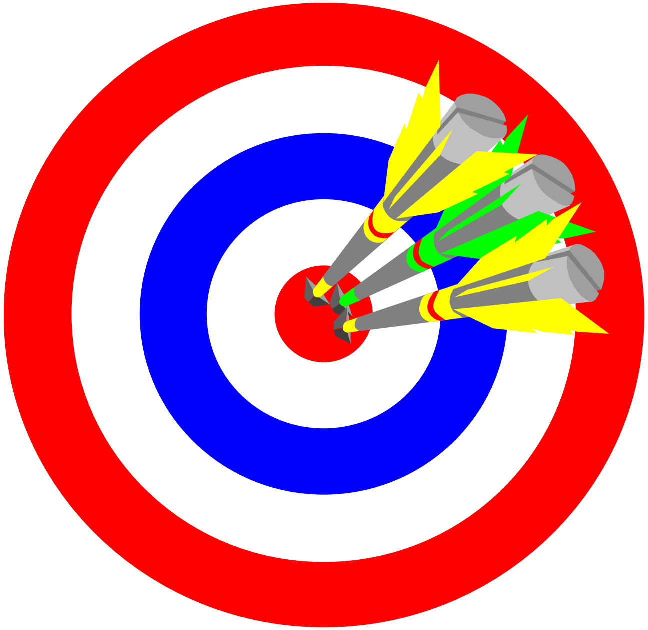
Chapter 7

Similarity

Objectives

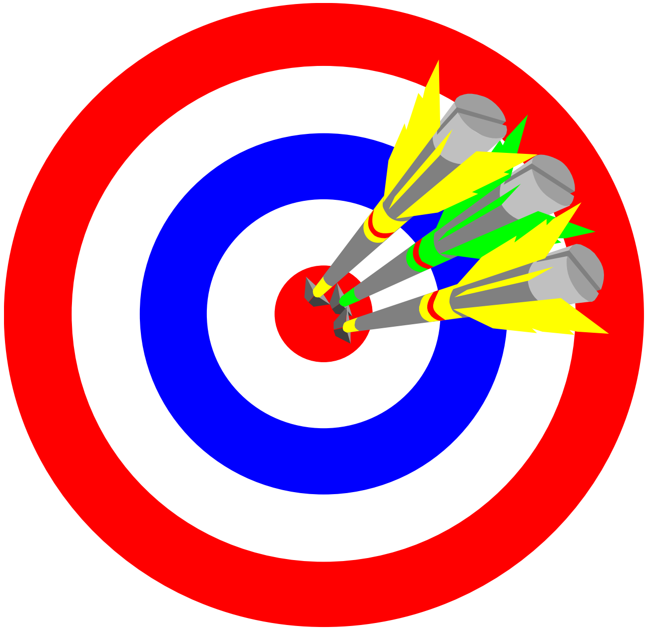


A. Use the terms defined in the chapter correctly.
B. Properly use and interpret the symbols for the terms and concepts in this chapter.
C. Appropriately apply the properties and theorems in this chapter.
D. Manipulate and solve ratios and proportions.
E. Identify and use similar polygons.
F. Use proportional lengths to solve for unknown values.

Section 7-1

Ratio and Proportion
Homework Pages 243-244:
2-32 evens
Excluding 8, 12, 28

Objectives



A. Use the terms ratio and proportion correctly.
B. Properly express ratios and proportions.
C. Properly setup and solve ratio and proportion problems, especially those referring to geometric shapes.

Ratios

- Ratio → a quotient expressed in *simplest form*.
  - The ratio of x to y can be written as either
    - x:y or
    - $\frac{x}{y}$
- Ratios are used to express the relationship between numbers or quantities
  - There are 3 feet in every yard
    - 3 feet:1 yard or $\frac{3\,feet}{1\,yard}$
  - The ratio of 4 to 5
    - 4:5 or $\frac{4}{5}$

Ratios

Ratios MUST always be expressed in their SIMPLEST form!

$$\frac{10}{50} \Rightarrow \frac{1}{5}$$

$$\frac{3x^2}{7x} = \frac{3 \bullet x \bullet x}{7 \bullet x} = \frac{3 \bullet x}{7} \bullet \frac{x}{x} = \frac{3x}{7} \bullet 1 \Rightarrow \frac{3x}{7}$$

$$\$10 \text{ for every } 25 \text{ miles} \Rightarrow \frac{\$10}{25\,miles} \Rightarrow \frac{\$2}{5\,miles}$$

Ratios

Be CAREFUL of units of measure!

What is the ratio between 26 centimeters and 2 meters?

$$\frac{26\ centimeters}{2\ meters} = \frac{26\ centimeters}{200\ centimeters} = \frac{13\ centimeters}{100\ centimeters} =$$

$$\frac{13\ centimeters}{1\ meter}$$

Ratios

- Ratios can also compare:
  - The lengths of line segments
  - The measurements of angles
  - The perimeters of figures
  - The area of figures
  - The volume of figures

Proportions

- Proportion → an equation stating that two ratios are equal.

Compare the ratio of a to b to the ratio of c to d.

$$\frac{a}{b} = \frac{c}{d} \text{ or } ad : bc$$

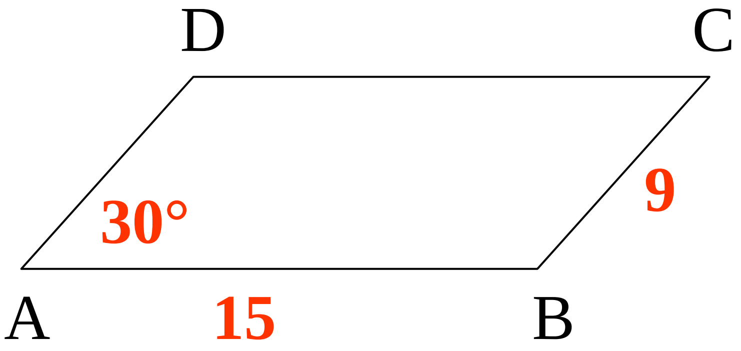
Also read as "a is to b as c is to d".

Also possible to have $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

Sample Problems

ABCD is a parallelogram. Find the value of each ratio.

1. AB : BC
3. m∠C : m∠D
5. AD : perimeter of ABCD



If x = 12, y = 10 and z = 24, then write each ratio in simplest form.

7. z to x
9. $\frac{x}{x+z}$
11. $\frac{y+z}{x-y}$
13. z : x : y

Sample Problems

Express the ratio of the height to the base in simplest form.

	15.	17.	19.
height	5 km	0.6 km	8 cm
base	45 km	0.8 km	50 mm

Write the algebraic ratio in simplest form.

21. $\frac{3a}{4ab}$

23. $\frac{3(x+4)}{a(x+4)}$

Sample Problem

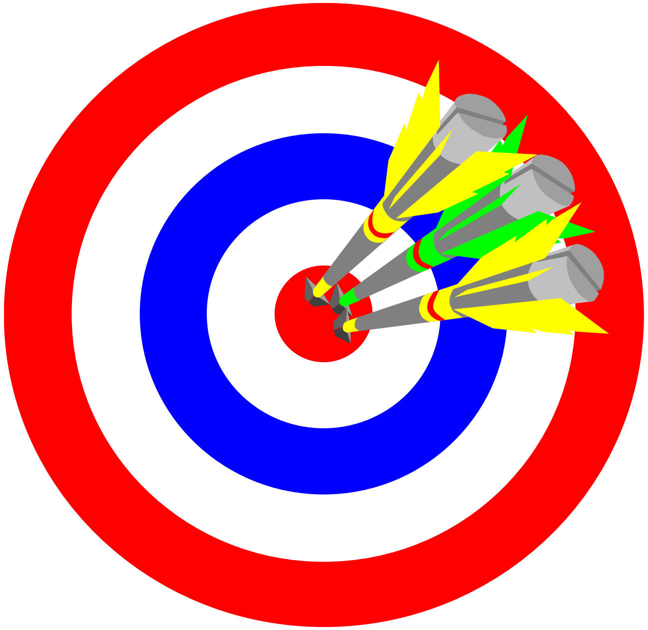
Find the measure of each angle.

25. The ratio of the measures of two supplementary angles is 11 : 4
27. The measures of the acute angles of a right triangle are in the ratio 5 : 7.
29. The measures of the angles of a hexagon are in the ratio 4 : 5 : 5 : 8 : 9 : 9.
31. The measures of the consecutive angles of a quadrilateral are in the ratio 5 : 7 : 11 : 13. Find the measure of each angle, draw a quadrilateral the satisfies the requirements, and explain why two sides must be parallel.

Section 7-2

Properties of Proportions
Homework Pages 247-248:
2-34 evens
Excluding 6, 16, 28

Objectives



A. Use the terms means, extremes, extended proportions and equivalent proportions correctly.
B. Properly identify and apply the properties of proportions.

Definitions

- Extremes → In the proportion a:b = c:d
  - a and d are called the ***extremes***
- Means → In the proportion a:b = c:d
  - b and c are called the ***means***
- Equivalent proportions → have equal means and equal extremes.
- Extended proportion → an equation stating that three or more ratios are equal

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$$

★Properties of Proportions

- To find equivalent proportions:
  - cross multiply (means-extremes)
  - switch values diagonally
  - flip both fractions
  - add the denominator of each fraction to the numerator of each fraction
- Extended proportions: the sum of all the numerators divided by the sum of all the denominators equals any one fraction in the extended proportion.

★ Properties of Proportions

Given $\frac{a}{b} = \frac{c}{d}$ then each of the following will be true:

1. $ad = bc$ — means-extremes
2. $\frac{a}{c} = \frac{b}{d}$ — switch one diagonal
3. $\frac{b}{a} = \frac{d}{c}$ — flip
4. $\frac{a+b}{b} = \frac{c+d}{d}$ — add bottom to the top

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then $\frac{a+c+e+\ldots}{b+d+f+\ldots} = \frac{a}{b} = \ldots$

Sample Problems

Complete each statement.

1. If $\frac{x}{3} = \frac{2}{5}$, then $5x = ?$

3. If $a : 3 = 7 : 4$, then $4a = ?$

5. If $\frac{a}{4} = \frac{b}{7}$, then $\frac{a}{b} = \frac{?}{?}$

7. If $\frac{x}{2} = \frac{y}{3}$, then $\frac{x+2}{2} = \frac{?}{?}$

Sample Problems

Find the value of x.

9. $\frac{x}{4} = \frac{3}{5}$

11. $\frac{2}{5} = \frac{3x}{7}$

13. $\frac{x+5}{4} = \frac{1}{2}$

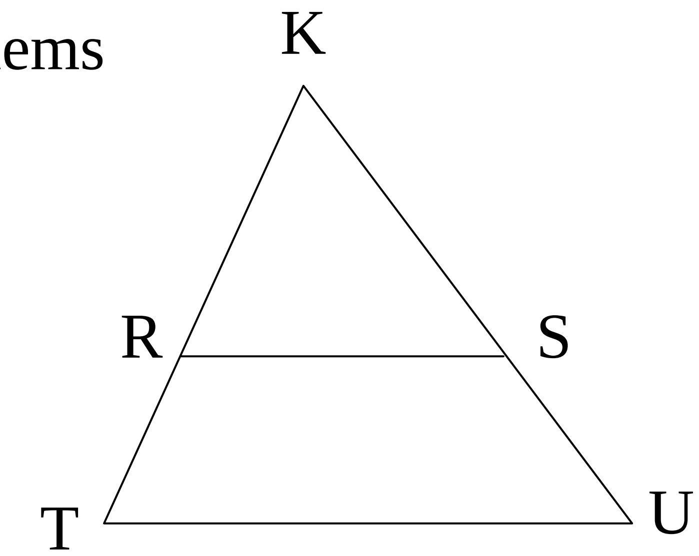
15. $\frac{x+2}{x+3} = \frac{4}{5}$

17. $\frac{x+3}{2} = \frac{2x-1}{3}$

19. $\frac{7}{6x-4} = \frac{9}{4x+6}$

Sample Problems

Complete the table if $\frac{KR}{RT} = \frac{KS}{SU}$.



	KR	RT	KT	KS	SU	KU
21.	12	9	?	16	?	?
23.	16	?	?	?	10	30
25.	?	?	12	10	5	?
27.	?	9	36	?	?	48

Sample Problems

Show that the proportions are equivalent.

29. $\frac{a+b}{b} = \frac{c+d}{d}$ and $\frac{a}{b} = \frac{c}{d}$

31. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ and $\frac{a}{b} = \frac{c}{d}$

Sample Problems

Find the value of x.

33. $\frac{x}{x+5} = \frac{x-4}{x}$

36. $\frac{x-1}{x-2} = \frac{x+4}{x+2}$

35. $\frac{x+1}{x-2} = \frac{x+5}{x-6}$

37. $\frac{x(x+5)}{4x+4} = \frac{9}{5}$

38. $\frac{x-1}{x+2} = \frac{10}{3x-2}$

Sample Problems

Find the values of x and y.

