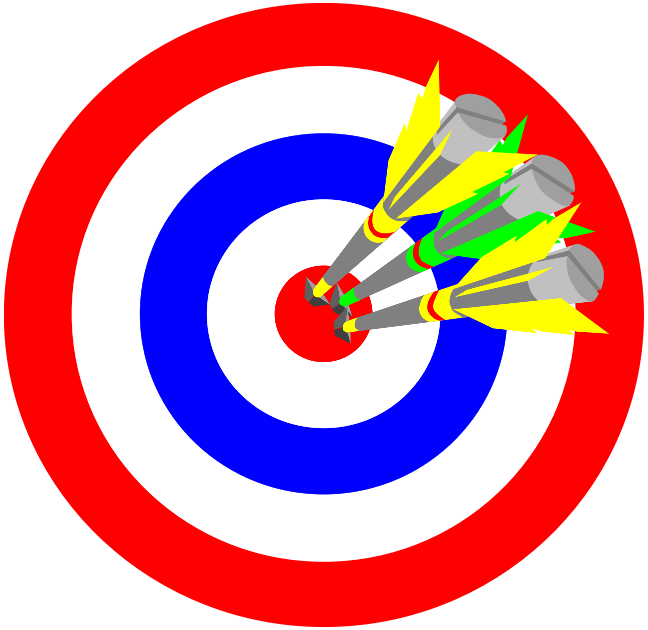
Section 5-5

Trapezoids

Homework Pages 192-194:

