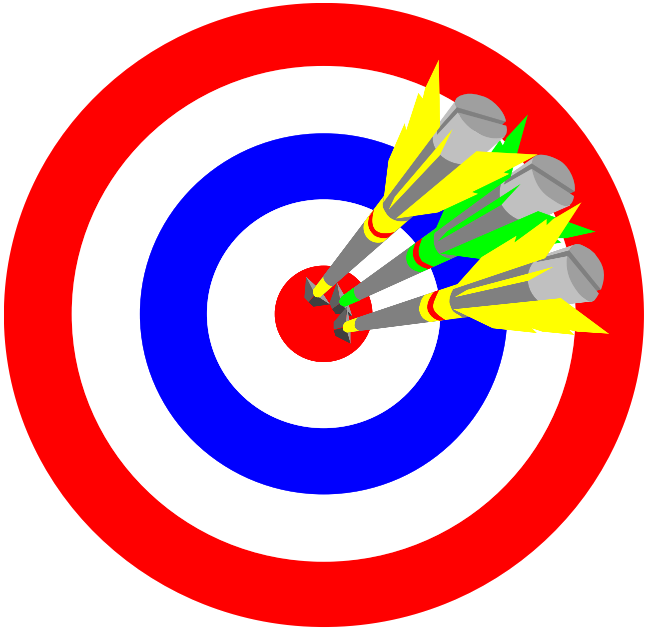
Chapter 6

Inequalities in Geometry

Objectives



A. Use the terms defined in the chapter correctly.
B. Properly use and interpret the symbols for the terms and concepts in this chapter.
C. Appropriately apply the properties and theorems in this chapter.
D. Identify and apply the properties of inequalities.
E. State the contrapositive and inverse of a conditional.
F. Identify logically equivalent statements.
G. Draw correct conclusions from given information
H. Write indirect proofs.

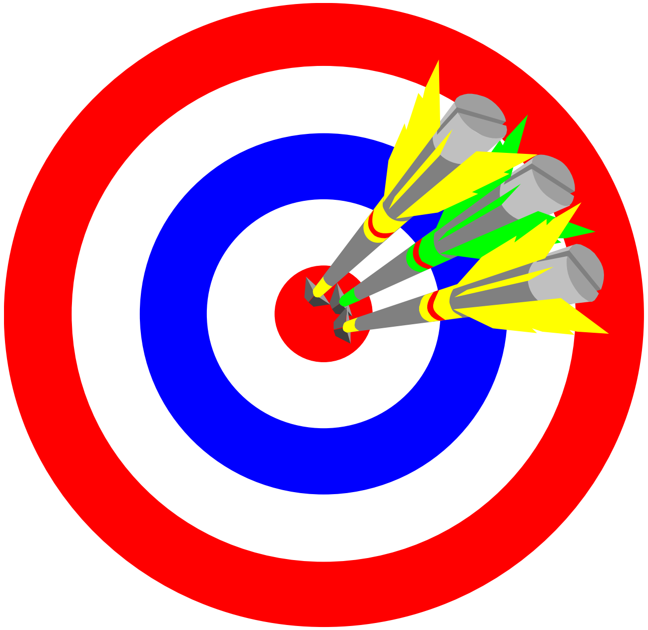
Section 6-1

Inequalities

Homework Pages 206-207:

2-12 evens

Objectives



A. Understand and apply the properties of inequalities.
B. Understand and apply the Exterior Angle Inequality Theorem.

★Properties of Inequality

- Addition: you may add the same amount to both sides of an inequality without changing the result (true/false) of the statement.
  - If a > b, then a + c > b + c
  - Further: If a > b and c >= d, then a + c > b + d
- Subtraction: you may subtract the same amount from both sides of an inequality without changing the result of the statement.
  - If a > b, then a - c > b – c
  - However: If a > b and c >= d, then it is NOT necessarily true that a - c > b – d
    - If a=2, b=1, c=10, d=3,
      - a > b, c >= d, but 2 -10 not greater than 1 -3

★Properties of Inequality

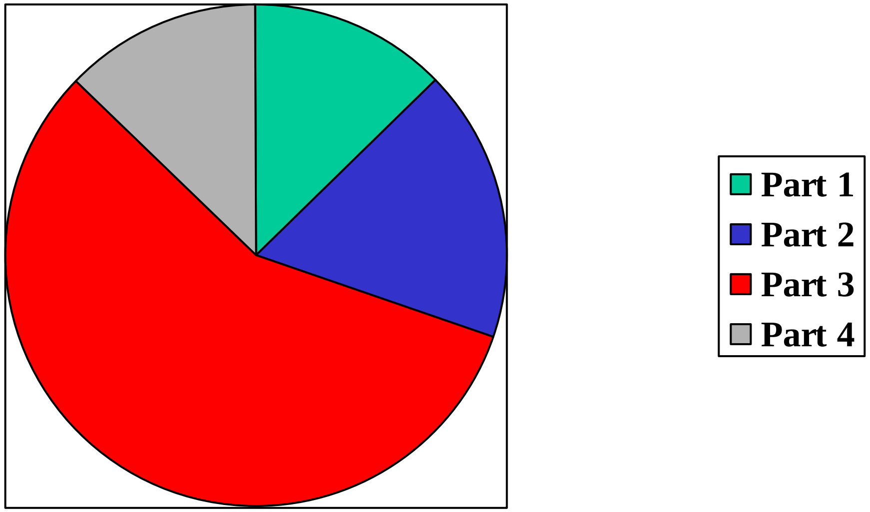
- Multiplication by positive numbers: You may multiply both sides of an inequality by the same positive number without changing the result of the statement.
  - If $a > b$ and $c > 0$, then $ac > bc$
  - Also: If $(a + d) > (b + e)$ and $c > 0$, then $(a + d)c > (b + e)c$
- Division by positive numbers: You may divide both sides of an inequality by the same positive number without changing the result of the statement.
  - If $a > b$ and $c > 0$, then $a/c > b/c$
  - Also: If $(a + d) > (b + e)$ and $c > 0$, then $(a + d)/c > (b + e)/c$

★Properties of Inequality

- Multiplication by negative numbers: You may multiply both sides of an inequality by the same negative number. However, <u>you must also reverse the inequality symbol</u> in order to produce valid results for the statement.
  - If $a > b$ and $c < 0$, then $ac < bc$
  - Also: If $(a + d) > (b + e)$ and $c < 0$, then $(a + d)c < (b + e)c$
- Division by negative numbers: You may divide both sides of an inequality by the same negative number. However, <u>you must also reverse the inequality symbol</u> in order to produce valid results for the statement.
  - If $a > b$ and $c < 0$, then $a/c < b/c$
  - Also: If $(a + d) > (b + e)$ and $c < 0$, then $(a + d)/c < (b + e)/c$

★Properties of Inequality

- Transitive: You can create chains of ordered values that either increase or decrease in value as you move along the chain.
  - If $a > b$ and $b > c$, then $a > c$
  - If $a < b$ and $b < c$, then $a < c$
- Part-to-Whole: Provided both parts have positive measurements, the whole is always larger than one of its parts.

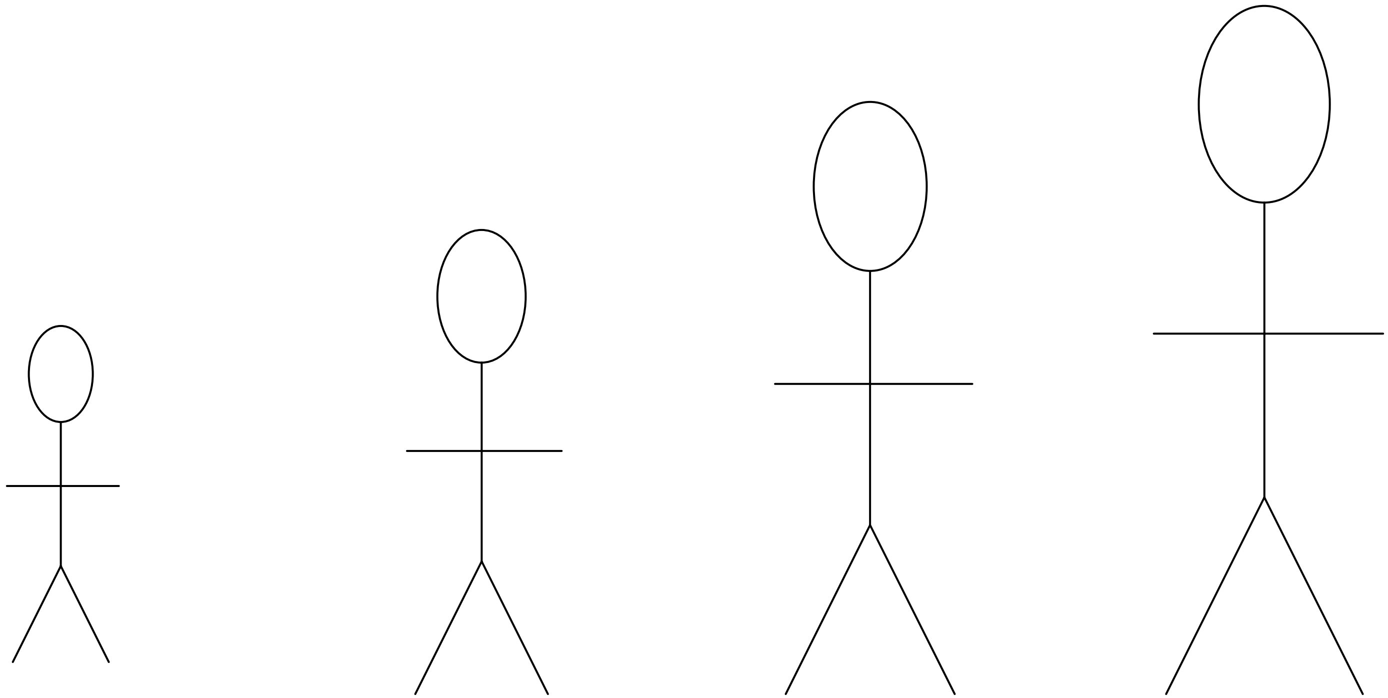


If $a + b + c = d$ and $a > 0$, $b > 0$, $c > 0$, then $a < d$ and $b < d$ and $c < d$.

Properties of Inequality: Example

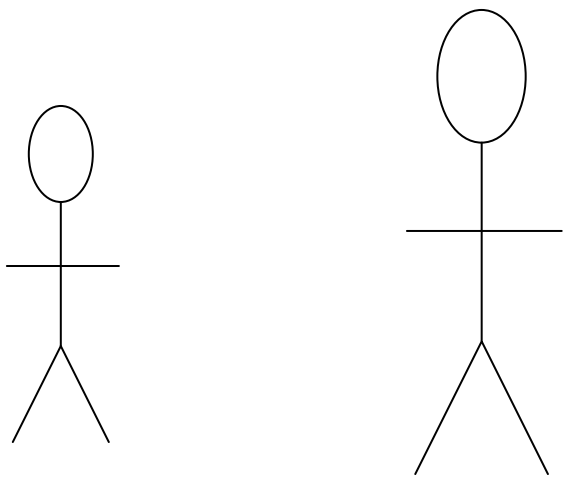
Given this inequality:	$\frac{3x+7}{2} > 5x-9$	
First, multiply both sides by a positive 2.	$2\left(\frac{3x+7}{2}\right) > 2(5x-9)$ $3x+7 > 10x-18$	Mult by (+)
Second, subtract 7 from both sides.	$3x+7-7 > 10x-18-7$ $3x > 10x-25$	Subtr
Third, subtract 10x from both sides.	$3x-10x > 10x-25-10x$ $-7x > -25$	Subtr
Fourth, divide both sides by - 7. Remember to reverse the inequality symbol.	$\frac{-7x}{-7} < \frac{-25}{-7}$ $x < \frac{25}{7}$	Div by (-)