39. $$\frac{y}{x-9} = \frac{4}{7}$$

$$\frac{x+y}{x-y} = \frac{5}{3}$$

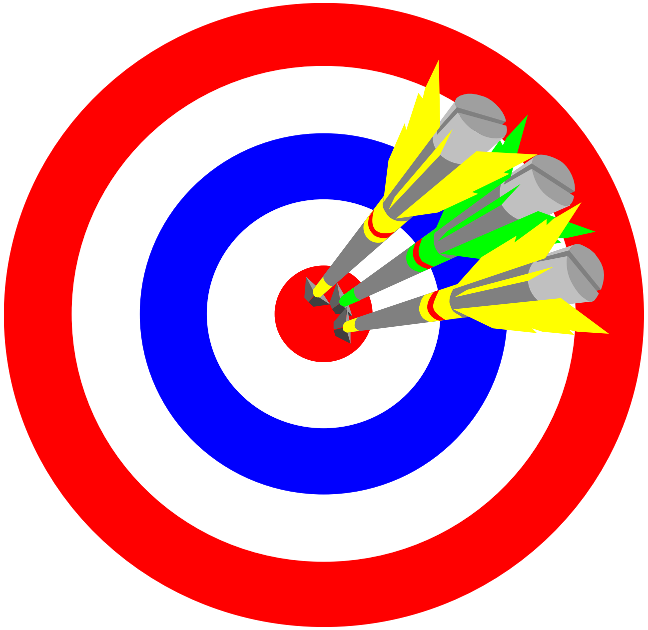
40. $$\frac{x-3}{4} = \frac{y+2}{2}$$

$$\frac{x+y-1}{6} = \frac{x-y+1}{5}$$

Section 7-3

Similar Polygons
Homework Pages 250-252:
2-34 evens
Excluding 10, 20, 30

Objectives



A. Understand and apply the terms 'similar polygons' and 'scale factor'.
B. Calculate scale factors correctly.
C. Apply scale factors to similar polygons.

Similar Polygons

- ★ Similar Polygons: Two or more polygons with vertices can be paired so that:
  - – ***corresponding*** angles are **congruent**
  - – ***corresponding*** sides are in **proportion**
    - In other words, their lengths have the same ratio.
- The symbol for similar is ~

$\Delta ABC \sim \Delta DEF$ reads "Triangle ABC is SIMILAR to Triangle DEF."

CP??

- Remember!

  CPCT → **C**orresponding **P**arts of **C**ongruent **T**riangles

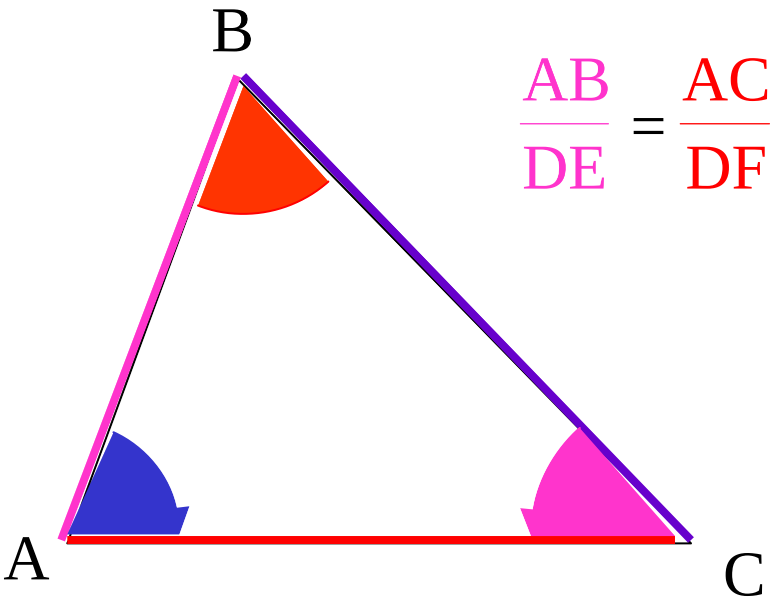
  CPCP → **C**orresponding **P**arts of **C**ongruent **P**olygons

- Also!

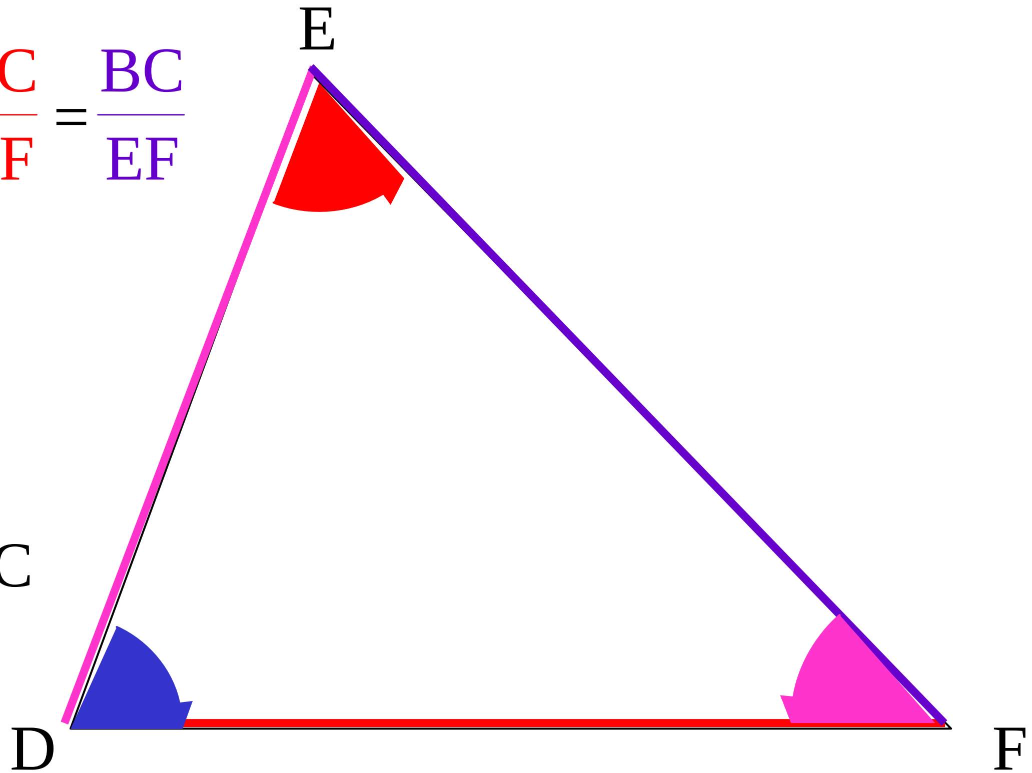
  CPST → **C**orresponding **P**arts of ***S****imilar* **T**riangles

  CPSP → **C**orresponding **P**arts of ***S****imilar* **P**olygons

★ Similar Polygons & CPST



$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

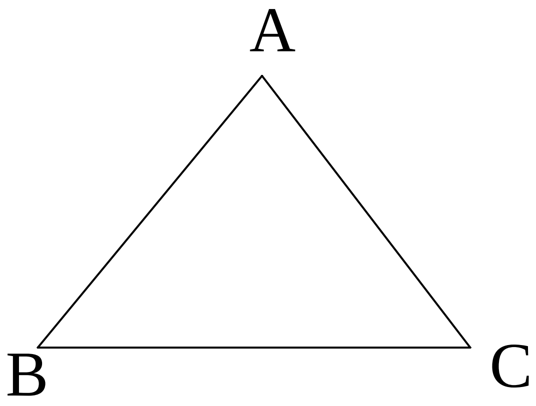


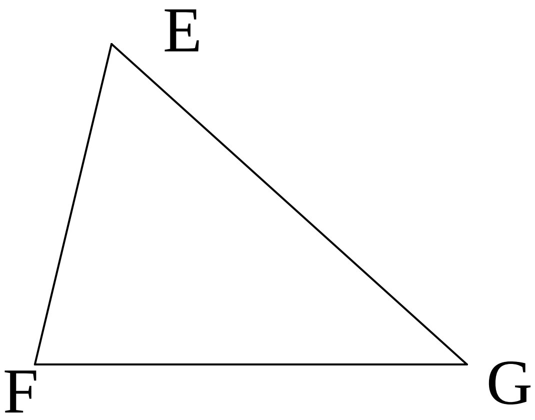
∠ A ≅ ∠ D, ∠ B ≅ ∠ E, ∠ C ≅ ∠ F

★ Similar Polygons & CPST

What could you conclude about the two triangles?

$$\text{If } AB = EF,\ BC = FG,\ AC < EG?$$





$$\text{Is } \frac{AB}{BC} = \frac{EF}{FG}?$$

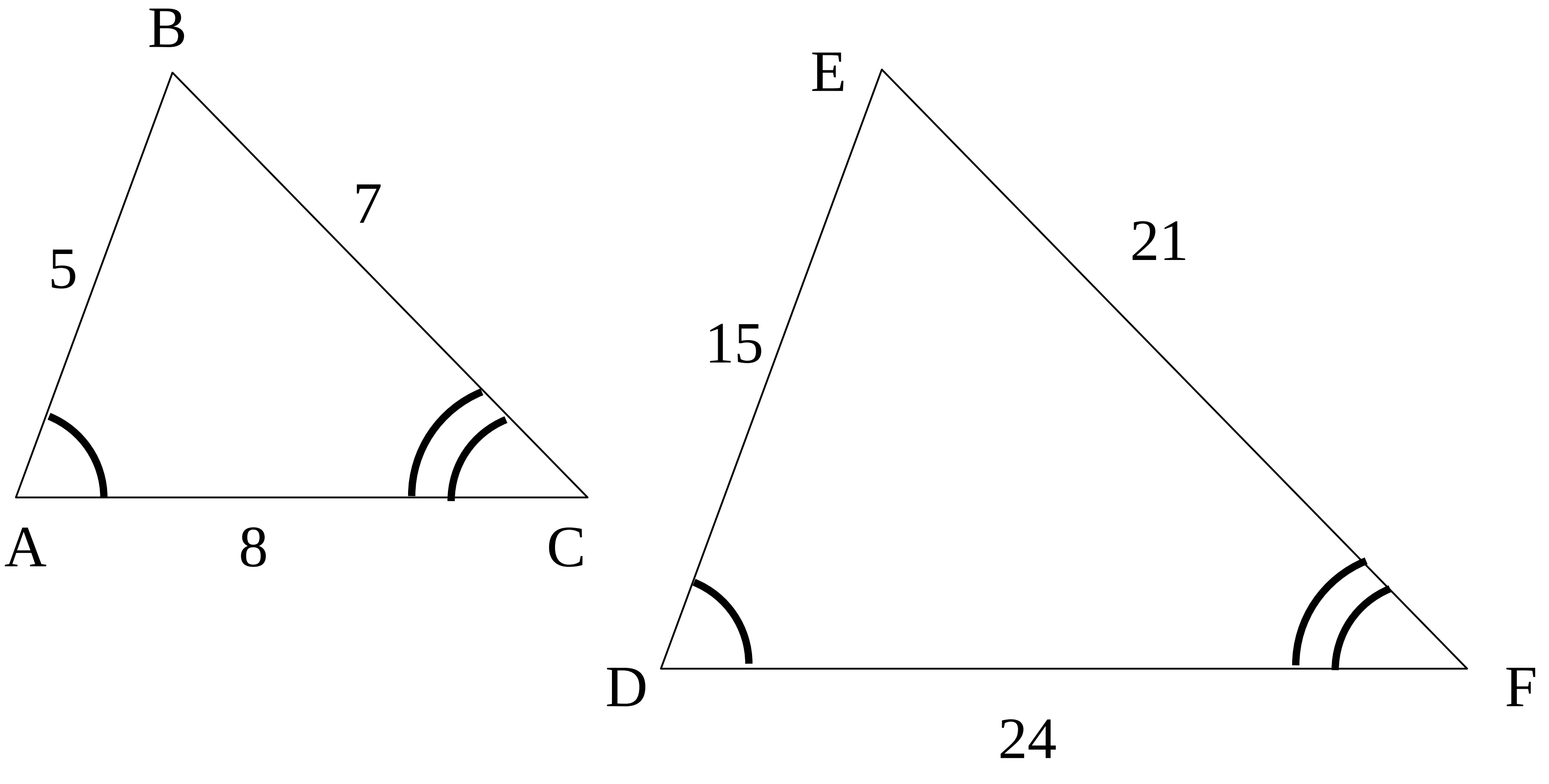
$$\text{Is } \frac{AC}{BC} = \frac{EG}{FG}?$$

$$\text{Is } \Delta ABC \sim \Delta EFG?$$

★ Scale Factor

★ Scale Factor: the ratio of corresponding sides for similar polygons

$$\frac{\text{small}}{\text{big}} = \frac{5}{15} = \frac{7}{21} = \frac{8}{24} = \frac{1}{3}$$