2-32 evens, excluding 12, 20, 24, 28

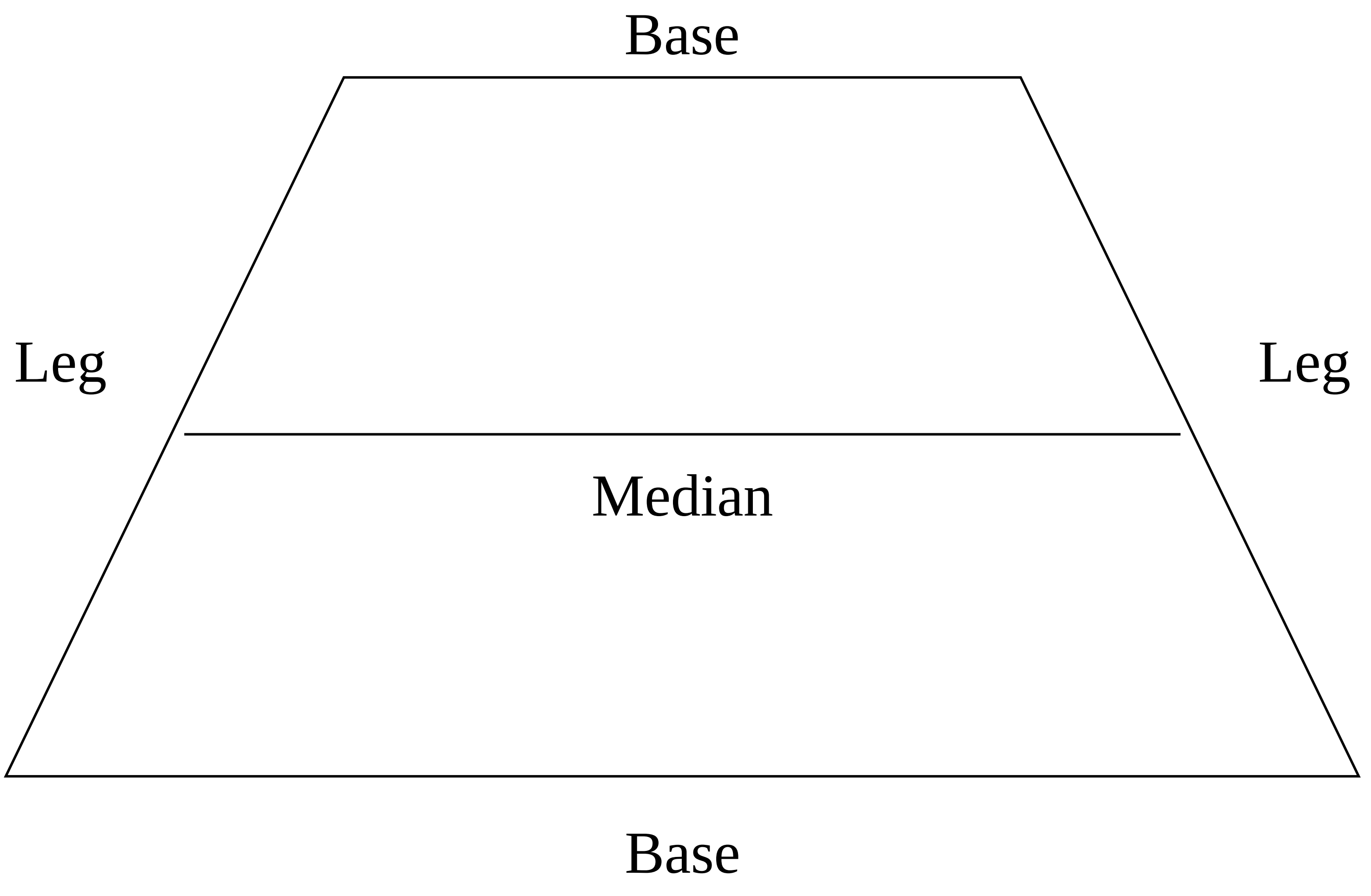
Objectives



A. Apply the definitions of trapezoid and isosceles trapezoid.
B. Identify the legs, bases, median and base angles of a trapezoid.
C. Understand and apply the theorems related to trapezoids.

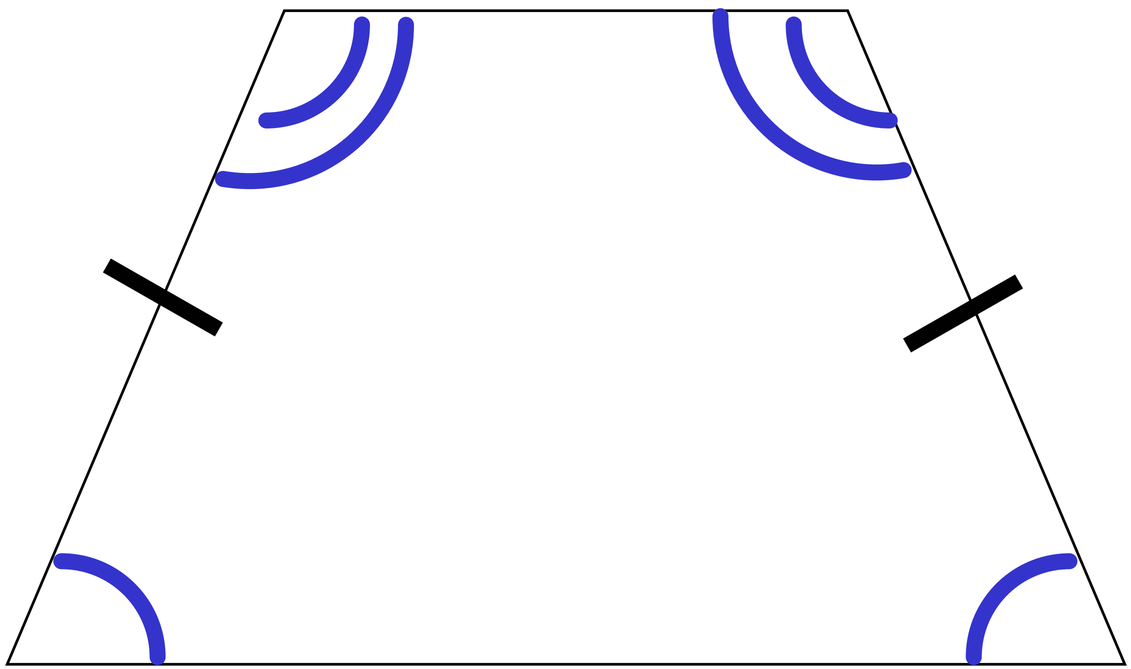
- ★ trapezoid: a quadrilateral with exactly one pair of parallel sides
- bases of a trapezoid: the parallel sides of a trapezoid
- legs of a trapezoid: the nonparallel sides of a trapezoid
- ★ isosceles trapezoid: a trapezoid with congruent legs
- median of a trapezoid: segment joining the midpoints of the legs
- Base angles of a trapezoid: formed by a base and a leg of the trapezoid