Properties of Inequality: Transitive



Bob < Jack < Tim < Andy

Properties of Inequality: Transitive



Bob $<$ Jack

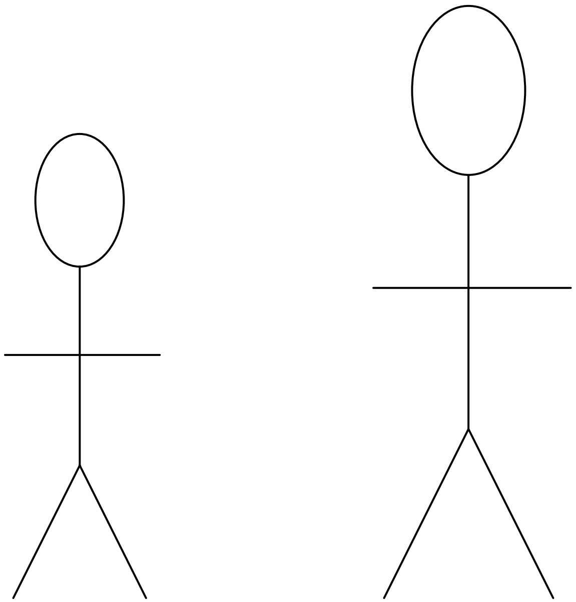
Properties of Inequality: Transitive

Bob < Tim

Properties of Inequality: Transitive

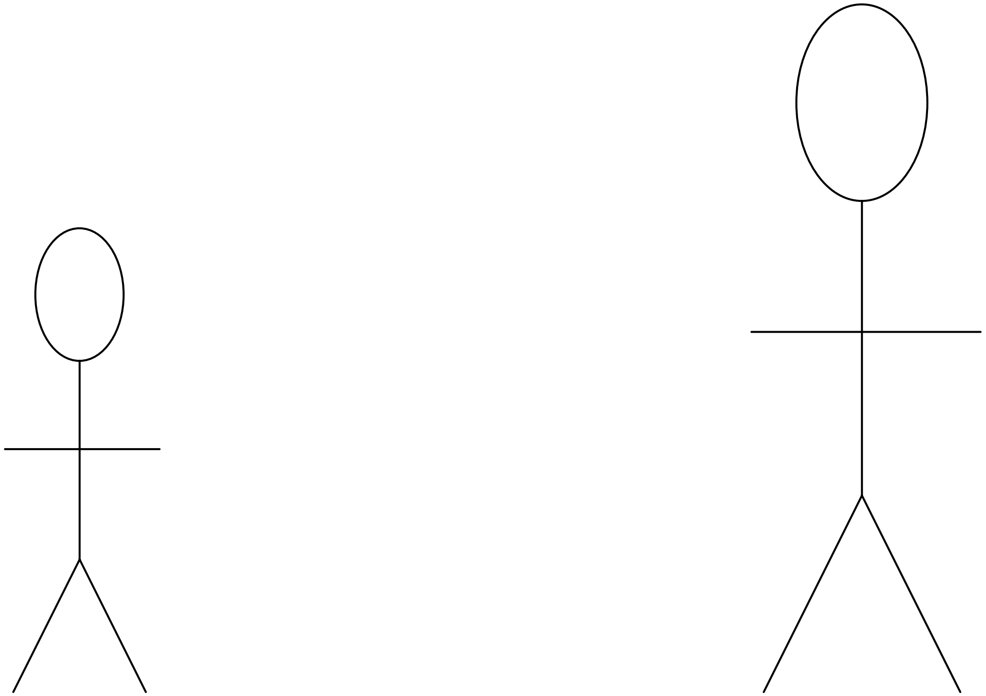
Bob < Andy

Properties of Inequality: Transitive



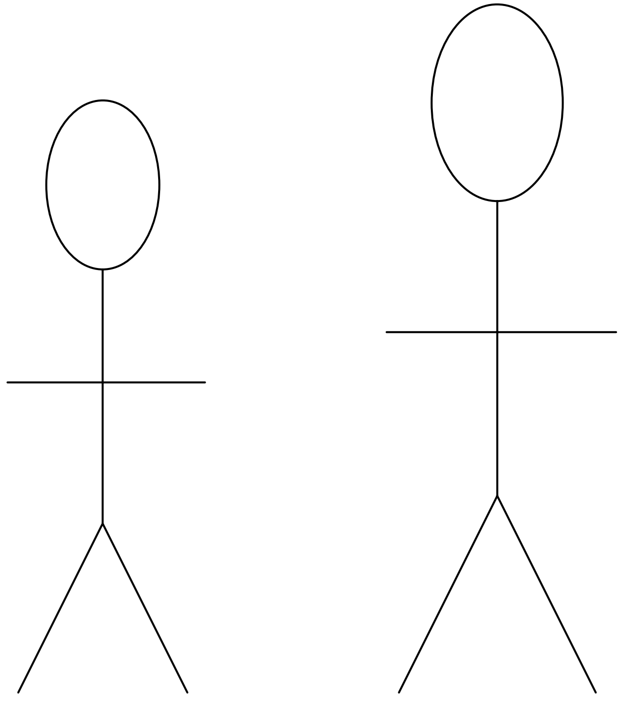
Jack < Tim

Properties of Inequality: Transitive



Jack < Andy

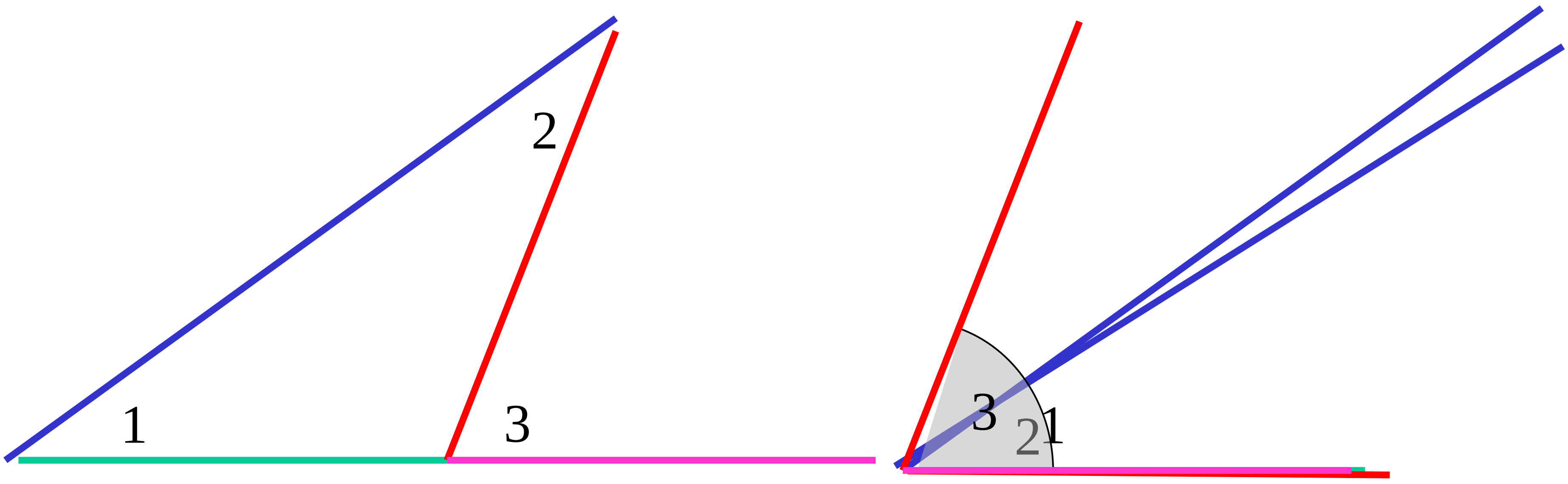
Properties of Inequality: Transitive



Tim < Andy

★ Theorem 6-1

The measure of the exterior angle of a triangle is greater than the measure of either remote interior angle.



$\angle 3 > \angle 1$

$\angle 3 > \angle 2$

Sample Problems

Some information about the diagram is given. Tell whether the other statements can be deduced from what is given.

1. Given: Point Y lies between points X and Z.

a. XY = ½XZ

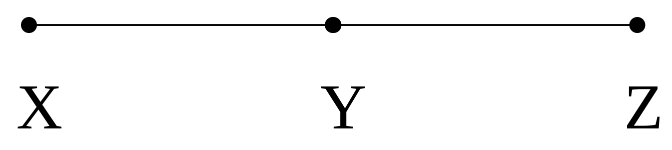
b. XZ = XY + YZ

c. XZ > XY

d. YZ > XY

e. XZ > YZ

f. XZ > 2XY



Sample Problems

3. Given: parallelogram ABCD; AC > BD

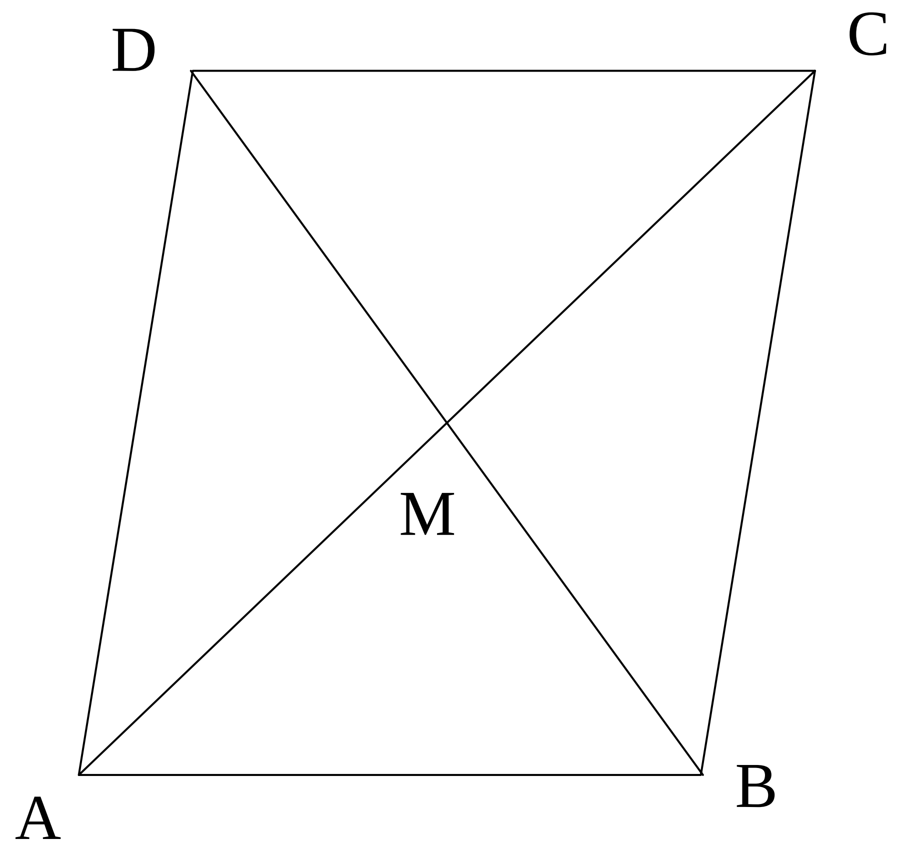
Determine which statements are true/false.

a. AB > AD

b. AM > MC

c. DM = MB

d. AM > MB



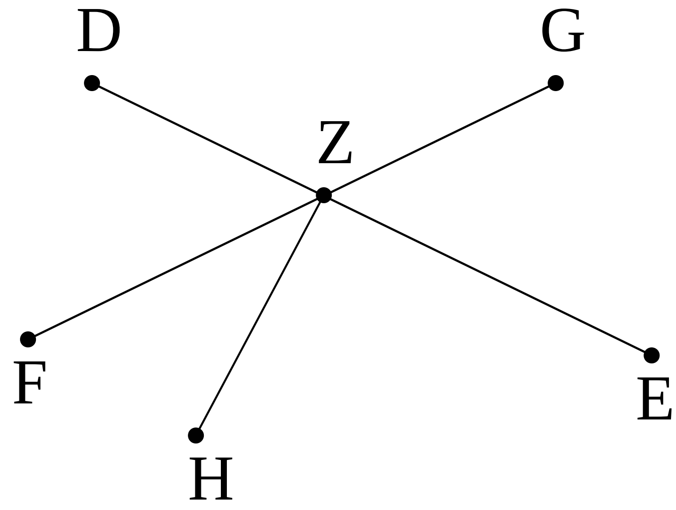
Sample Problems

5. When some people are given j > k and l > m, they carelessly conclude that j + k > l + m. Find values for j, k, l and m that show this conclusion is false.

7. Write the reasons that justify the statements.

Given: DE, FG and ZH contain point Z

Prove: m ∠ DZH > m ∠ GZE

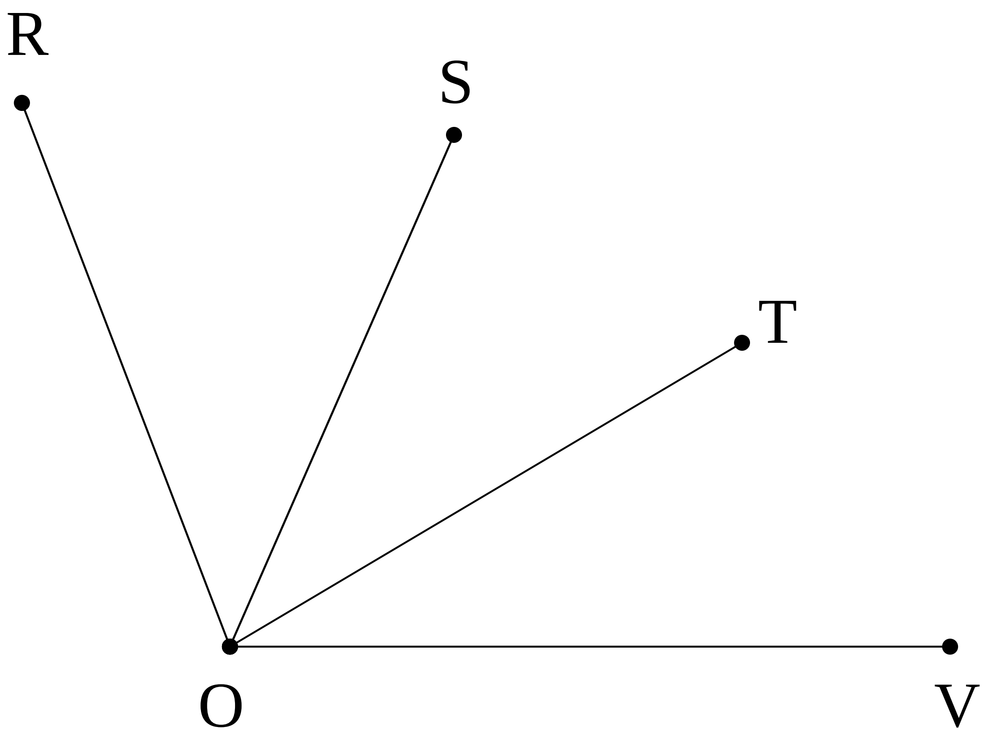


1. m ∠ DZF = m ∠ GZE
2. m ∠ DZH = m ∠ DZF + m ∠ FZH
3. m ∠ DZH > m ∠ DZF
4. m ∠ DZH > m ∠ GZE