Sample Problems

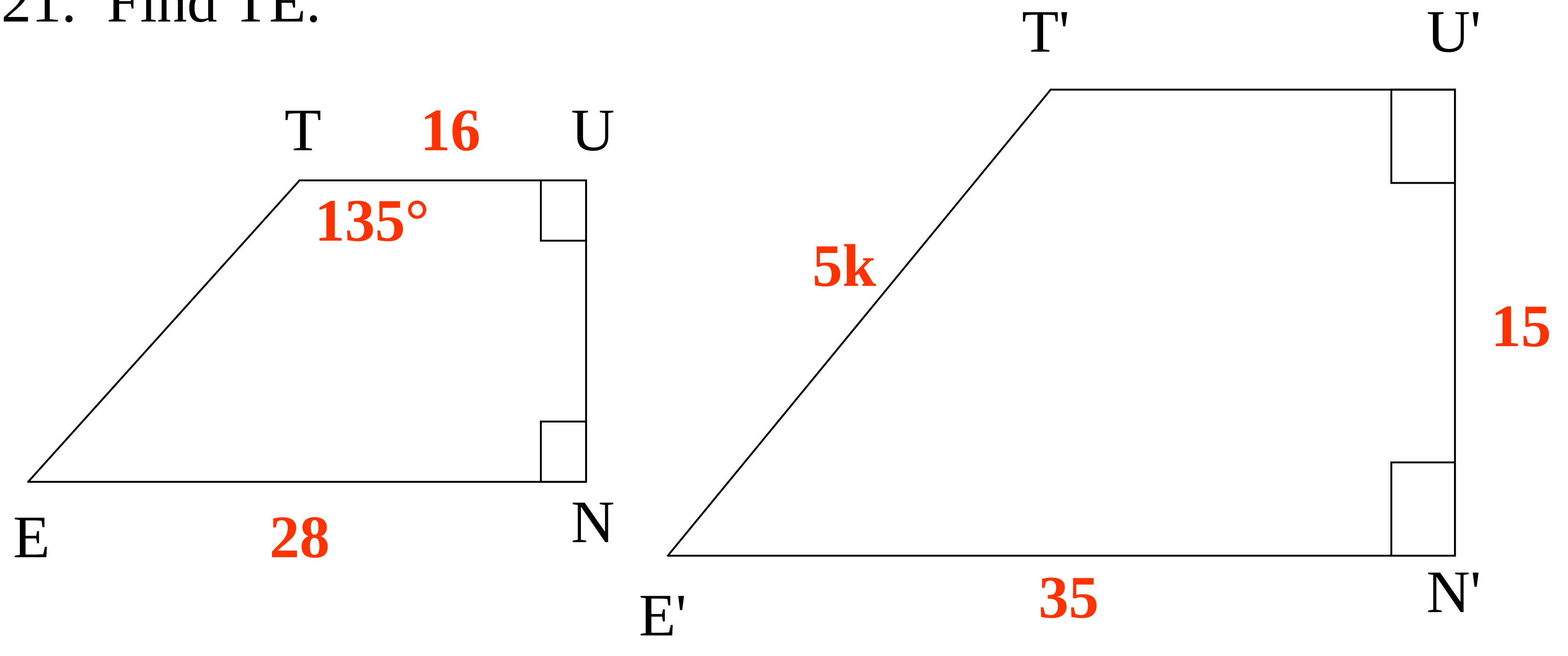
Tell whether the two polygons are always, sometimes or never similar.

1. two equilateral triangles
3. two isosceles triangles
5. two squares
7. two rhombuses
9. two regular hexagons
11. a right triangle and an acute triangle
13. a right triangle and a scalene triangle

Sample Problems

TUNE ~ T'U'N'E'

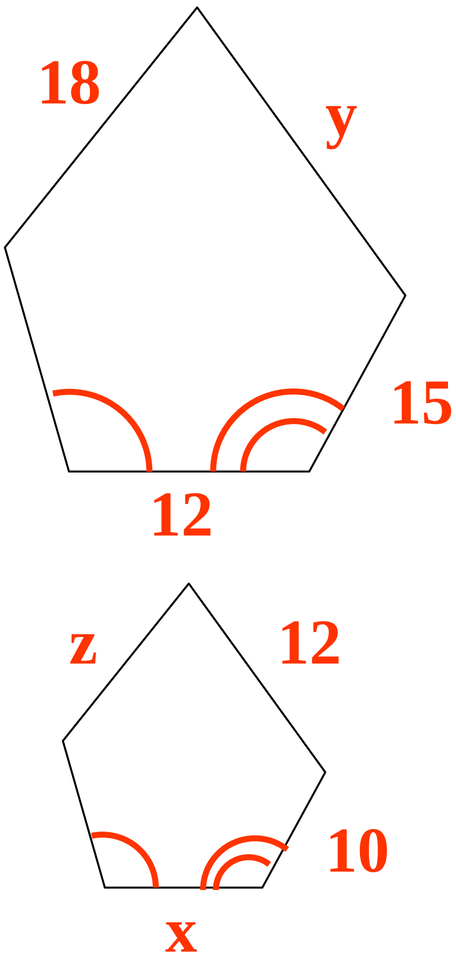
15. What is the scale factor of TUNE to T'U'N'E'?
17. Find m $\angle$ T'.
19. Find UN.
21. Find TE.



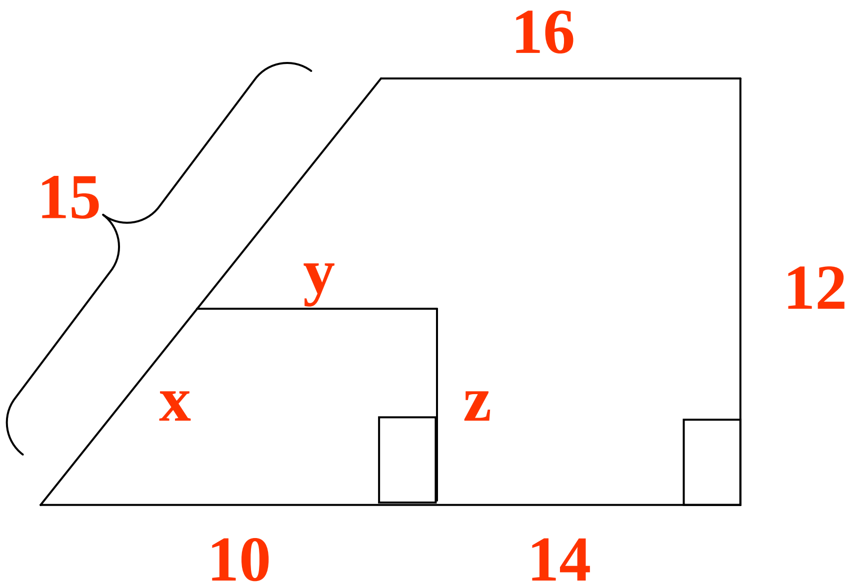
Sample Problems

The polygons are similar. Find the values of x, y and z.

25.

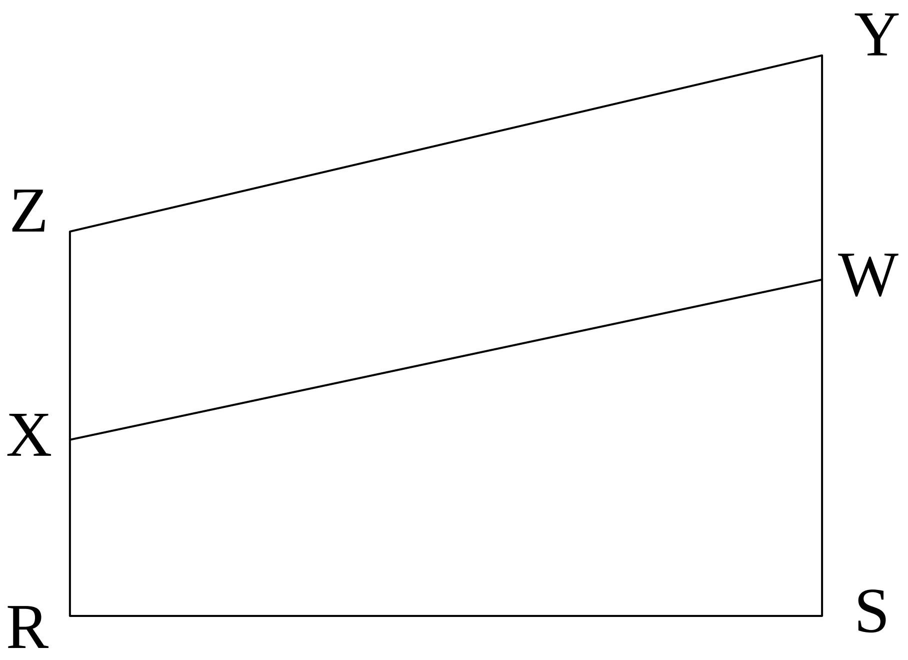


27.



Sample Problems

29. Draw two equiangular hexagons that are clearly not similar.
31. Explain how you can tell that RSWX is not similar to RSYZ.



Sample Problems

Plot the points on graph paper. Draw ABCD and A'B'.

Locate points C' and D' so that ABCD ~ A'B'C'D'

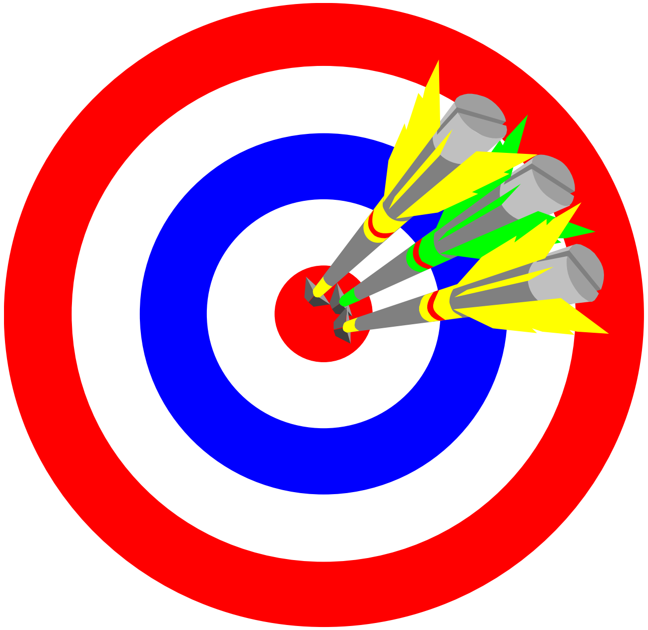
33. A(0, 0) B(4, 0) C(2, 4) D(0, 2) A' (7, 2) B' (7, 0)

35 WHAT is a figure such that WHAT ~ HATW. Find the measure of each angle. What special kind of shape must it be?

Section 7-4

A Postulate for Similar Triangles
Homework Pages 257-259:
2-26 evens
Excluding 20

Objectives



A. Understand and properly apply the Angle-Angle Similarity Postulate.

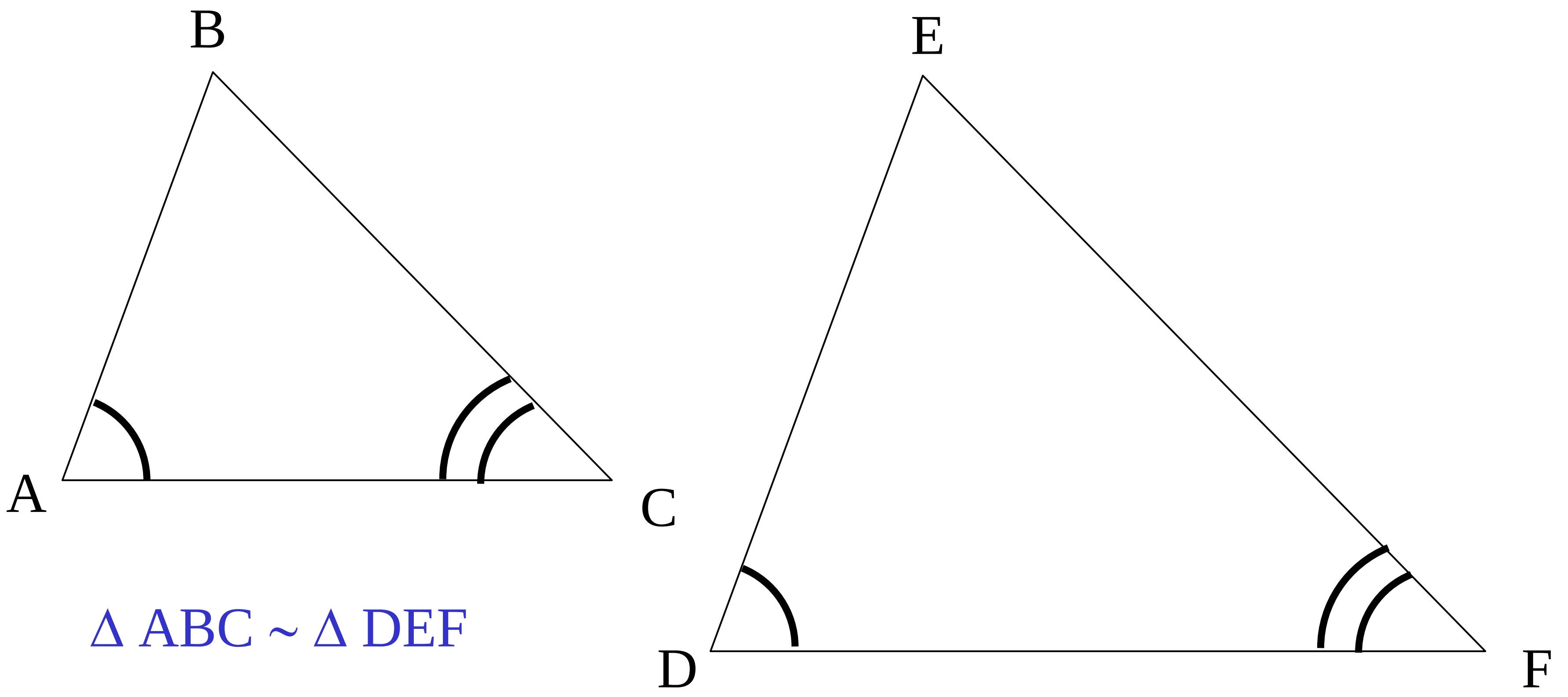
B. Use the Angle-Angle Similarity Postulate to conclude other information about similar triangles.