★ Parts of a Trapezoid



★ Theorem 5-18

The base angles of an isosceles trapezoid

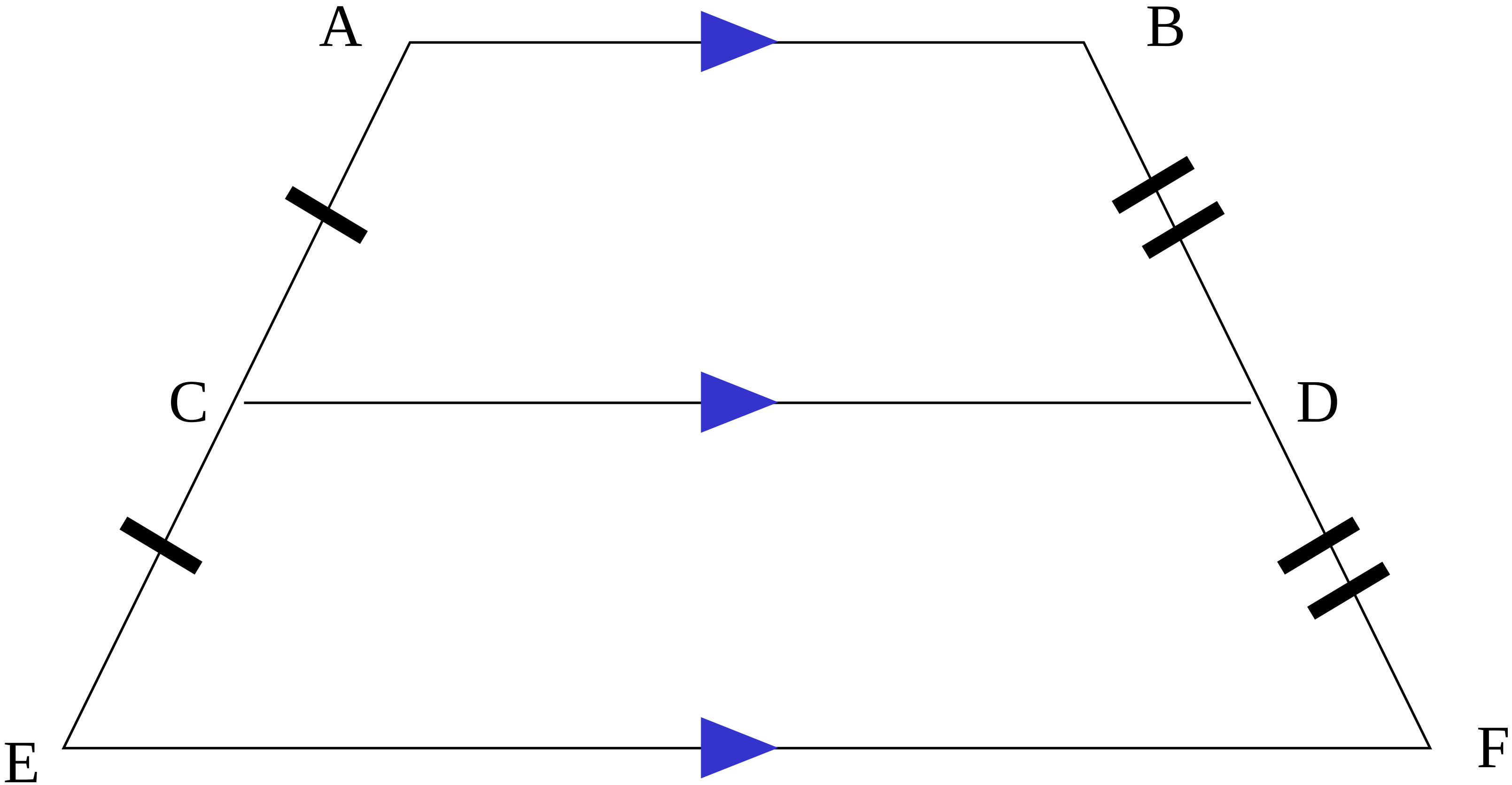


are congruent.