Sample Problems

9. Given: m ∠ ROS > m ∠ TOV

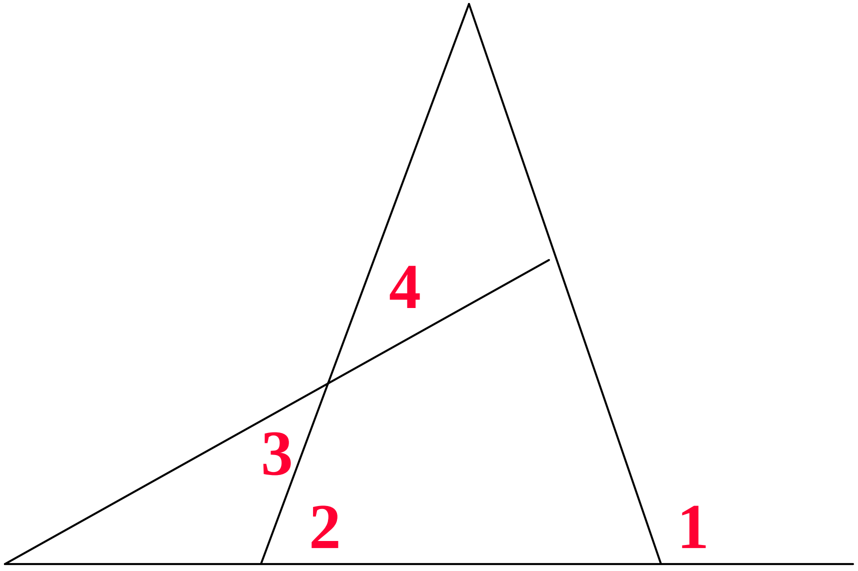
   Prove: m ∠ ROT > m ∠ SOV



Sample Problems

11. Given: the diagram

Prove: m ∠ 1 > m ∠ 4



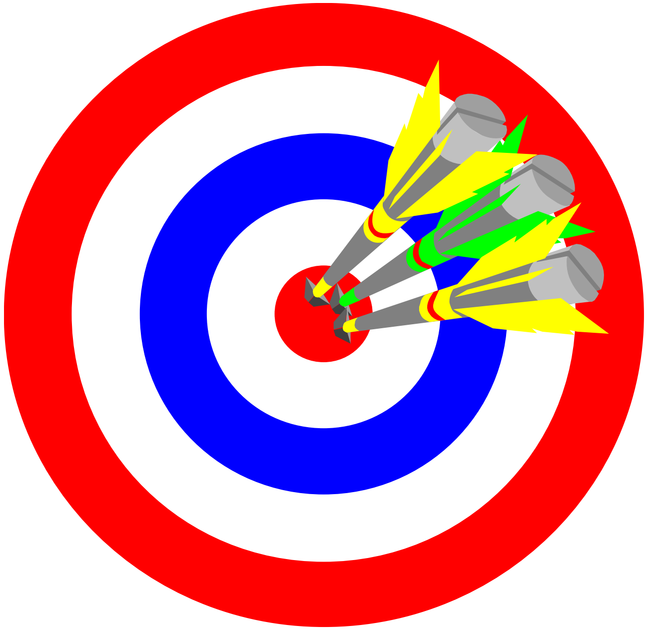
Section 6-2

Inverses and Contrapositives

Homework Pages 210-212:

2-18 evens

Objectives



A. Define the terms ‘contrapositive’ and ‘inverse’ in regards to conditional statements.
B. State the contrapositive and inverse of a conditional.
C. Define and identify logically equivalent statements.
D. Interpret and apply Venn diagrams correctly.

Definitions

- If-Then form of a Conditional:
  - If hypothesis, then conclusion.
- Converse of a Conditional:
  - Switches hypothesis and conclusion of the conditional.
  - If conclusion, then hypothesis.
- Inverse of a Conditional:
  - Negates the parts of the conditional.
  - If NOT hypothesis, then NOT conclusion.
  - NOTE: The mathematical symbols for NOT can be the caret (^) or the exclamation mark (!).
    - If ^ hypothesis, then ^ conclusion.
    - If ! hypothesis, then ! conclusion.
  - REMEMBER! Two negatives make a positive!

Definitions

- Contrapositive of a Conditional:
  - Switches *and* negates the parts of the conditional
  - If NOT conclusion, then NOT hypothesis.
- Logical equivalents:
  - Two statements that have the same meaning but are written differently
    - The conditions that make statement 1 true also make statement 2 true, and vice versa
    - The conditions that make statement 1 false also make statement 2 false, and vice versa
  - Examples:
    - conditional & contrapositive
    - converse & inverse

★ Logic Statements & Logical Equivalents

Conditional:

If you have me for class, then you have homework every night.

Converse:

If you have homework every night, then you have me for class.

Inverse:

If you don't have me for class, then you don't have homework every night.

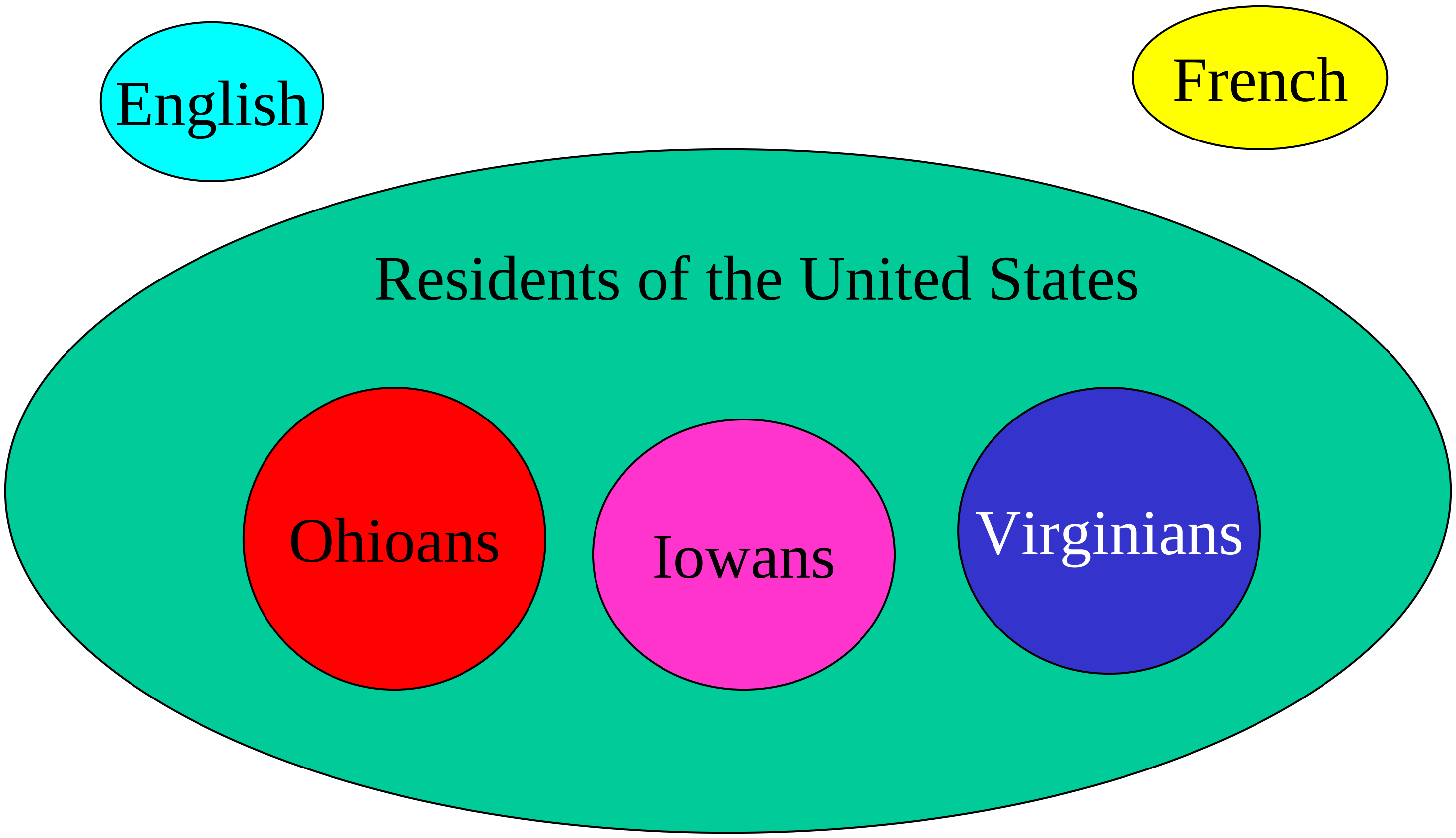
Contrapositive:

If you don't have homework every night, then you don't have me for class..

Venn Diagrams

- Venn diagrams are:
  - Pictorial representations of mathematical sets
  - Used to clarify conditional statements
  - Consist of circles to represent sets of objects

Basic Venn Diagram



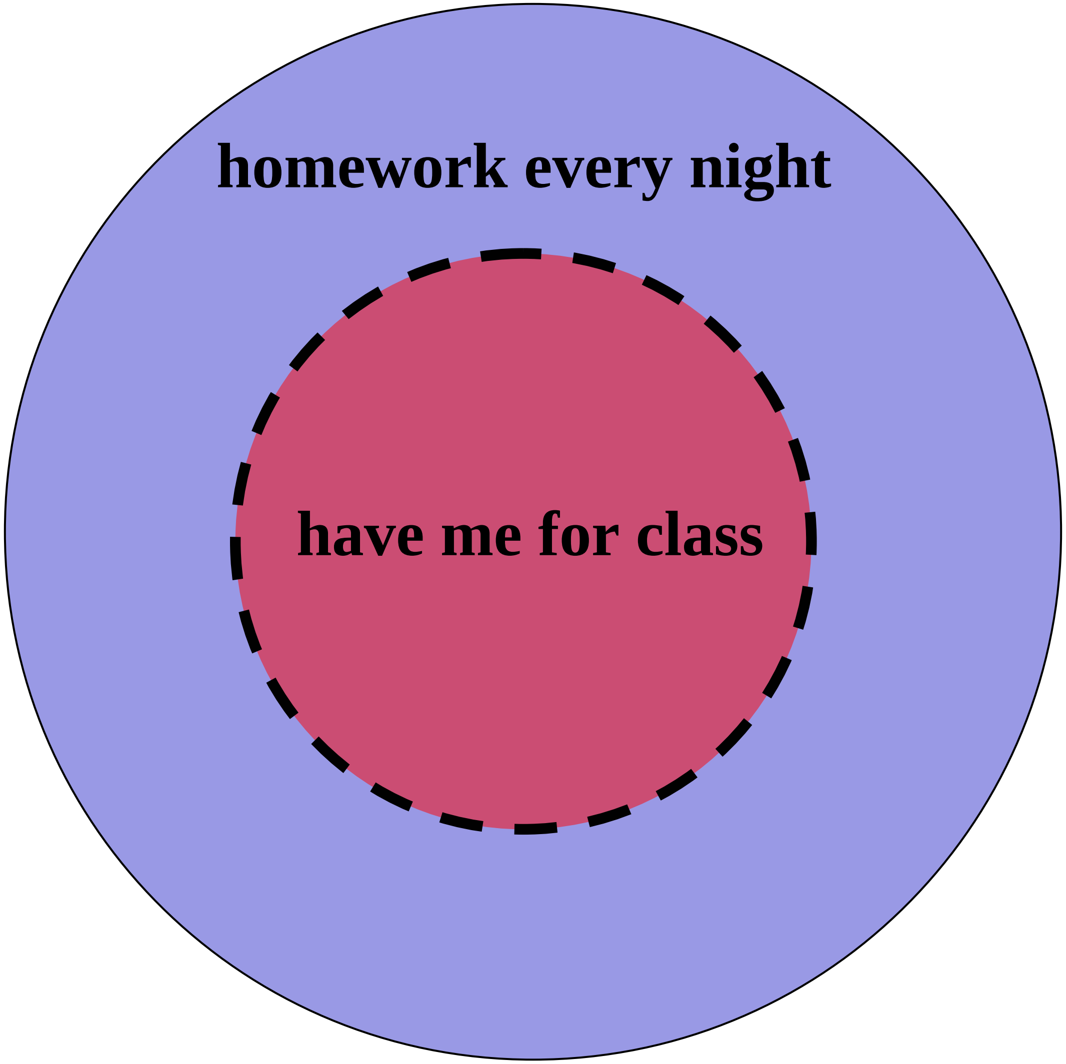
'Ohioans' are people that have permanent residence in that state.

Venn Diagrams for Conditionals

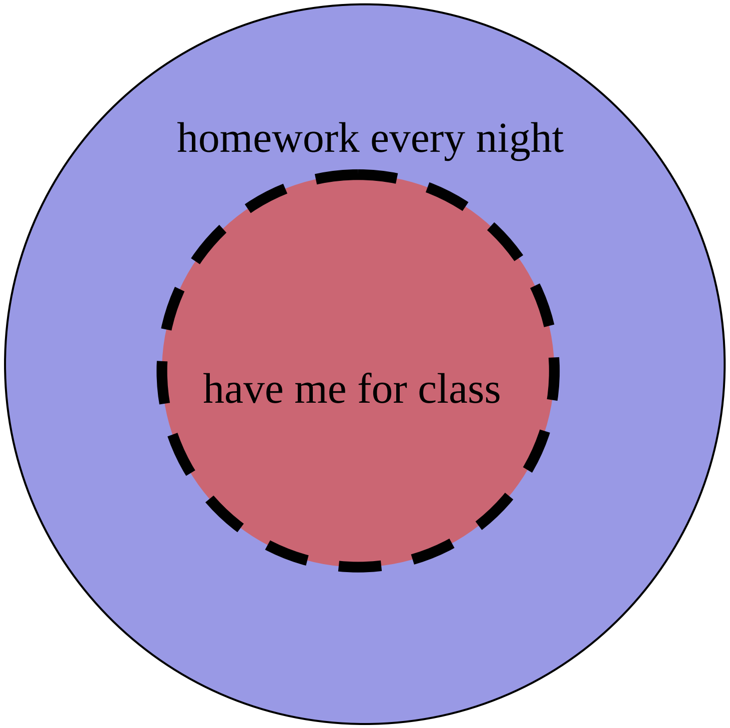
1. Place the hypothesis of the conditional in the inner circle.
2. Place the conclusion of the conditional in the outer circle.
3. If the conditional is true, the hypothesis will always ‘live in’ the confines of the conclusion.

Venn Diagrams

If you **have me for class**, then you have **homework every night**.



Venn Diagrams



If you **have me for class**, then you have **homework every night**.