Activity

Complete the activity provided by your teacher.

★ Postulate 15

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

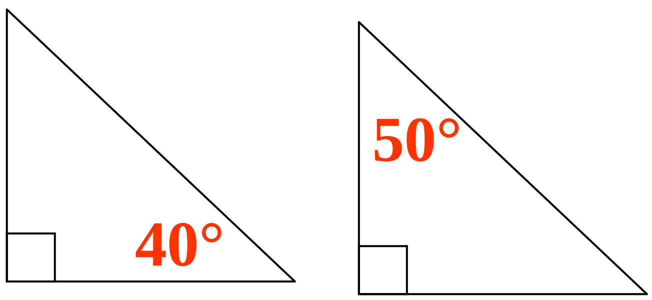


Δ ABC ~ Δ DEF

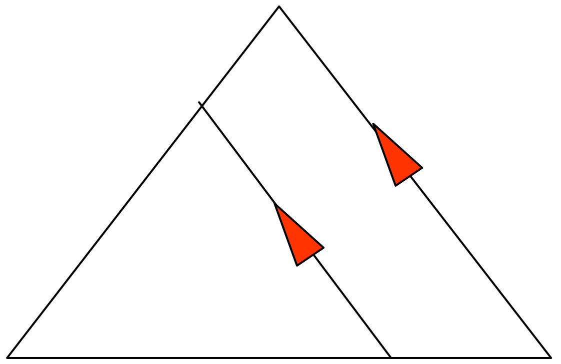
Sample Problems

Tell whether the triangles are similar or not.

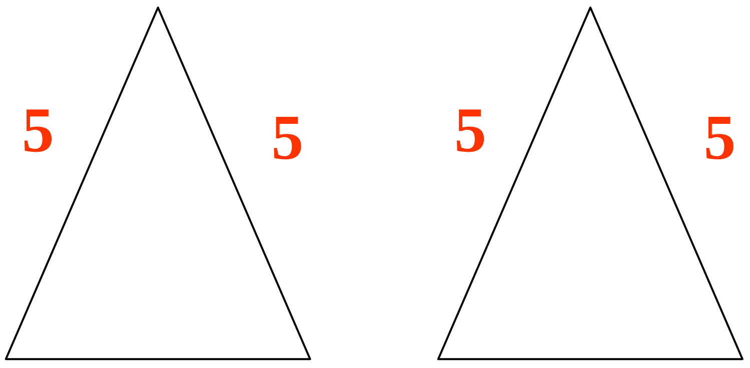
1.



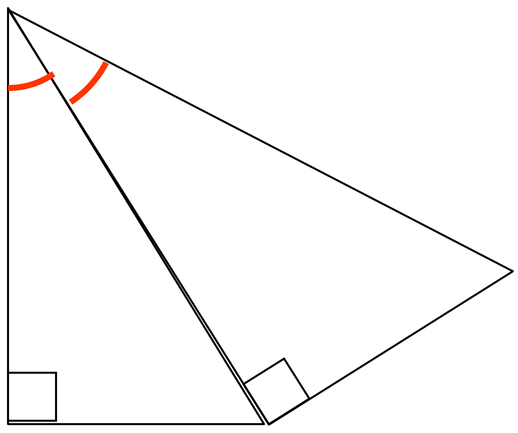
3.



5.

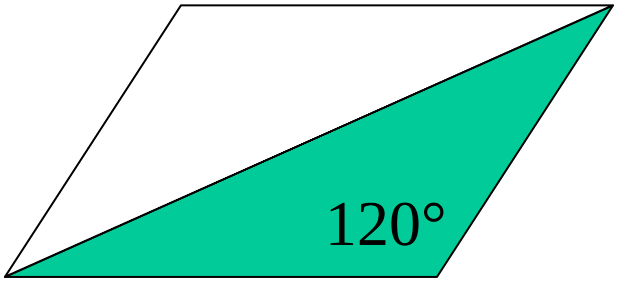


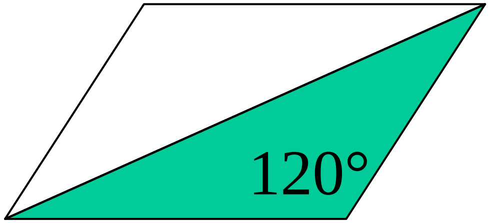
7.



Sample Problems

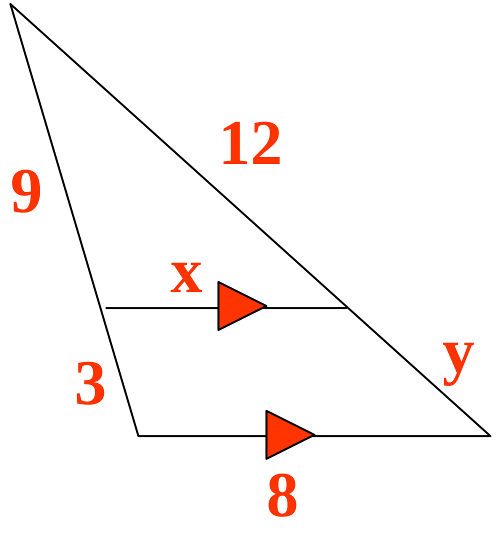
9.



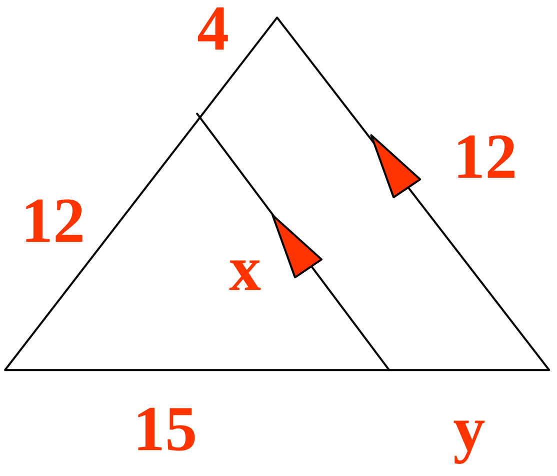


Find the values of x and y.

11.

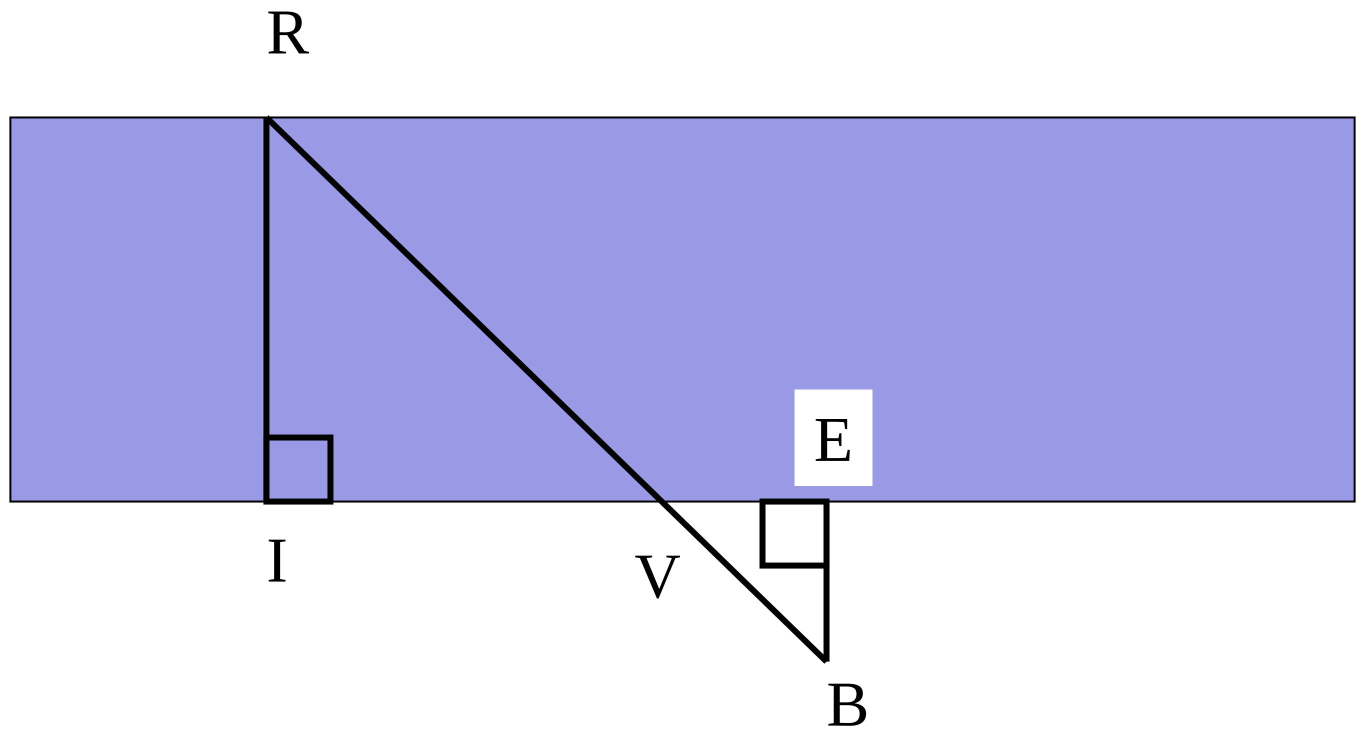


13.

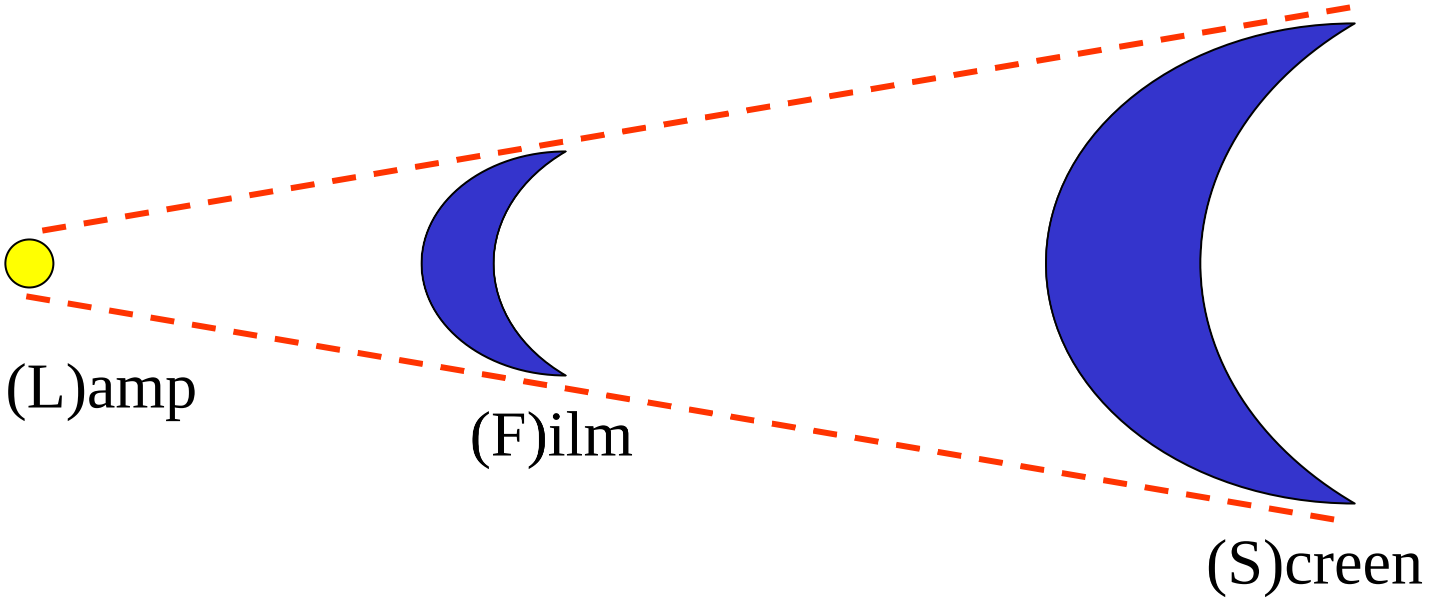


Sample Problems

If VI = 36 m, VE = 20 m, and EB = 15 m, find the width, RI, of the river.