Note that the base angles are PAIRED in specific manner!

★ Theorem 5-19

The median of a trapezoid



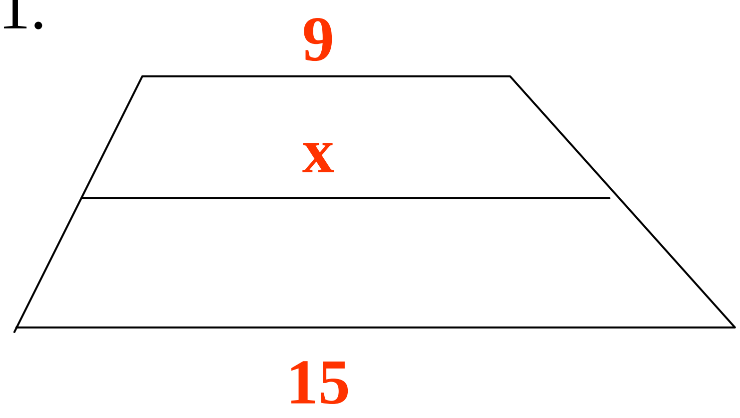
is parallel to the bases.

has a length equal to the average of the base lengths. $CD = \frac{AB + EF}{2}$

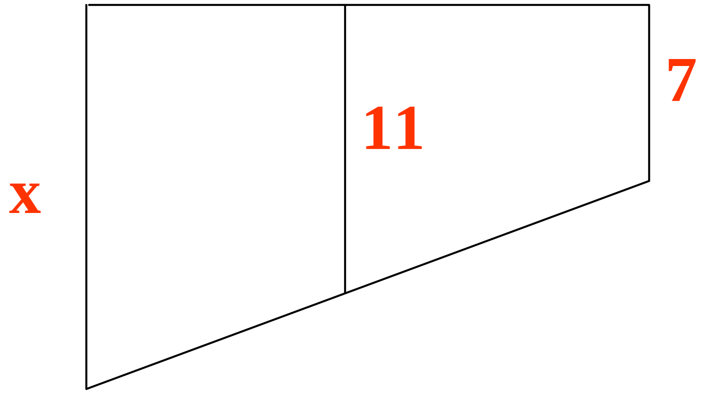
Sample Problems Section 5-5

Each diagram shows a trapezoid and its median. Find x.

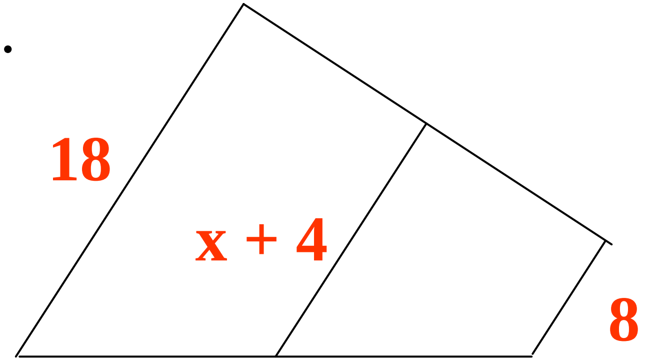
1.



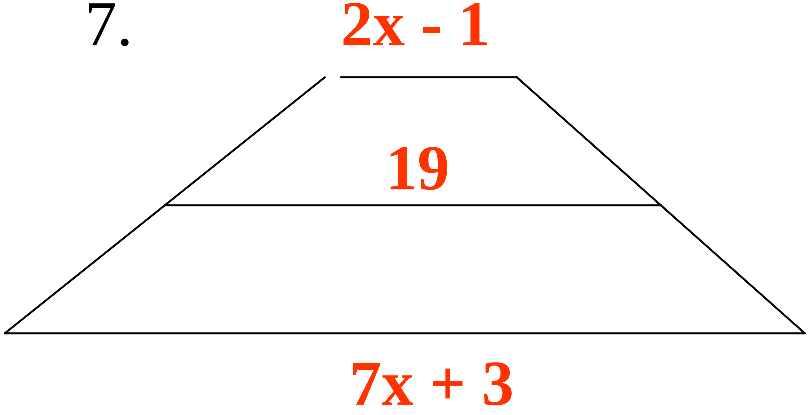
3.