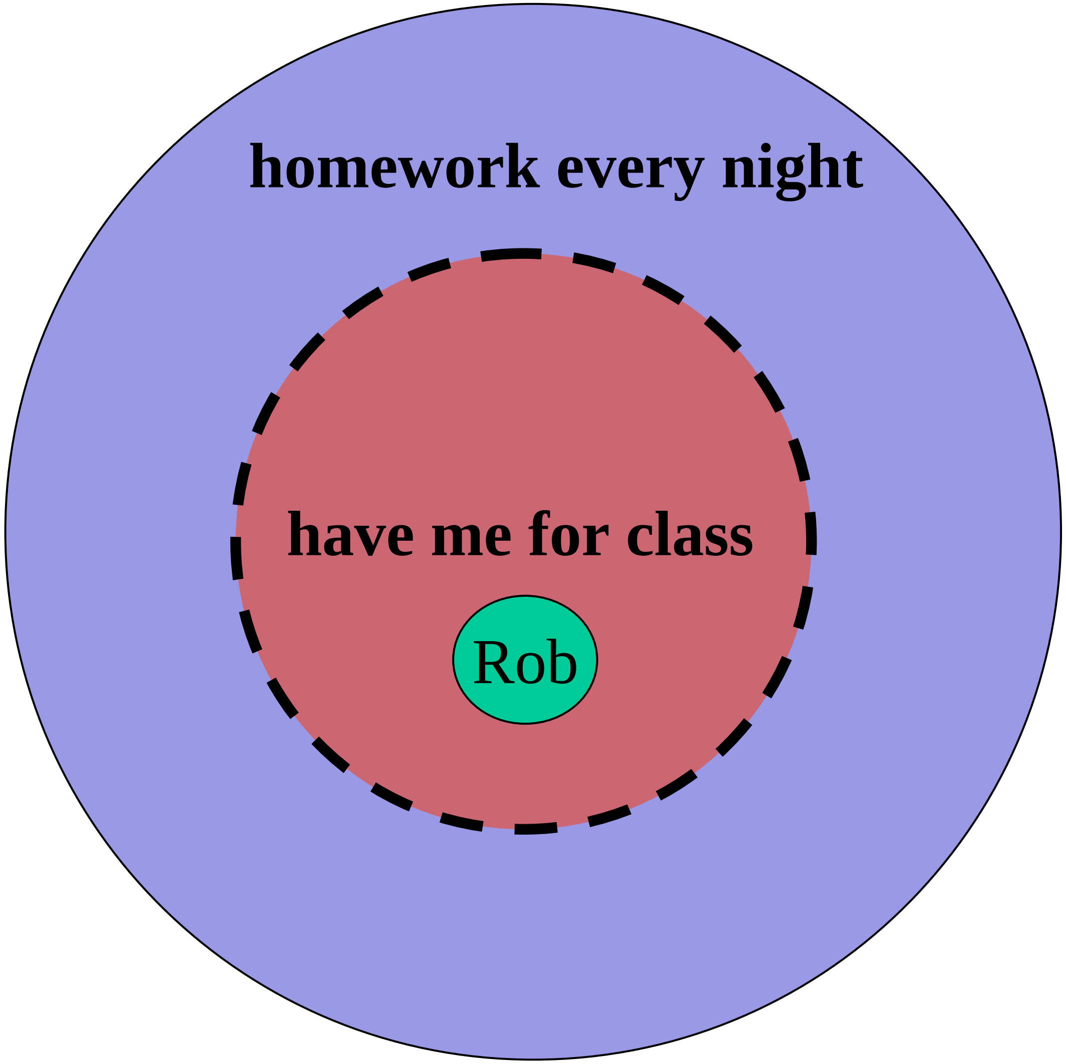
Inner circle is a part of the outer circle just like a room is part of your house.

To be the opposite of a characteristic is to be outside the circle labeled with that characteristic.

Any time a statement can be represented by two marks on the Venn diagram your result is No Conclusion.

Venn Diagrams

If you **have me for class**, then you have **homework every night**.



Statement to be tested:

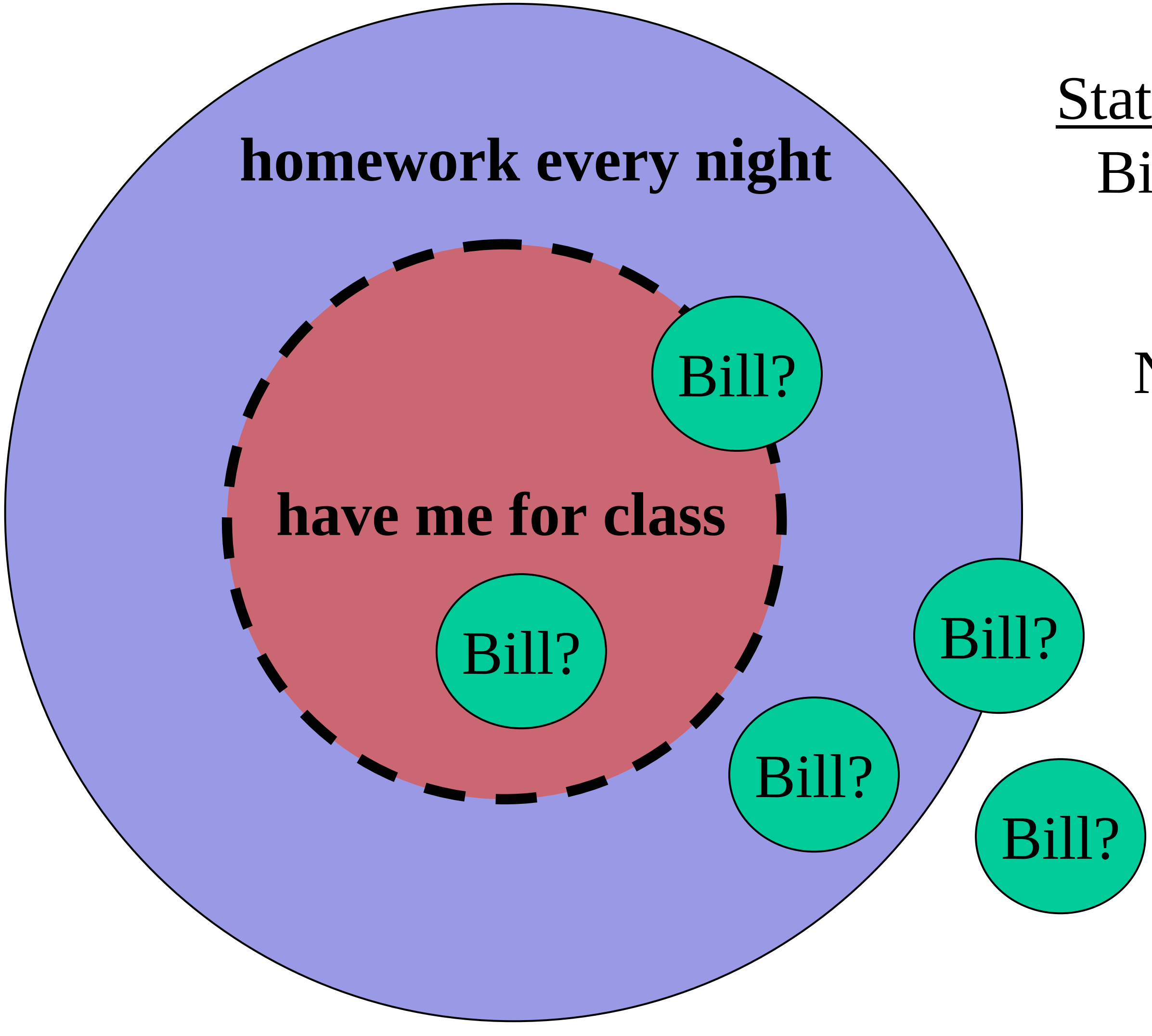
Rob is in my 6th period class.

Result:

Rob has homework every night

Venn Diagrams

If you **have me for class**, then you have **homework every night**.

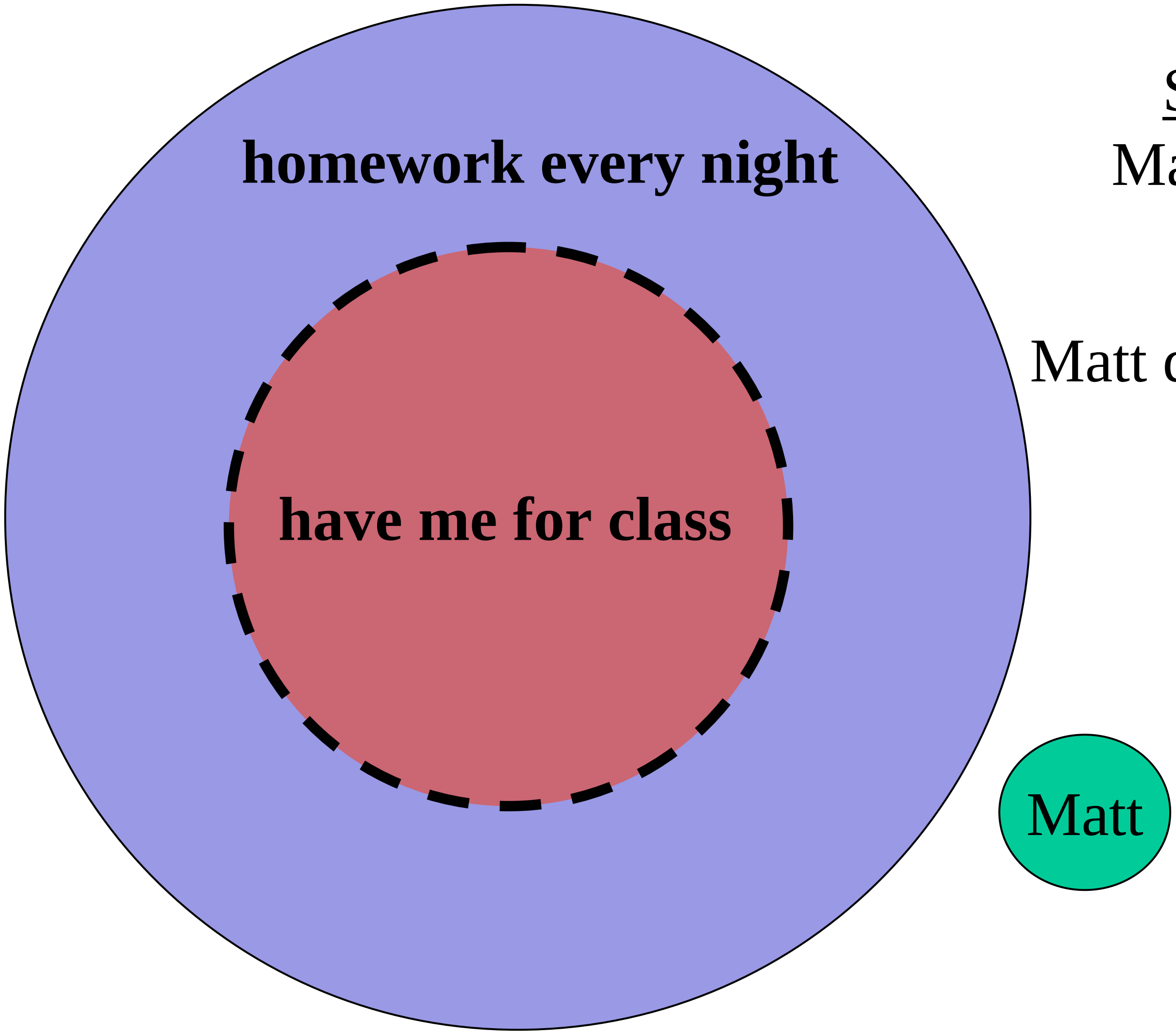


Statement to be tested:
Bill has Mr. Brady.

Result:
No conclusion

Venn Diagrams

If you **have me for class**, then you have **homework every night**.

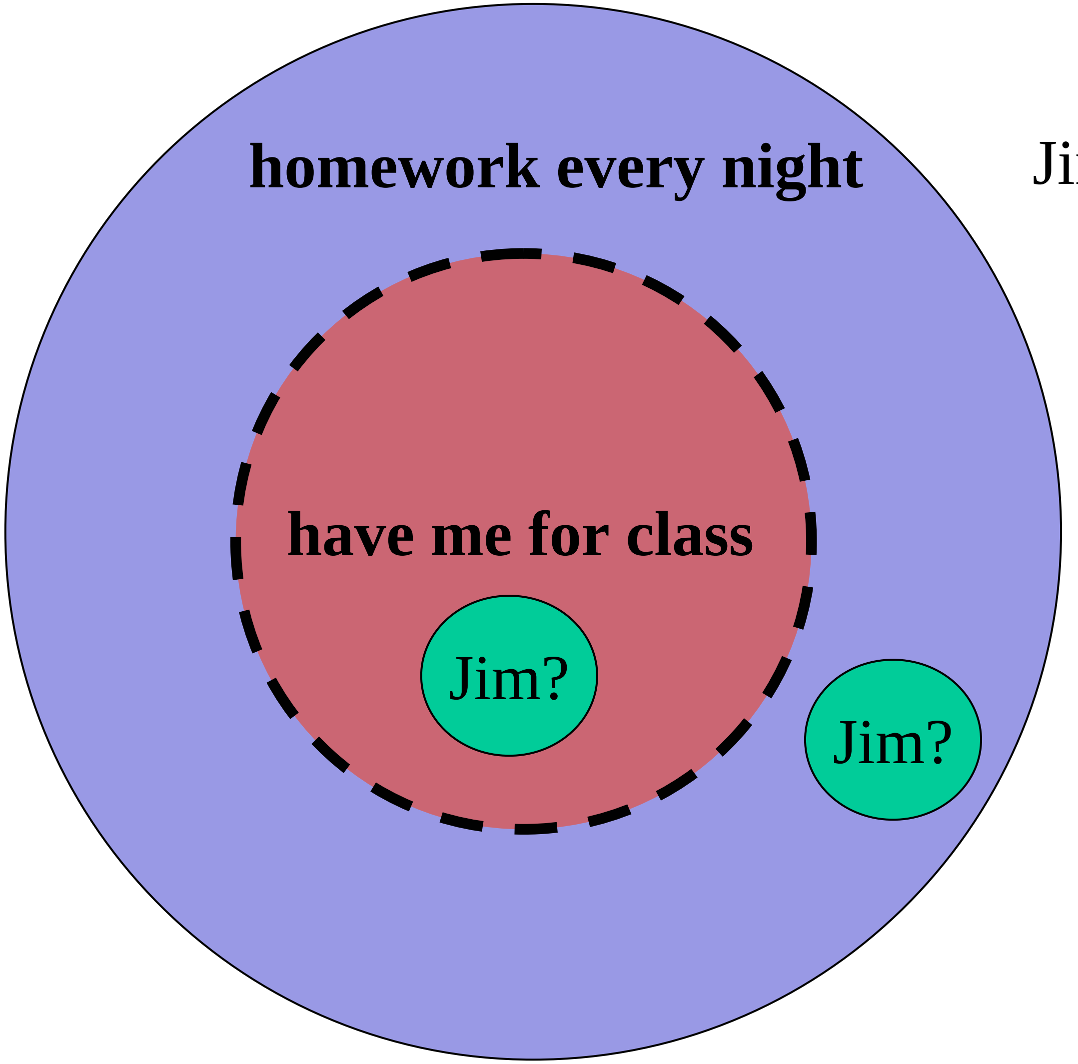


Statement to be tested:
Matt never has homework.