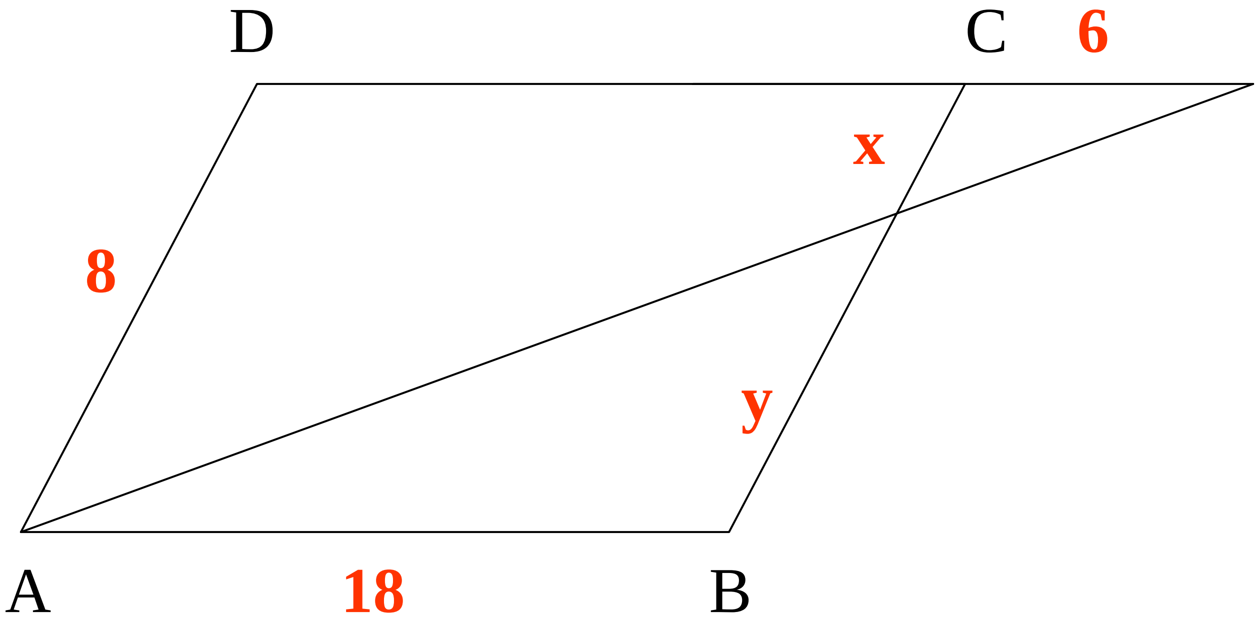
Sample Problem



The diagram show a film being projected on a screen. LF = 6 cm and LS = 24 m. The screen image is 2.2 m tall. How is the film image?

Sample Problem

19. ABCD is a parallelogram. Find x and y.

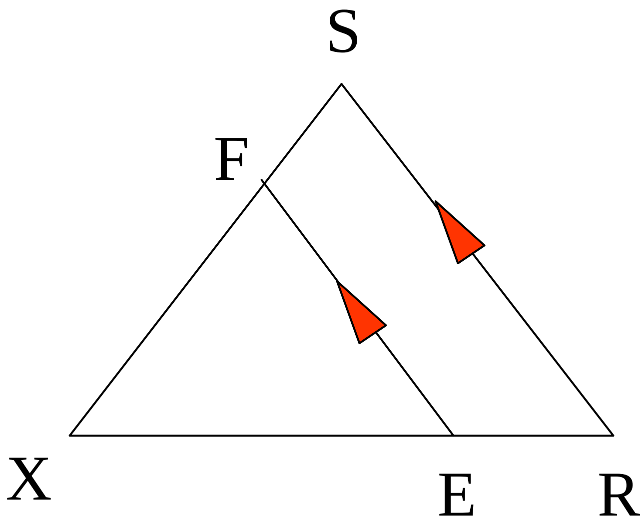


Sample Problems

21. Given: EF || RS

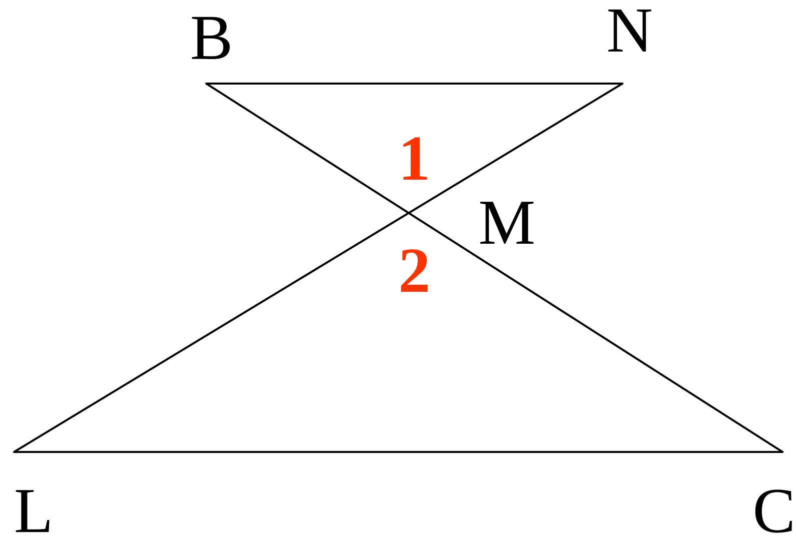
Prove: (a) Δ FXE ~ Δ SXR

(b) $\frac{FX}{SX} = \frac{EF}{RS}$



23. Given: ∠ B ≅ ∠ C

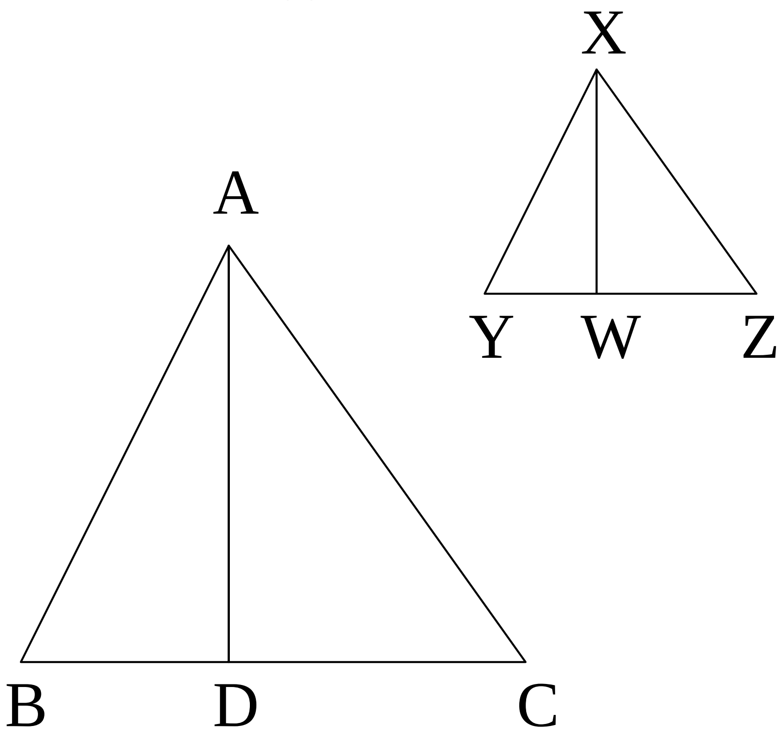
Prove: (NM)(CM) = (LM)(BM)



Sample Problems

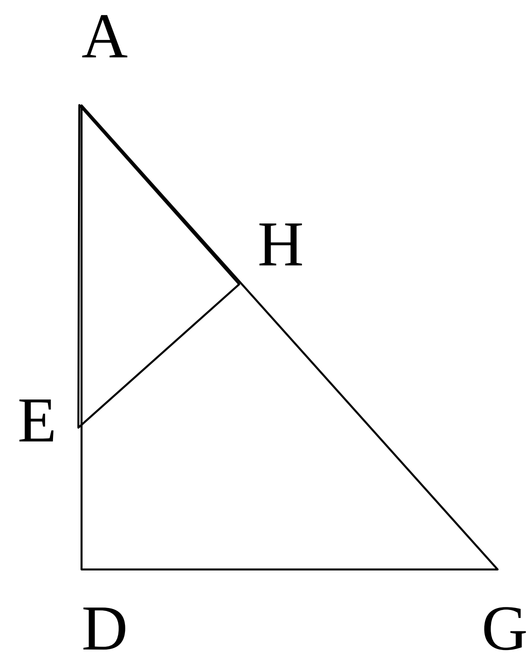
25. Given: $\Delta ABC \sim \Delta XYZ$;
AD & XW are altitudes

Prove: $$\frac{AD}{XW} = \frac{AB}{XY}$$



27. Given: $AH \perp EH$
$AD \perp DG$

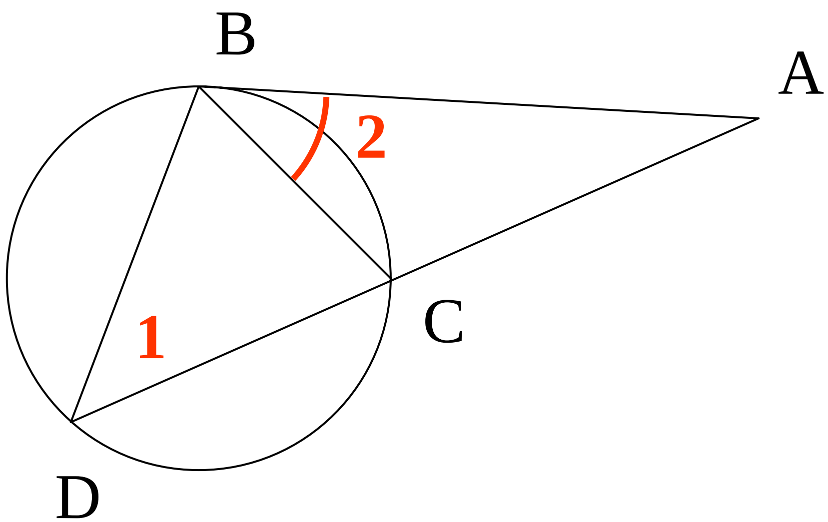
Prove: $(AE)(DG) = (AG)(HE)$



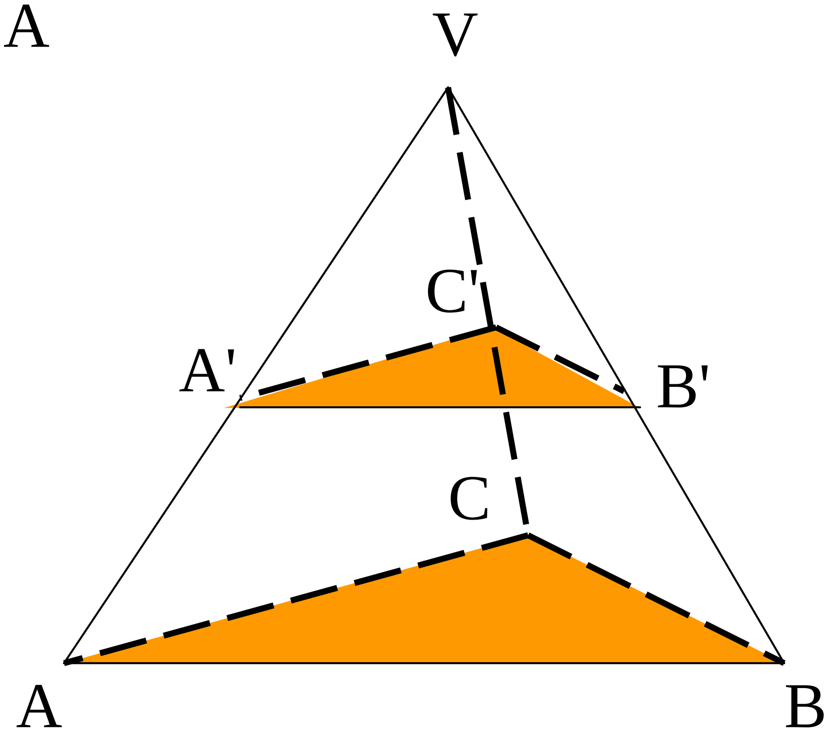
Sample Problems

29. Given: $\angle 1 \cong \angle 2$

Prove: $(AB)^2 = (AD)(AC)$



31. If VA' = 10, VA = 25, AB = 20, BC = 14, and AC = 16. Find the Perimeter of Δ A'B'C'.



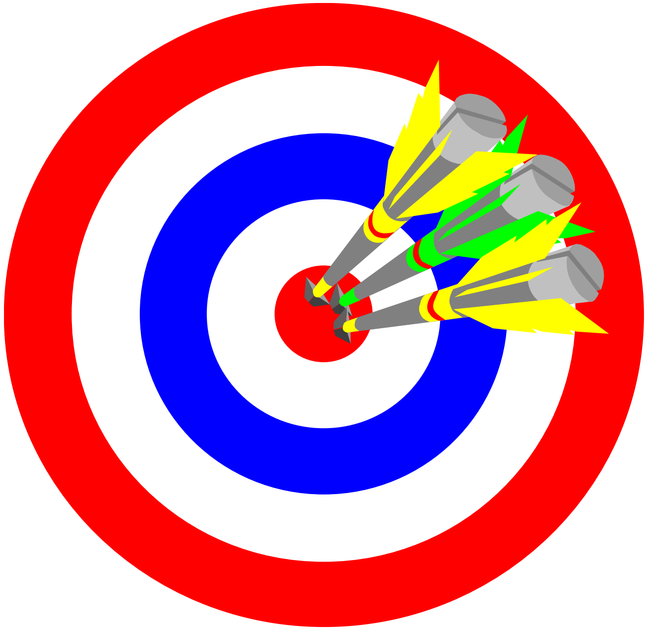
Section 7-5

Theorems for Similar Triangles

Homework Pages 266-267:

2-18 evens

Objectives



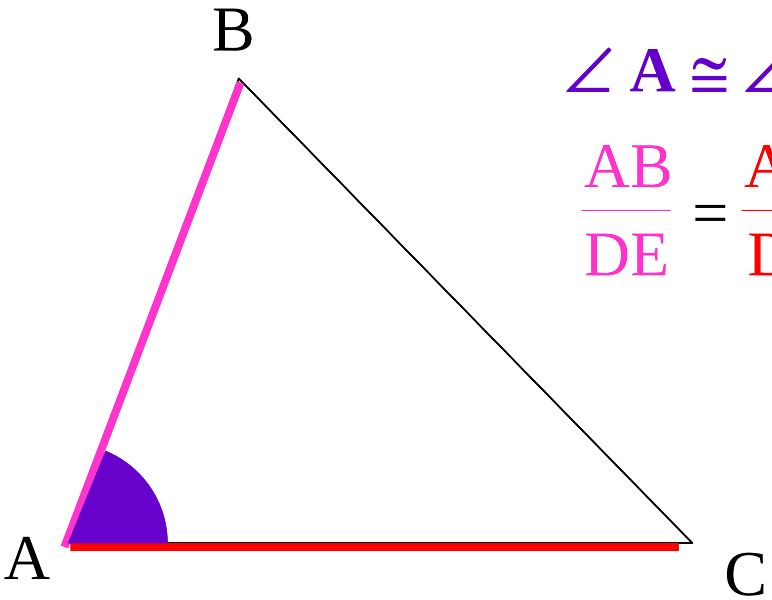
A. Understand and properly apply the Side-Angle-Side and the Side-Side-Side Similarity Theorems.

B. Use the SAS and SSS Similarity Theorems to conclude other information about similar triangles.

C. Determine which of the similarity postulates and theorems to use under varying circumstances.

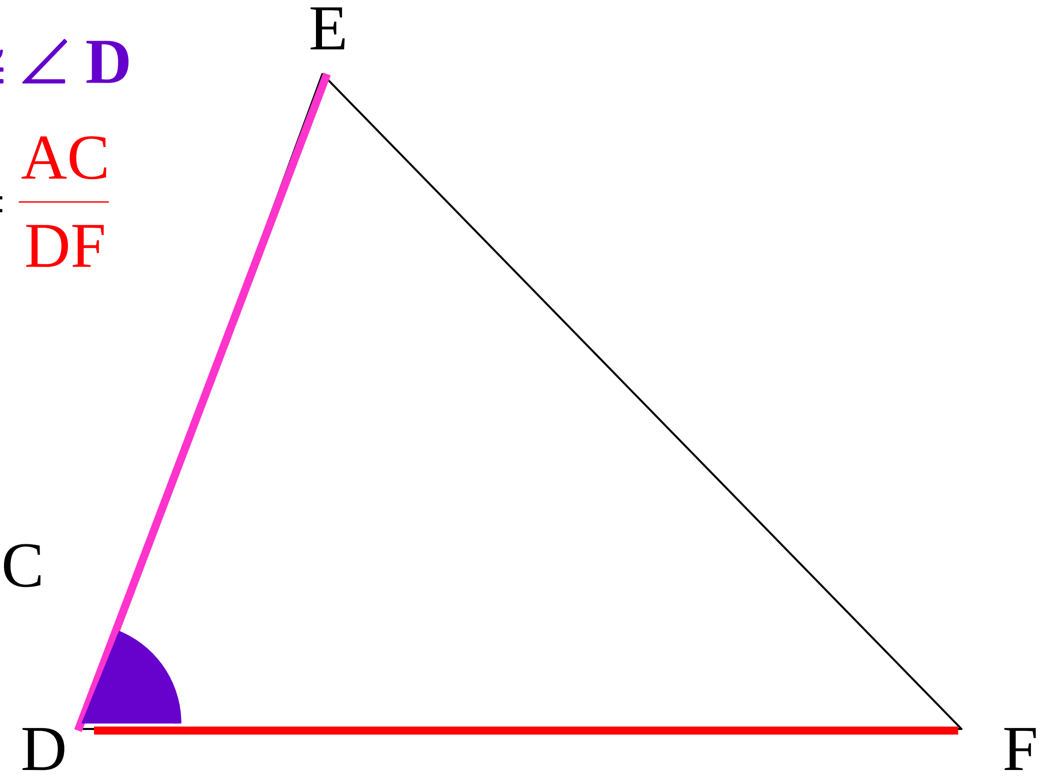
★ Theorem 7-1 SAS Similarity

If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar.



$$\angle A \cong \angle D$$

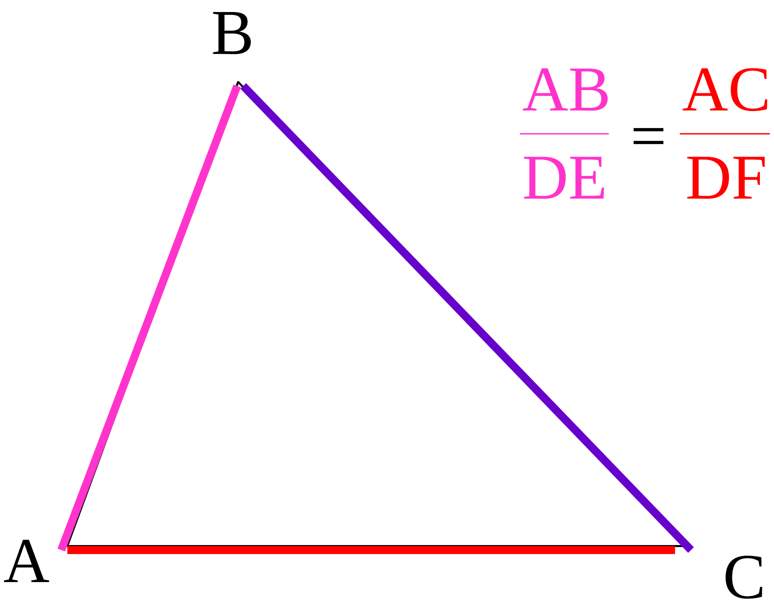
$$\frac{AB}{DE} = \frac{AC}{DF}$$



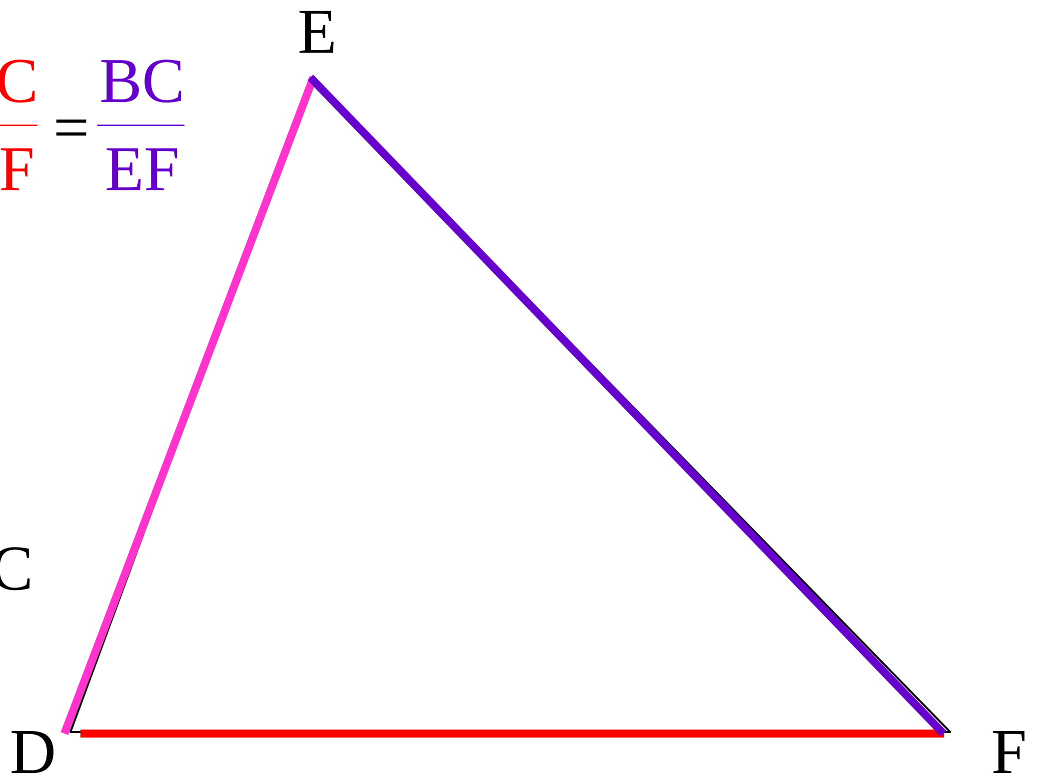
$\Delta ABC \sim \Delta DEF$

★ Theorem 7-2 SSS Similarity

If the sides of two triangles are in proportion,
then the triangles are similar.



$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$



$\Delta ABC \sim \Delta DEF$

Which To Use?

- You have one postulate (AA Similarity), two theorems (SAS and SSS Similarity), and a definition (CPST) to use to determine triangle similarity AND to conclude things about various parts of similar triangles.
- REMEMBER! Once you have shown two triangles to be similar, you can use CPST to deduce many other things about the relationships of various parts of the triangles, such as:
  - All three angles of the first triangle must be congruent to all three angles of the second triangle.
  - The proportion of any two sides of the first triangle must be the same as the proportion of the corresponding sides in the second triangle.

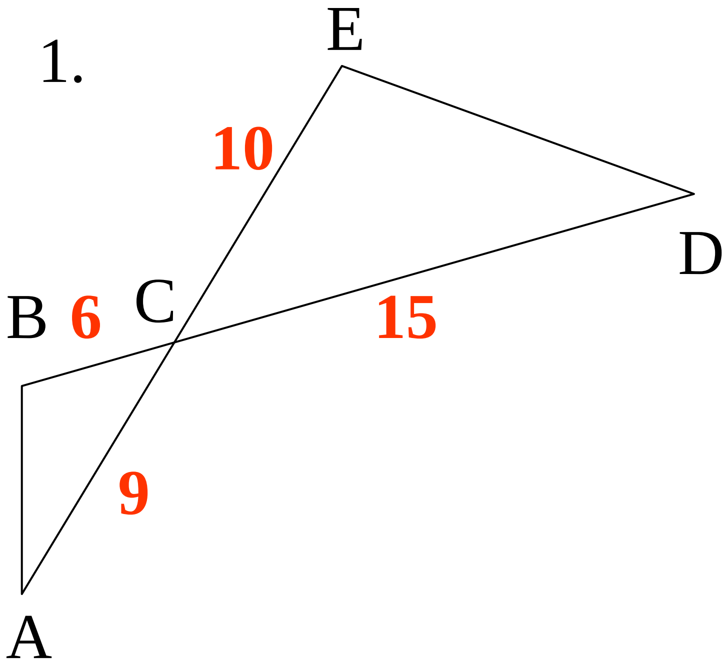
Which To Use?

- Use AA Similarity when:
  - Two angles of one triangle are congruent to two angles of a second triangle
- Use SAS Similarity when:
  - An angle of one triangle is congruent to an angle of a second triangle, AND
  - The sides that make up the angle in the first triangle are proportional to the sides that make up the angle in the second triangle.
- Use SSS Similarity when:
  - You have two sides of one triangle that are proportional to the two corresponding sides of a second triangle.

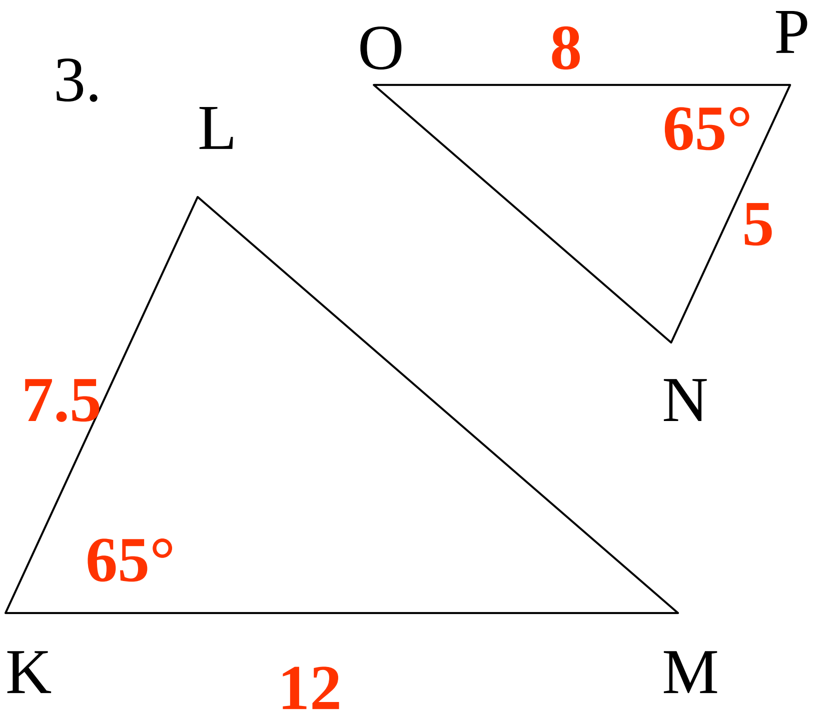
Sample Problems

Name two similar triangles. What postulate or theorem justifies your answer?