5.



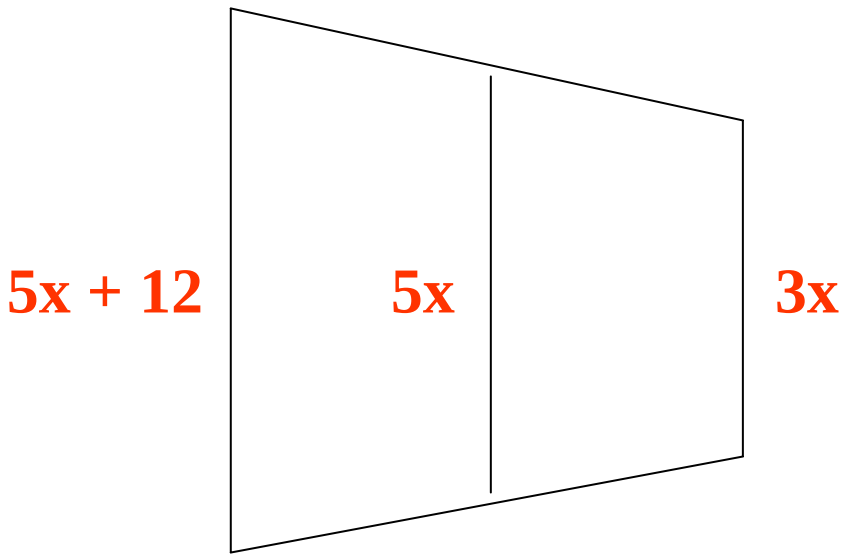
7.



Sample Problems Section 5-5

Each diagram shows a trapezoid and its median. Find x.

9.



11. Two congruent angles of an isosceles trapezoid have measures 3x + 10 and 5x - 10. Find the value of x and then give the measures of all angles of the trapezoid.

Sample Problems Section 5-5

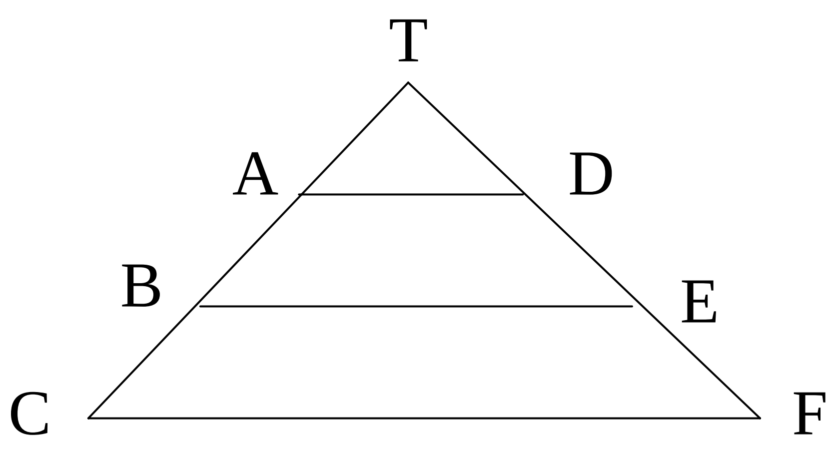
TA = AB = BC and TD = DE = EF

13 Write an equation that relates AD, BE and CF.

15. If BE = 26, then AD = ? and CF = ?

17. If AD = x + 3, BE = x + y and CF = 36, then x = ? and y = ?

19. Tony makes up a problem for the figure, setting AD = 5 and CF = 17. Katie says “You can’t do that.” Explain.



Sample Problems Section 5-5

Draw the quadrilateral named. Join, in order, the midpoints of the sides. What type of special quadrilateral do you appear to get?