Result:
Matt does not have me for class.

Venn Diagrams

If you **have me for class**, then you have **homework every night**.



Statement to be tested:
Jim has homework every night.

Result:
No conclusion

Sample Problems

Write (a) the contrapositive and (b) the inverse of each statement.

1. If $n = 17$, then $4n = 68$.
3. If x is not even, then $x + 1$ is not odd.

For each statement tell whether it is true or false, then write its contrapositive , converse, and inverse and tell whether each of these is true or false.

5. If I live in Los Angeles, then I live in California.
7. If AM = MB, then M is the midpoint of AB.
9. If $-2n < 6$, then $n > -3$.

Sample Problems

Reword the given statement in if-then form and illustrate it with a Venn diagram. What can you conclude by using the given statement together with each additional statement?

11. Given: All senators are at least 30 years old.

a. Jose Avila is 48 years old.

b. Rebecca Castelloe is a senator.

c. Constance Brown is not a senator.

d. Ling Chen is 29 years old.

13. Given: If it is not raining, then I am happy.

a. I am not happy.

b. It is not raining.

c. I am overjoyed.

d. It is raining.

Sample Problems

15. Given: If two angles are vertical angles, then they are congruent.

a. $\angle 1 \cong \angle 2$

b. $m\angle ABC \neq m\angle DBF$

c. $\angle 3$ and $\angle 4$ are adjacent angles

d. RS and TU intersect at V

17. Given: The diagonals of a rectangle are congruent.

a. PQRS is a rectangle

b. In quad. ABCD, AC = BD

c. WXYZ is not a rectangle.

d. In quad. STAR, SA > TR.

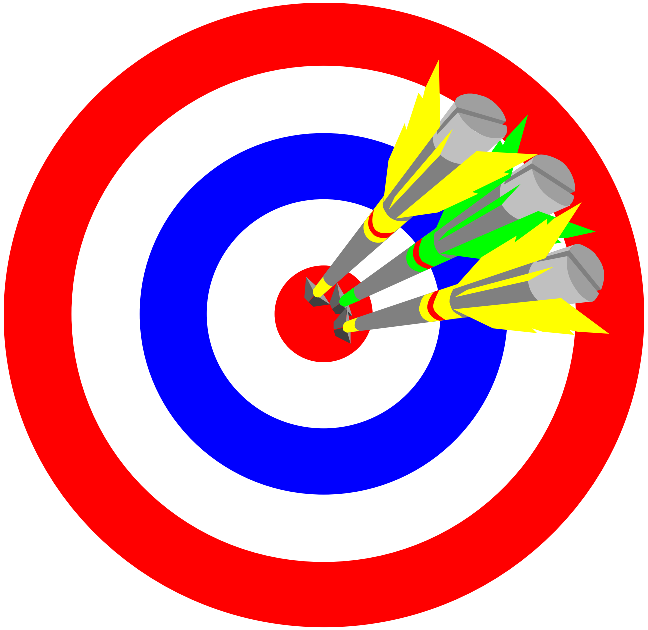
Section 6-3

Indirect Proof

Homework Pages 216-217:

2-18 evens, skip #6

Objectives



A. Understand the term 'indirect proof'.

B. Understand the steps in performing an indirect proof.

C. Apply indirect proof concepts.

Definition

An indirect proof is a method of proving a conditional statement to be true by FIRST assuming that the conclusion is not true. From there, make logical steps to show that if the conclusion is not true, some other statement that is known to be true MUST be false. Therefore, the original conclusion can ONLY be false if some other known true statement is ALSO false. From this, conclude that the original conclusion must be true.

Indirect Proof

How an Indirect Proof Works:

- A conditional must either be true or false.
- An indirect proof works by showing that the statement cannot be false without leading to a contradiction of a true statement.
- Since the statement cannot be false it must therefore be true.

How to Write an Indirect Proof:

1. Temporarily assume that the opposite of the “proof” statement (conclusion) is true.
2. Working from this assumption, write a normal proof until you reach a statement that contradicts either the given, a postulate, theorem, corollary, definition, or property.
3. The reason for the “proof” statement is the contradiction.

Indirect Proof Example

Given: $m\angle X \neq m\angle Y$

Prove: $\angle X$ and $\angle Y$ are NOT both right angles.

1. $m\angle X \neq m\angle Y$	1. Given
2. Angle X and Angle Y are both right angles.	2. Temporarily assume conclusion is not true. (opposite is true)
3. $m\angle X = 90°; \quad m\angle Y = 90°$	3. Definition of right angles (assuming step 2 is true)
4. $m\angle X = m\angle Y$	4. Substitution
5. $\angle X$ and $\angle Y$ are NOT both right angles.	5. Step 4 contradicts the given information. Therefore, original assumption in step 2 must be false.

Indirect Proof Example

$$\text{Given}: 2x + 3 \neq 17$$

$$\text{Prove}: x \neq 7$$

1. $2x + 3 \neq 17$	1. Given
2. $x = 7$	2. Temporarily assume conclusion is not true. (opposite is true)
3. $2(7) + 3 = 17$	3. Substitution
4. $x \neq 7$	4. Step 3 contradicts the given information. Therefore, assumption in step 2 must be false.

Sample Problems

Write the correct first step of the indirect proof.

1. If m $\angle$ A = 50, then m $\angle$ B = 40
3. If a $\neq$ b, then a - b $\neq$ 0.
5. If EF $\neq$ GH, then EF and GH aren't parallel.

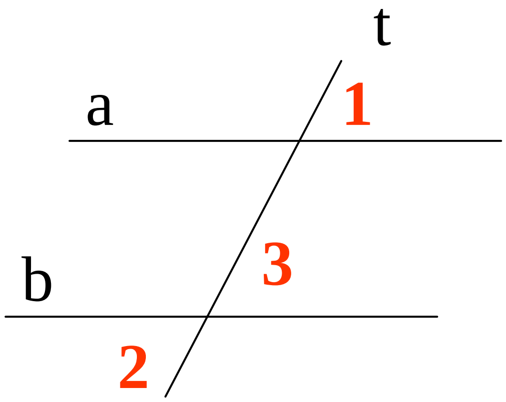
Write an indirect proof.

7. Given: Δ XYZ; m $\angle$ X = 100

   Prove: $\angle$ Y is not a right angle.
9. Given: Transversal t cuts lines a and b;

   m $\angle$ 1 $\neq$ m $\angle$ 2

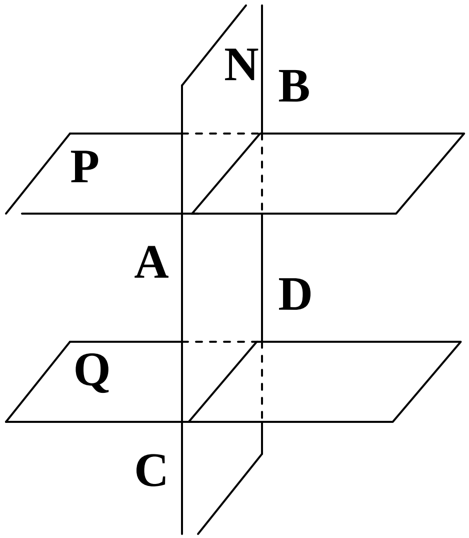
   Prove: a is not parallel to b



Sample Problem

11. Given: AB is not parallel to CD

Prove: planes P and Q intersect

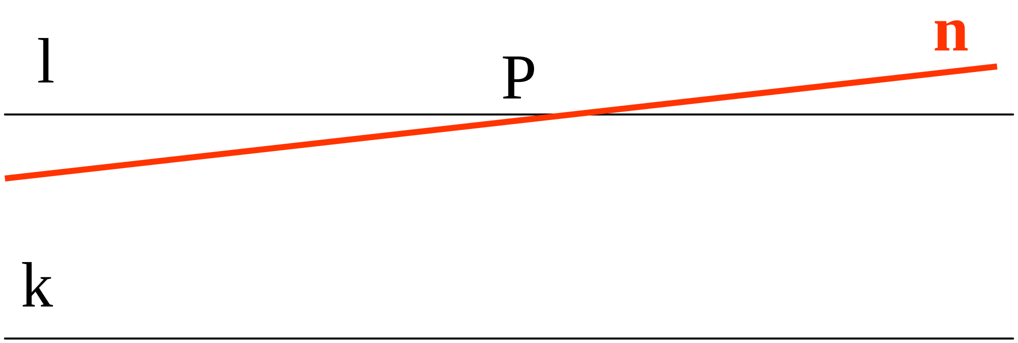


13. Given: quad EFGH in which m ∠ EFG = 93; m ∠ FGH = 20; m ∠ GHE = 147; m ∠ HEF = 34

Prove: EFGH is not a convex quadrilateral

Sample Problems

15. Given: coplanar lines l, k, n;
n intersects l in P; l || k
Prove: n intersects k



17. Prove that there is no regular polygon with an interior angle whose measure is 155.

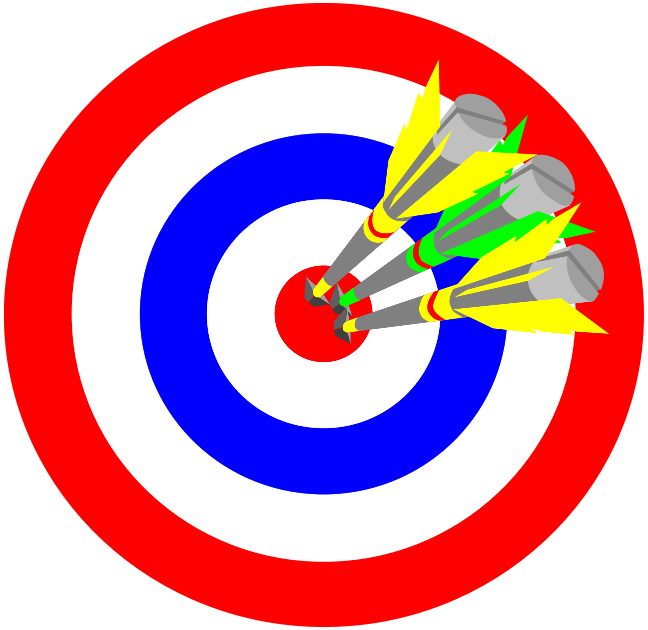
Section 6-4

Inequalities for One Triangle

Homework Pages 222-223:

2-18 evens

Objectives

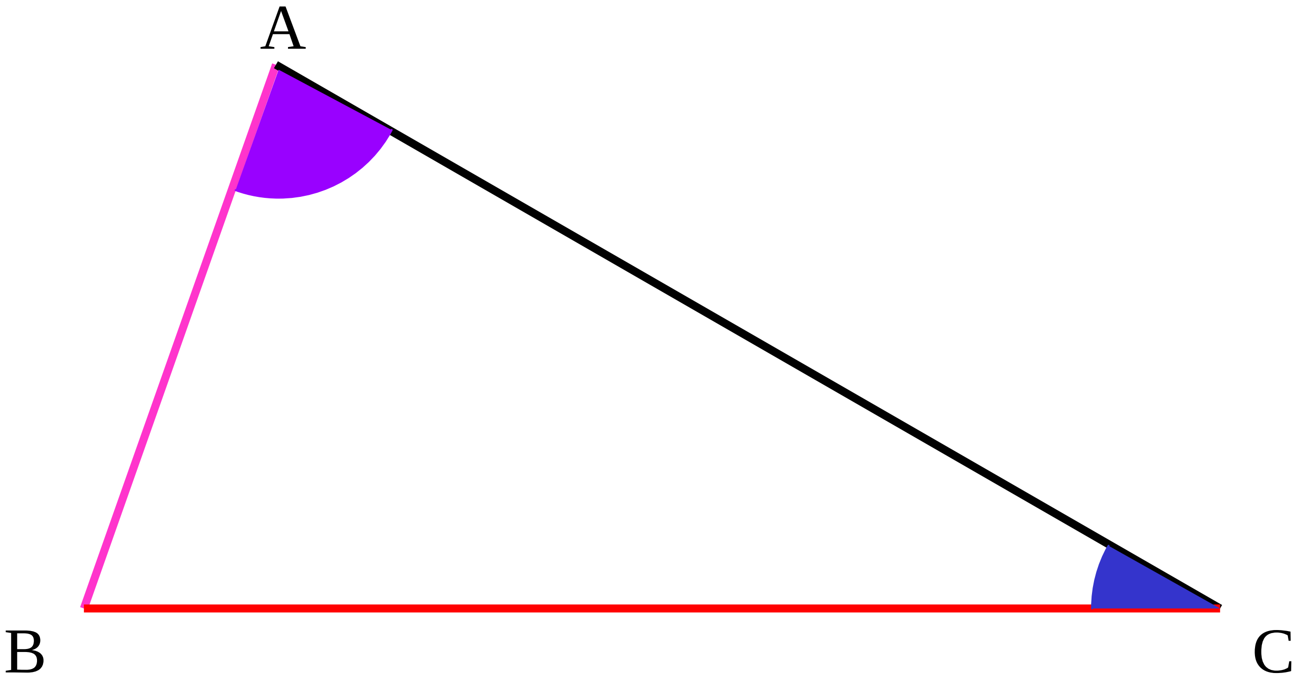


A. Understand and apply the theorems and corollaries concerning inequalities of one triangle.

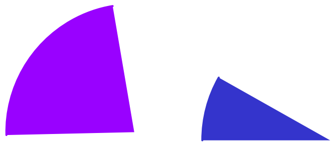
B. Apply indirect reasoning to proof of these theorems and corollaries.