1.

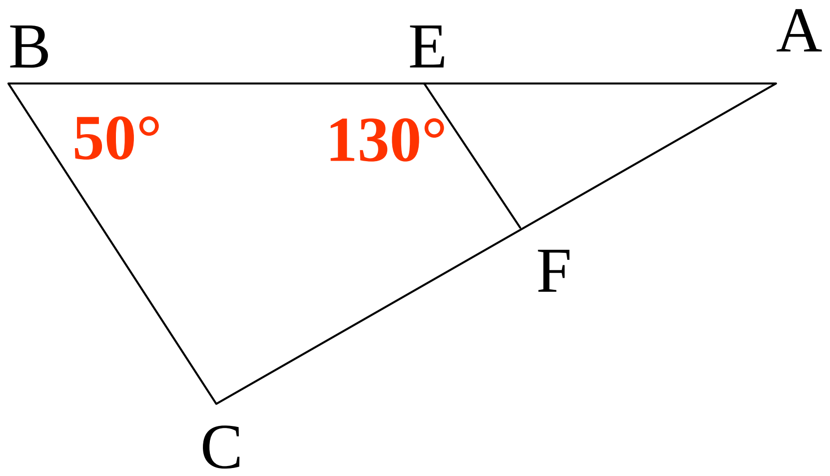


3.



Sample Problems

5.



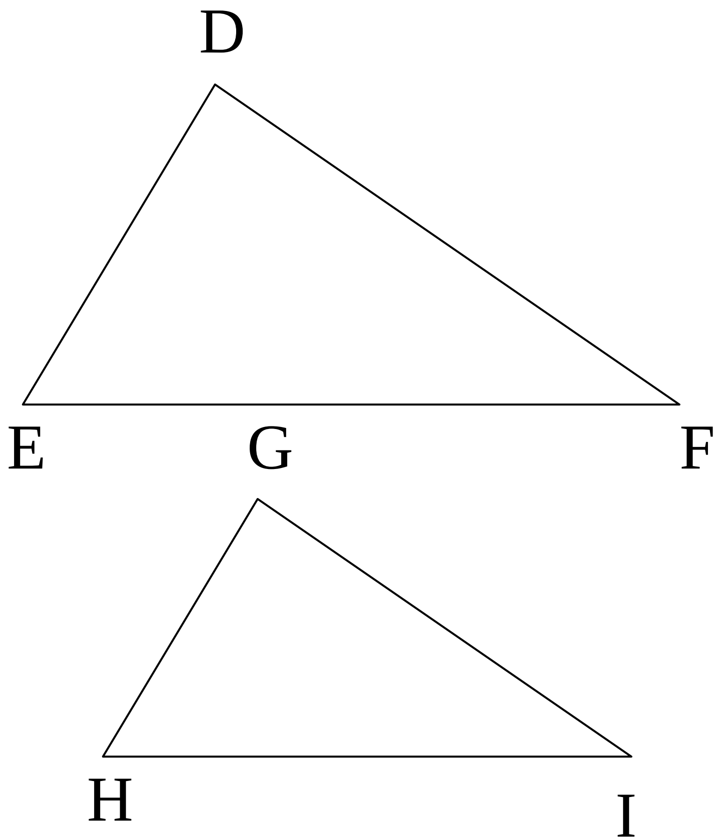
One triangle has vertices A, B, and C. Another has vertices T, R and I. Are the two triangles similar? If so, state the similarity and the scale factor.

	AB	BC	AC	TR	RI	TI
7.	6	8	10	9	12	15
9.	6	8	10	20	25	15

Sample Problems

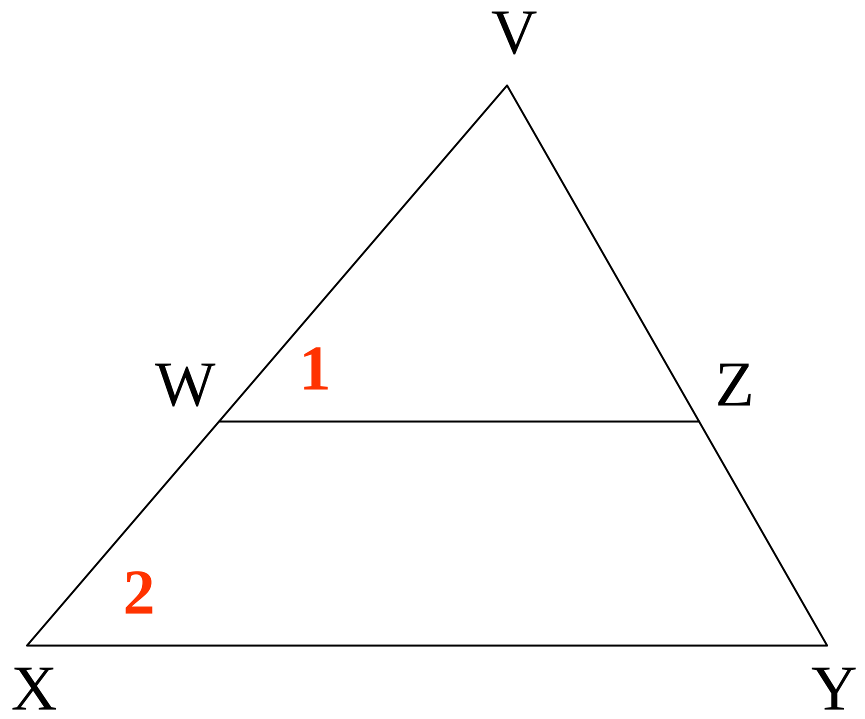
11. Given: $\frac{DE}{GH} = \frac{DF}{GI} = \frac{EF}{HI}$

Prove: $\angle E \cong \angle H$



13. Given: $\frac{VW}{VX} = \frac{VZ}{VY}$

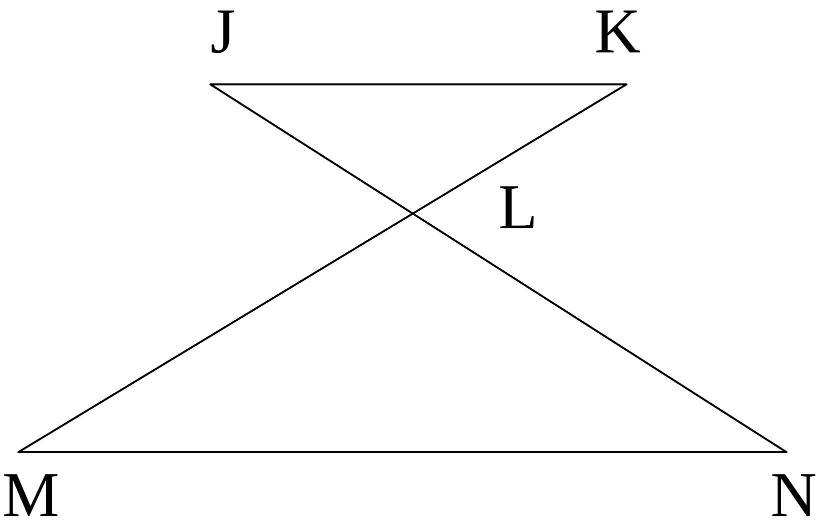
Prove: $WZ \parallel XY$



Sample Problems

15. Given: $\frac{JL}{NL} = \frac{KL}{ML}$

Prove: $\angle J \cong \angle N$



Sample Problems

Draw and label a diagram. List, in terms of the diagram, what is given and what is to be proved. Then write a proof.

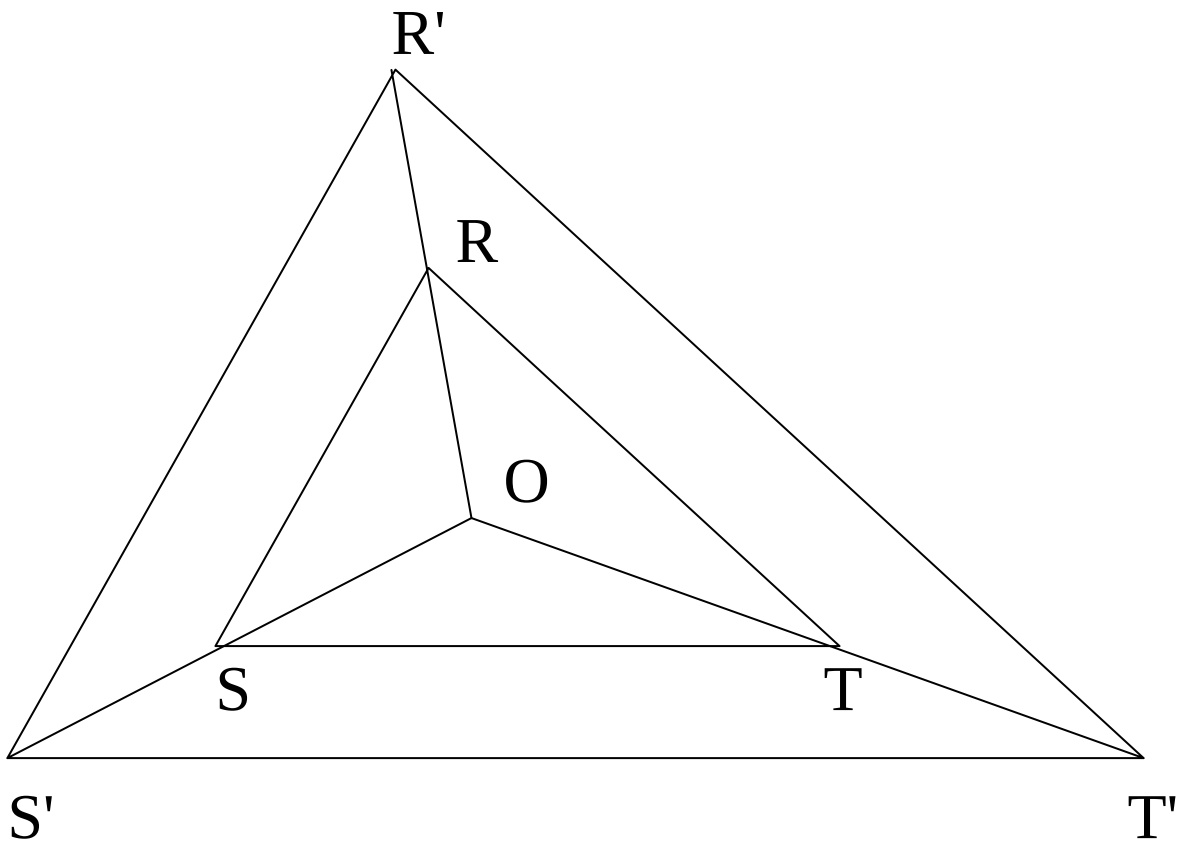
17. If two triangles are similar, then the lengths of corresponding medians are in the same ratio as the lengths of corresponding sides.

19. If the vertex angle of an isosceles triangle is congruent to the vertex angle of another isosceles triangle, then the triangles are similar.