21. rhombus
23. isosceles trapezoid
25. quadrilateral with no congruent sides
27. Prove Theorem 5 - 18

A kite is a quadrilateral with two pairs of congruent sides but opposite sides are not congruent.

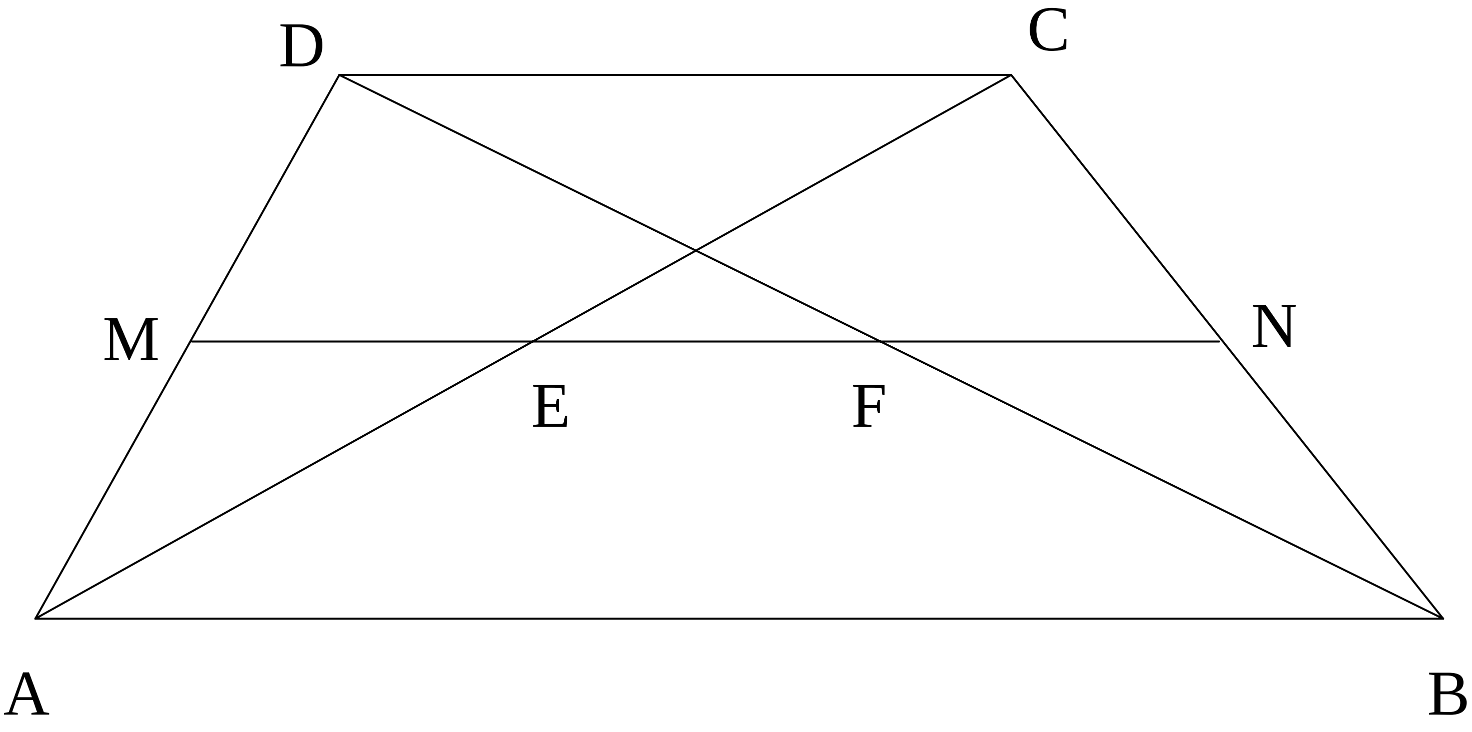
29a. Draw a convex kite. State and prove what you can about the diagonals.

29b. Repeat part (a) but draw a non-convex kite.

Sample Problems Section 5-5

ABCD is a trapezoid with median MN.

31. Prove that EF = ½(AB - DC)



Chapter 5

Quadrilaterals

Review

Homework Page 199:

2-18 evens