★ Theorem 6-2

If **one side** of a triangle is longer than a **second side**,

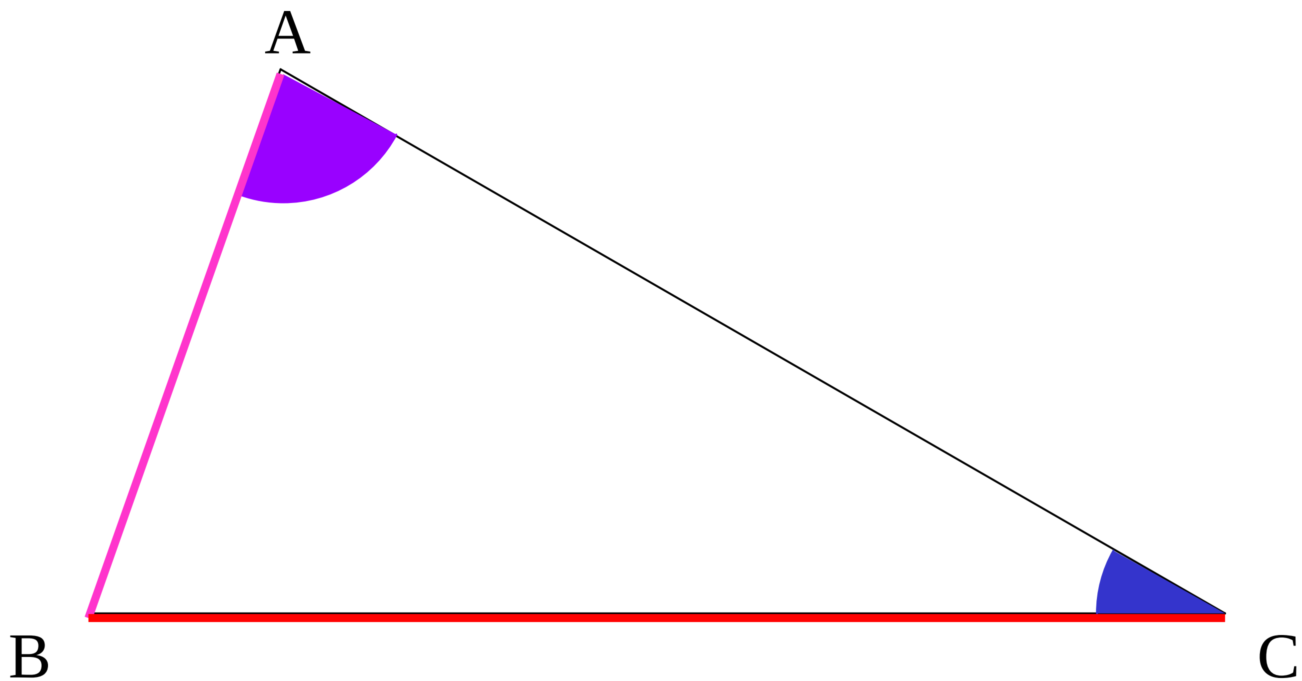


then the **angle opposite the first side** is larger than the **angle opposite the second side**.



★ Theorem 6-3

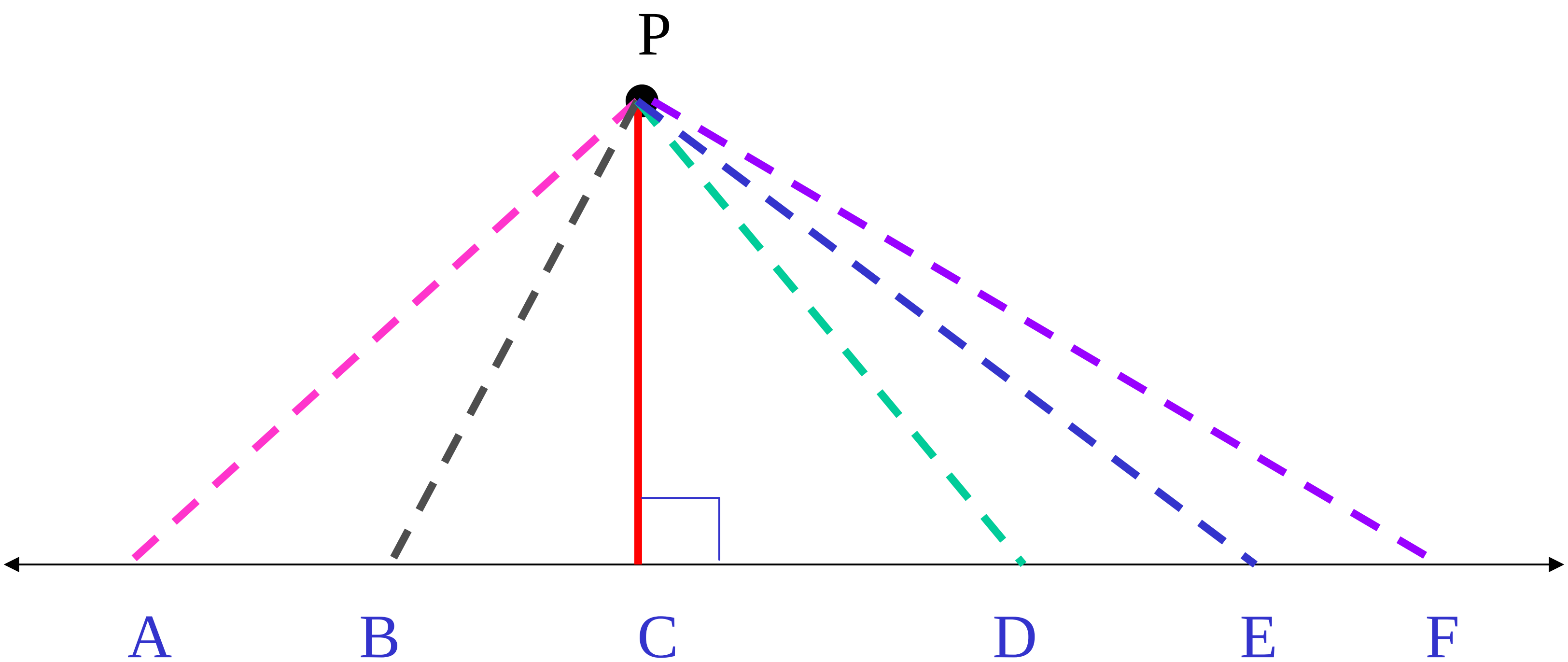
If **one angle** of a triangle is larger than a **second angle**,



then the **side opposite the first angle** is larger than the **side opposite the second angle**.

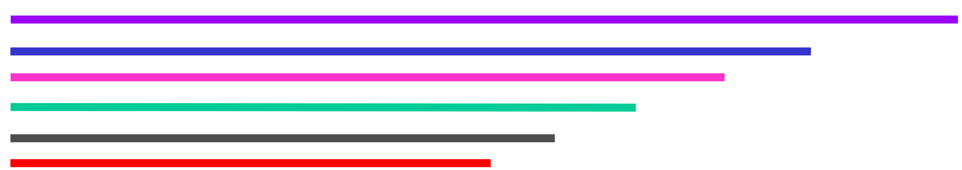
Corollary 1 Theorem 6-3

The perpendicular segment from a point to a line is



the shortest segment from the point to the line.

PC < PA, PC < PB, PC < PD,
PC < PE, PC < PF



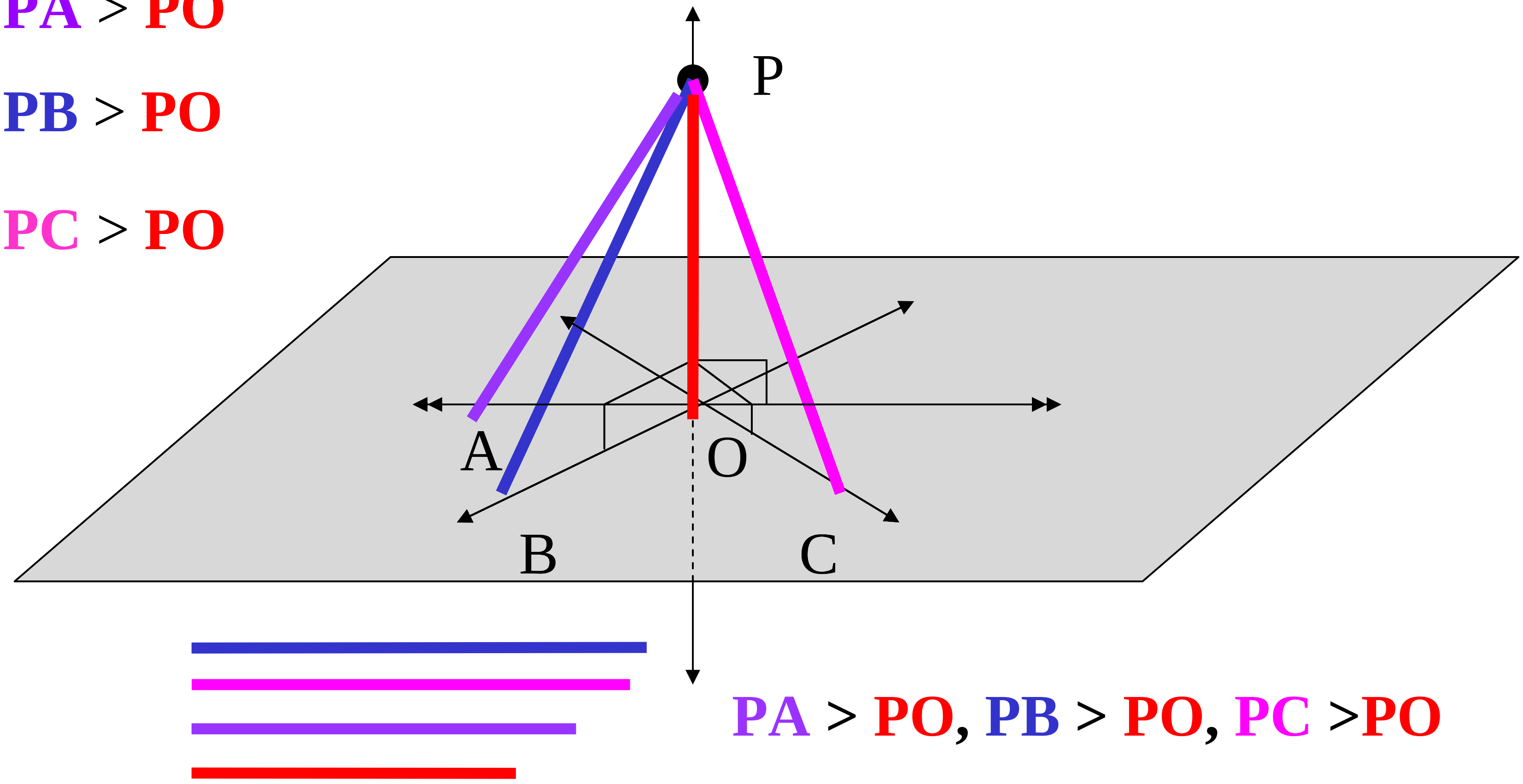
Corollary 2 Theorem 6-3

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

PA > **PO**

PB > **PO**

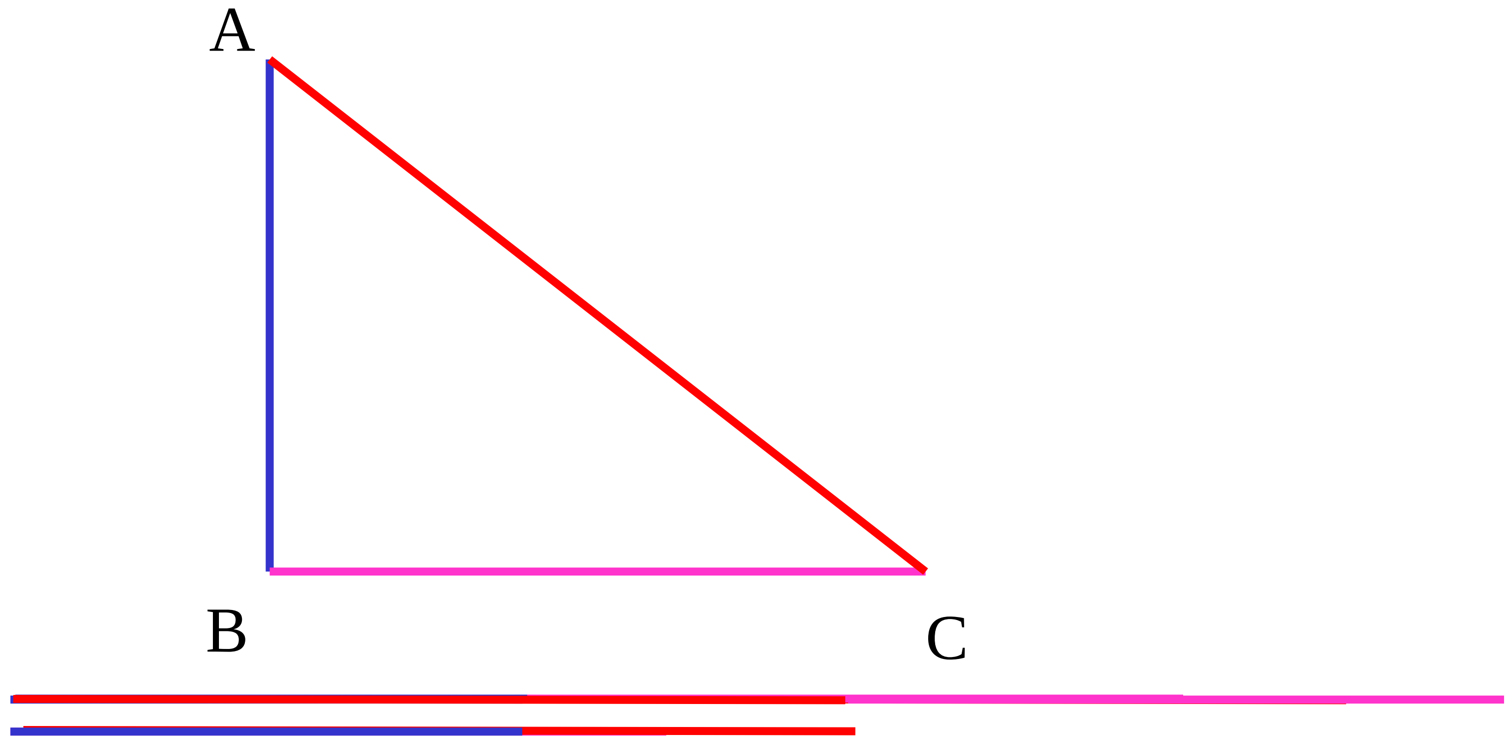
PC > **PO**



PA > **PO**, **PB** > **PO**, **PC** >**PO**

★ Theorem 6-4

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



AC + BC > AB

AB + BC > AC

AB + AC > BC

Sample Problems

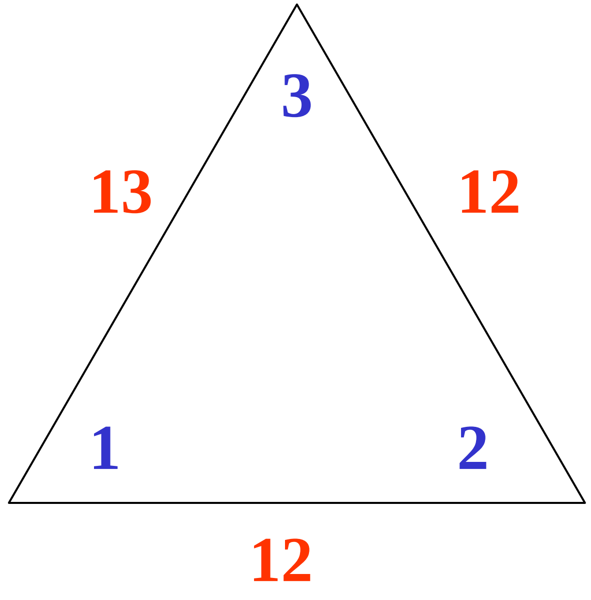
The lengths of two sides of a triangle are given. Write the numbers that best complete the statement: The length of the third side must be greater than ? but less than ?

1. 6, 9
3. 100, 100
5. a, b (where $a > b$)

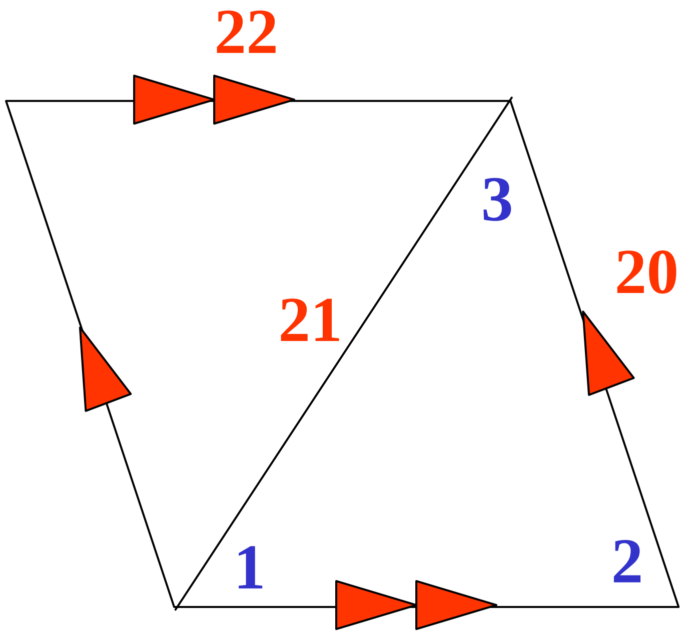
Sample Problems

The diagrams are not drawn to scale. Which numbered angle is the largest?

7.

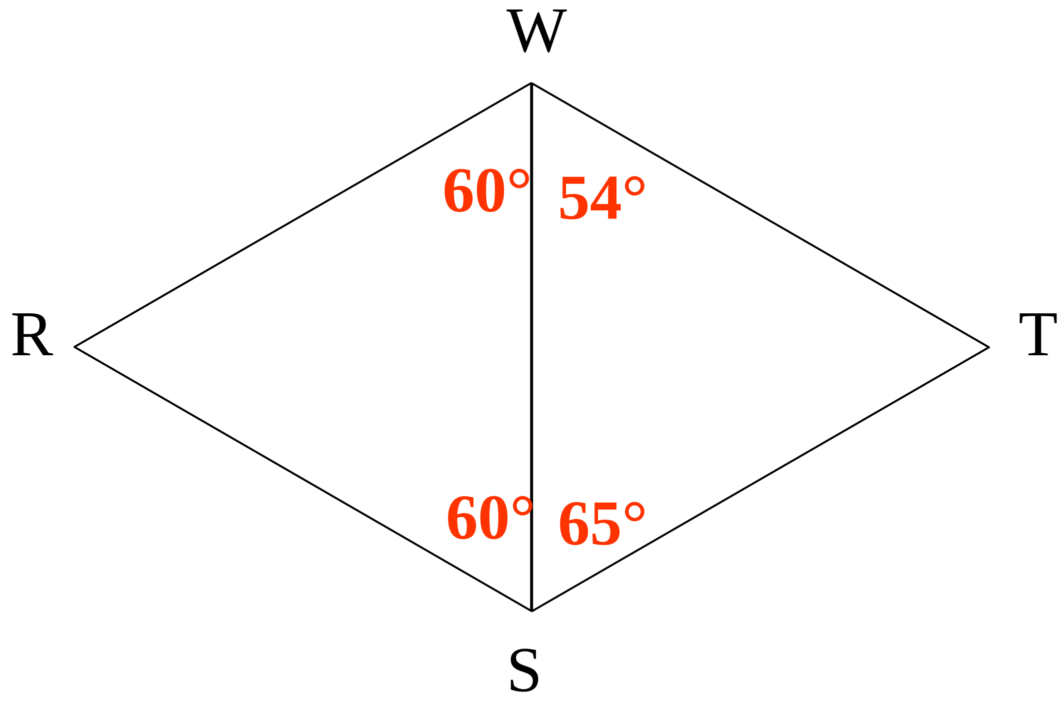


9.



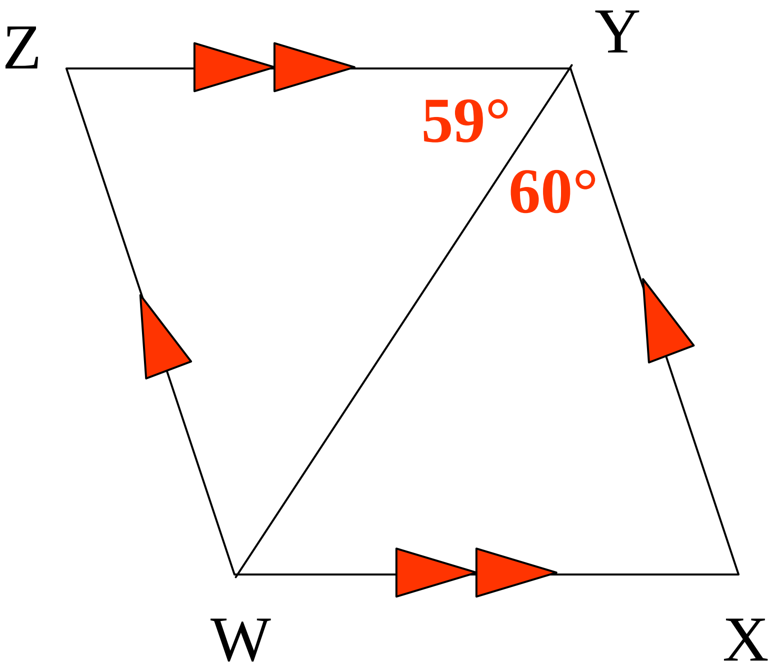
Sample Problems

The diagrams are not drawn to scale. Which segment shown is the longest?



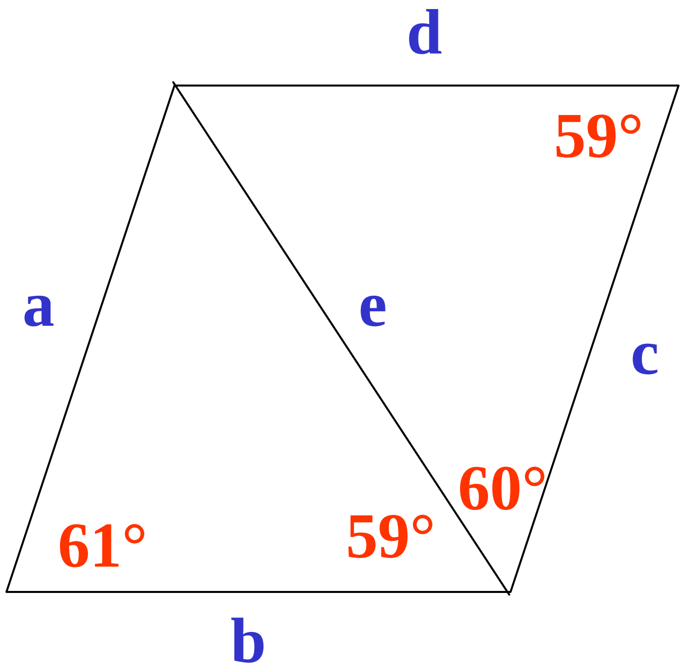
Sample Problems

The diagrams are not drawn to scale. Which segment shown is the longest?



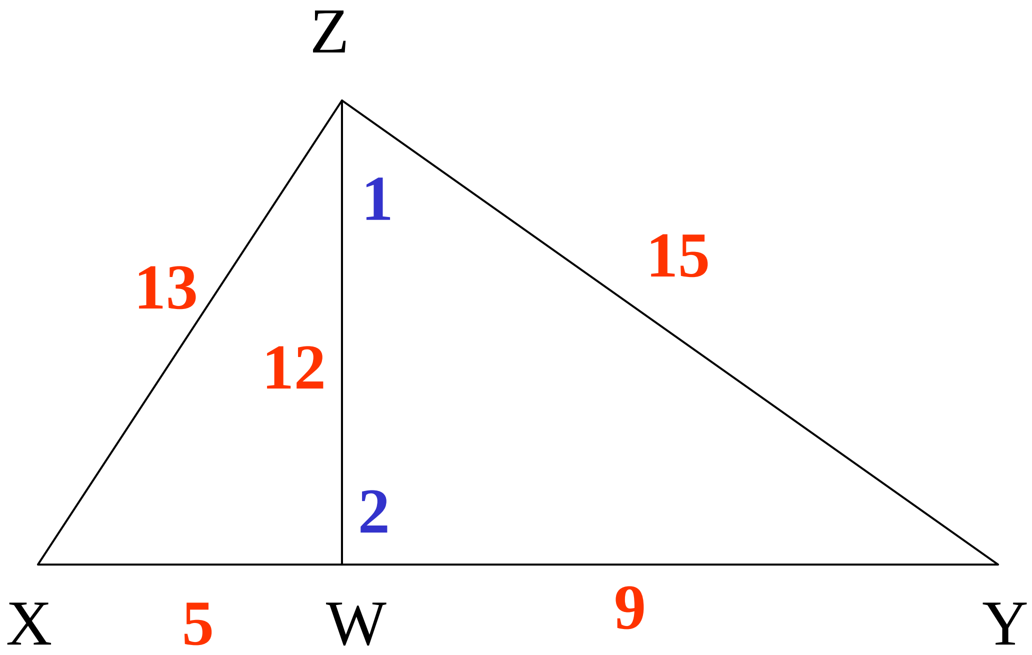
Sample Problems

Use the lengths a, b, c, d, and e to complete: __>__>__>__>__



Sample Problems

The diagram is not drawn to scale. Use m ∠ 1, m ∠ 2, m ∠ X, m ∠ Y, and m ∠ XZY to complete: __>__>__>__>__



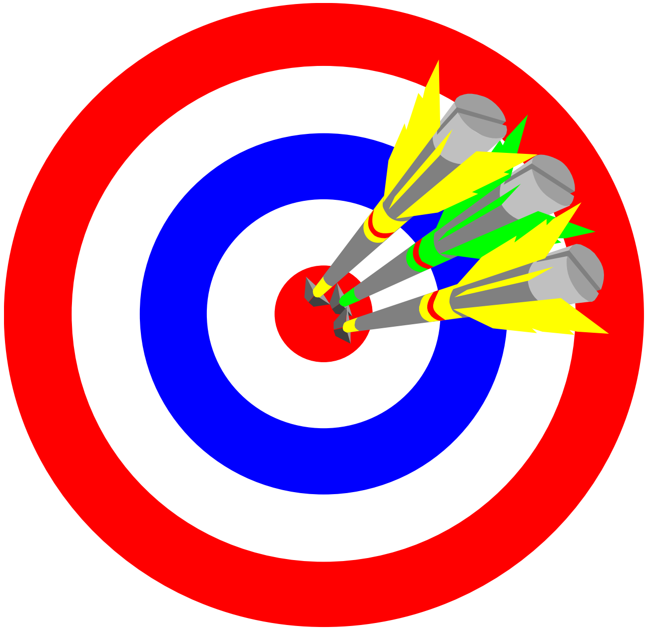
Section 6-5

Inequalities for Two Triangles

Homework Pages 231-232:

2-12 evens

Objectives

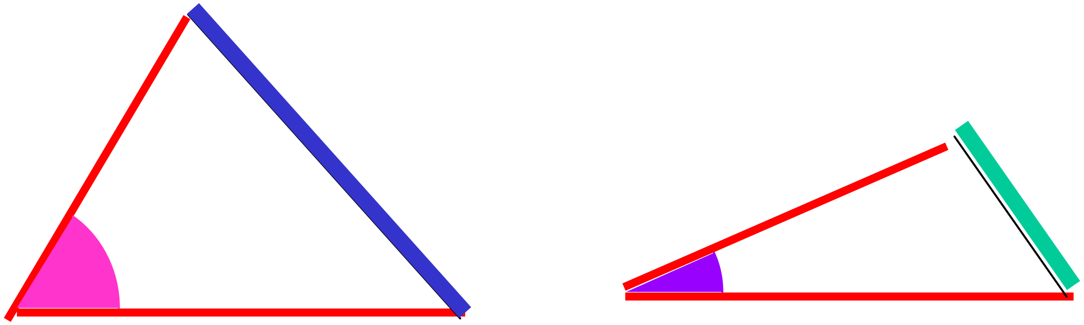


A. Understand and apply the SAS Inequality Theorem.

B. Understand and apply the SSS Inequality Theorem.

★ Theorem 6-5: SAS Inequality

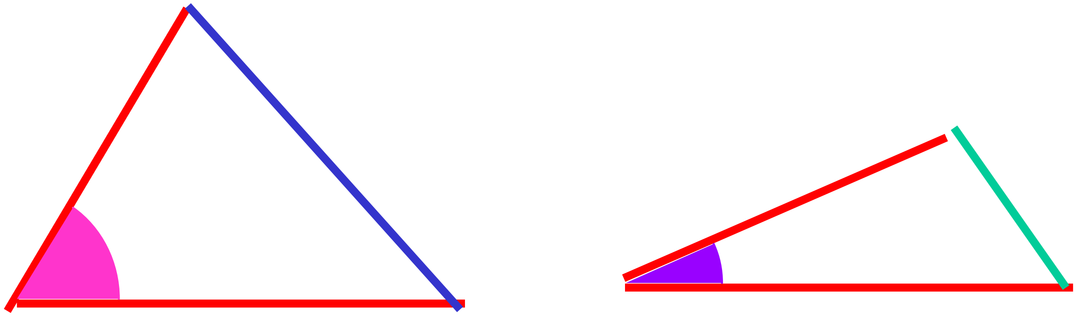
If **two sides** of one triangle are congruent to **two sides** of another triangle, but the **included angle of the first triangle** is larger than the **included angle of the second**,



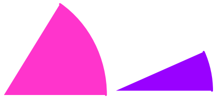
then the **third side** of the first triangle is longer than the **third side** of the second triangle.

★ Theorem 6-6: SSS Inequality

If **two sides** of one triangle are congruent to **two sides** of another triangle, but the **third side** of the first triangle is larger than the **third side** of the second,



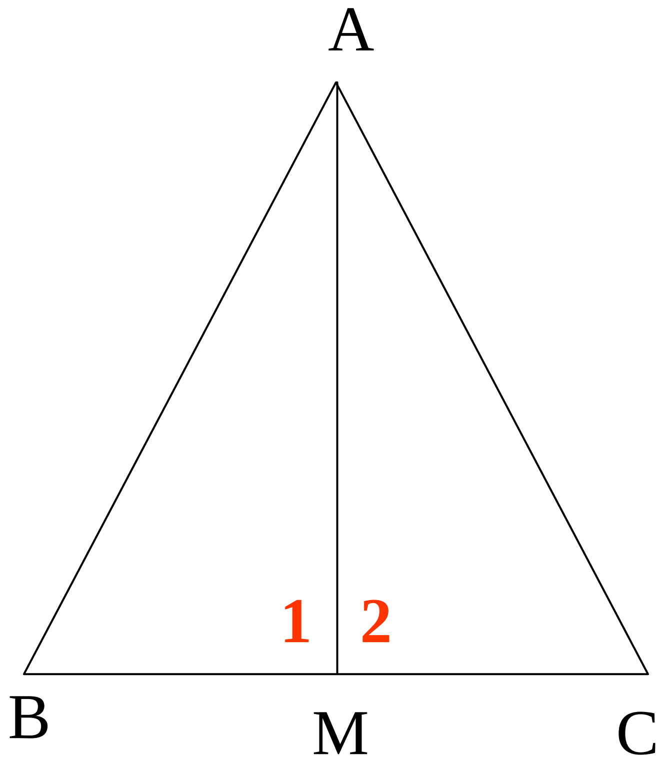
then the **included angle of the first triangle** is larger than the **included angle of the second triangle**.



Sample Problems

What can you deduce? Name the theorem that supports your answer.

1. Given: AM is the median of Δ ABC; AB > AC

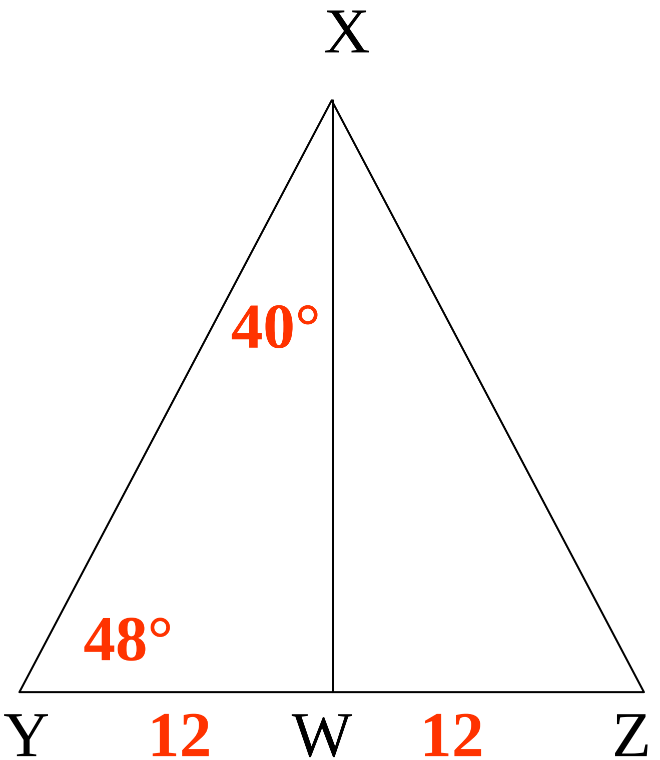


Sample Problems

Complete the statement by writing <, =, or >.

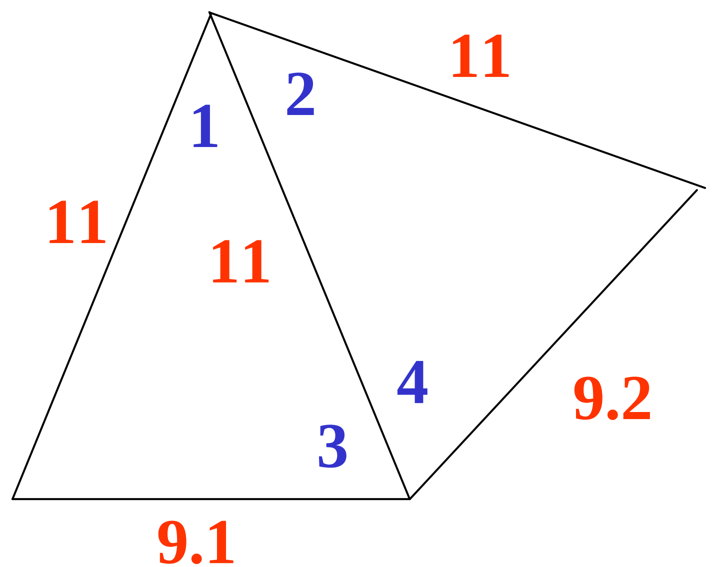
3. XY _____ XZ

XW _____ 12



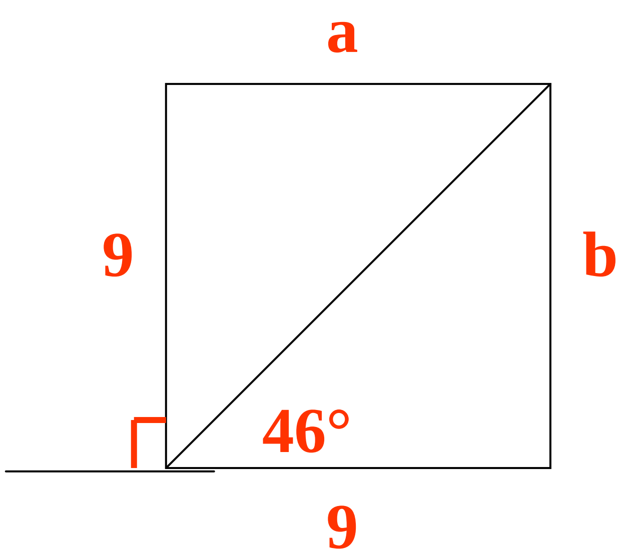
5. m ∠ 1 _____ m ∠ 2

m ∠ 3 _____ m ∠ 4

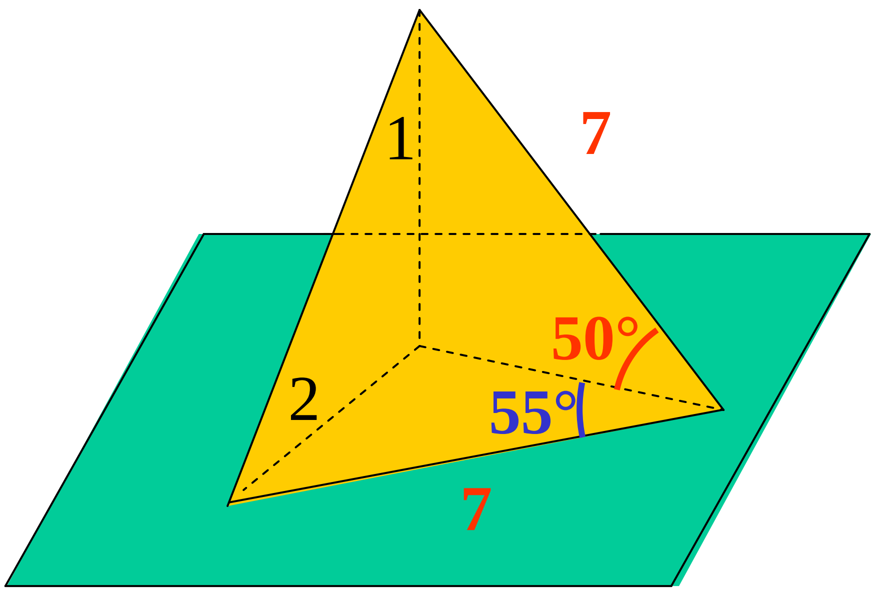


Sample Problems

7. a _____ b



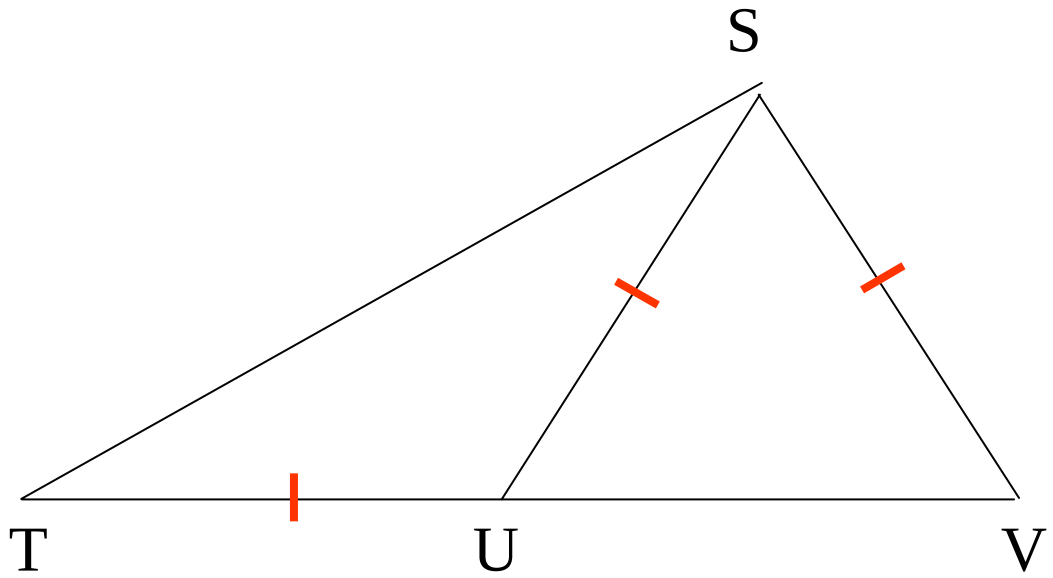
9. m ∠ 1 _____ m ∠ 2



Sample Problems

11. Given: TU = US = SV

    Prove: ST > SV



Chapter 6

Inequalities in Geometry

Review

Homework Pages 236-237:

2-16 evens