Sample Problems

21. Given: OR' = 2(OR); OS' = 2(OS); OT' = 2(OT)

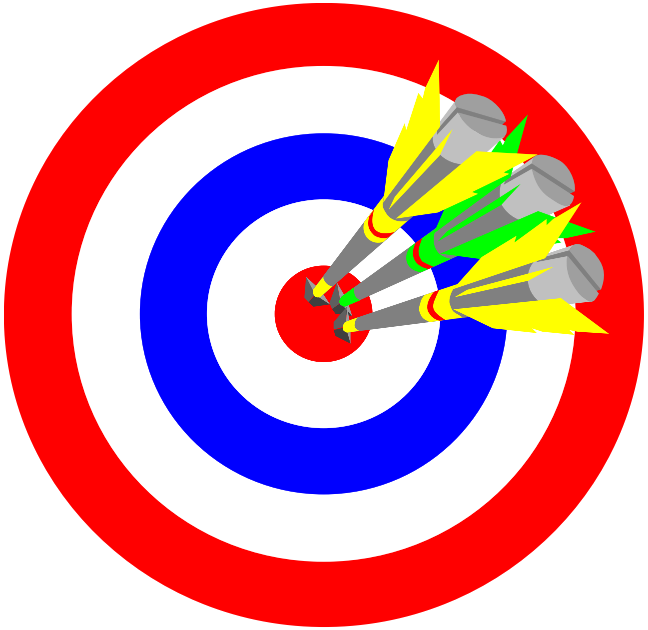
    Prove: Δ RST ~ Δ R'S'T'



Section 7-6

Proportional Lengths
Homework Pages 272-273:
2-26 evens
Excluding 8, 16, 20

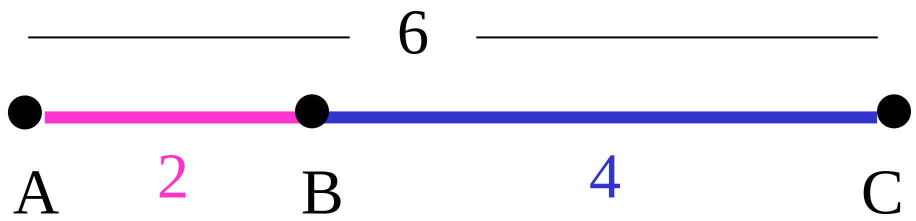
Objectives



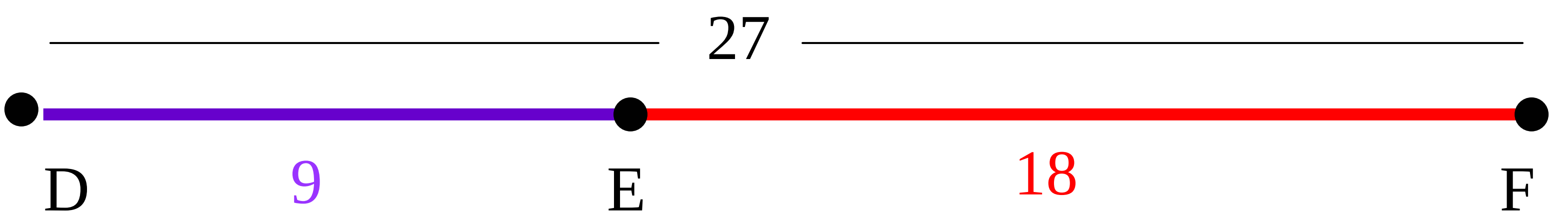
A. Understand and apply the term ‘divided proportionally’ correctly.
B. Understand and properly apply the Triangle Proportionality Theorem and Corollary.
C. Understand and properly apply the Triangle Angle-Bisector Theorem.
D. Use the Triangle Proportionality and Triangle Angle-Bisector Theorems and corollary to conclude other information about similar triangles.

- divided proportionally: the quotient of the pieces of one segment equals the quotient of the pieces of another segment

★ Divided Proportionally

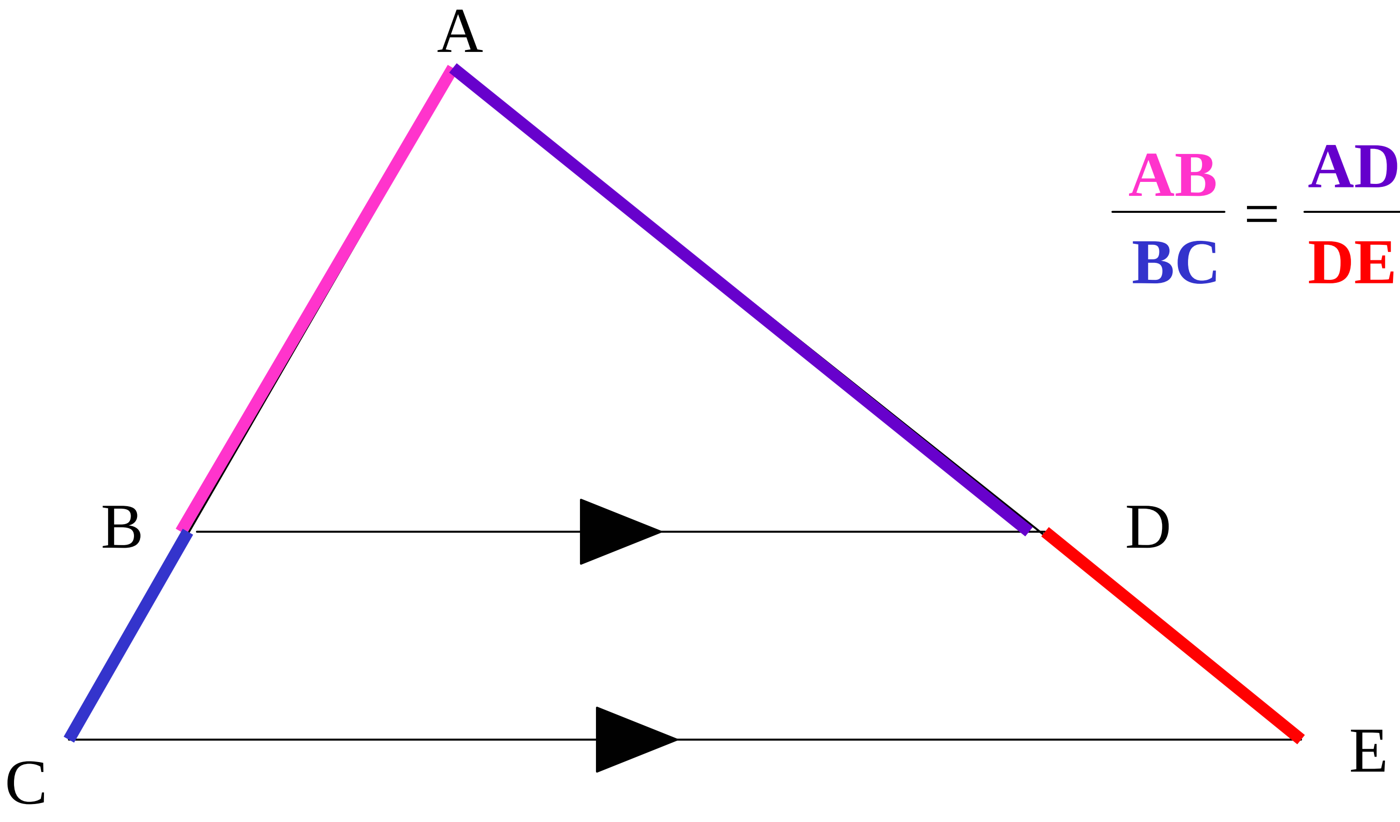


$$\frac{AB}{BC} = \frac{DE}{EF} \qquad \frac{2}{4} = \frac{9}{18}$$



★ Theorem 7-3

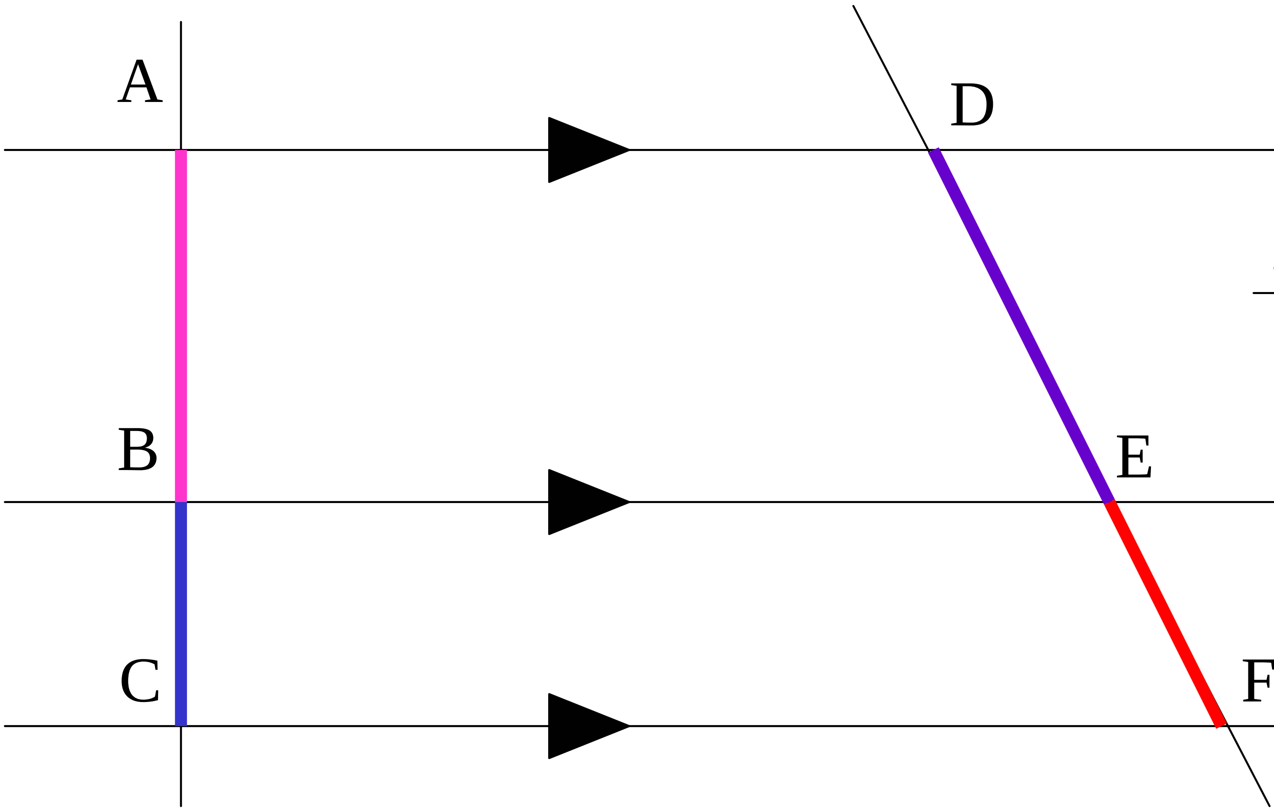
If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.



$$\frac{AB}{BC} = \frac{AD}{DE}$$

★ Corollary 1 Theorem 7-3

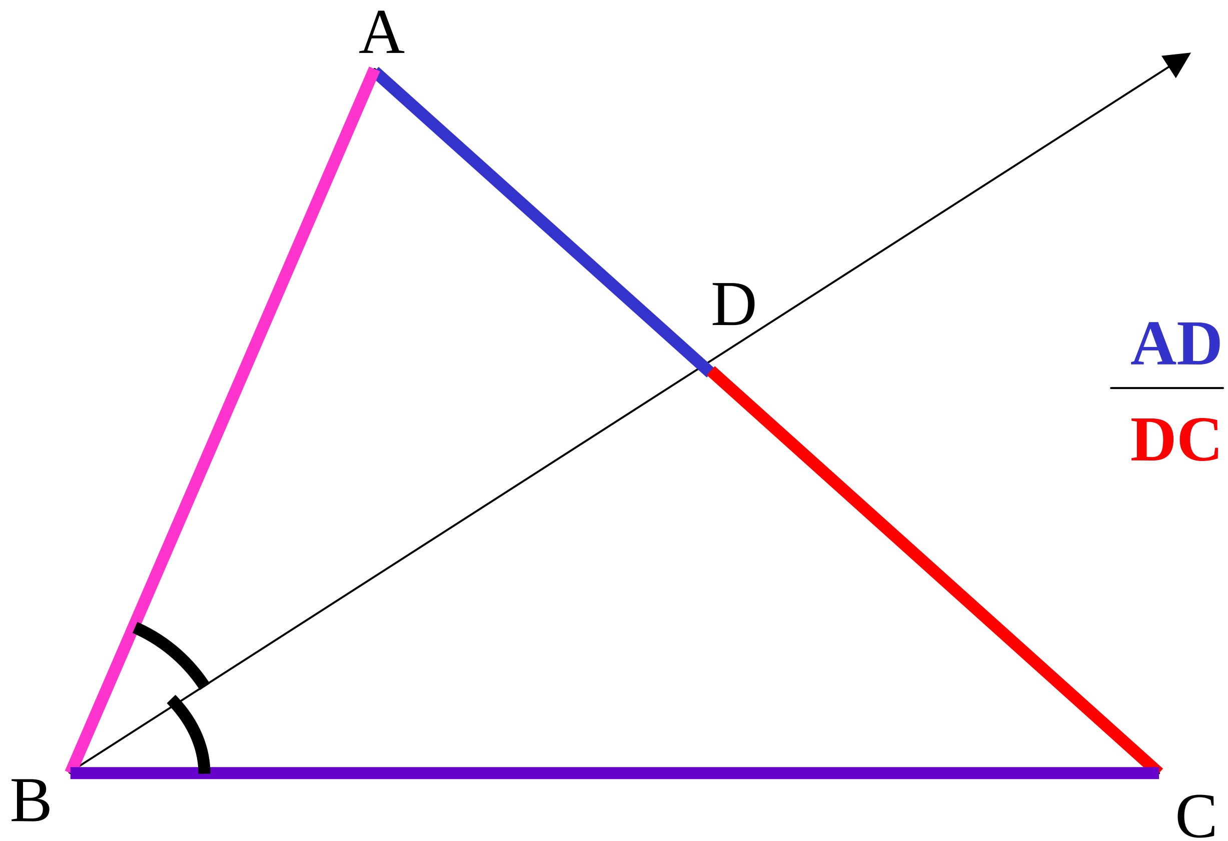
If three parallel lines intersect two transversals,
then they divide the transversals proportionally.



$$\frac{AB}{BC} = \frac{DE}{EF}$$

★ Theorem 7-4

If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.



$$\frac{AD}{DC} = \frac{AB}{BC}$$

Sample Problems

Tell whether the proportion is correct.

a. $\frac{r}{s} = \frac{a}{b}$

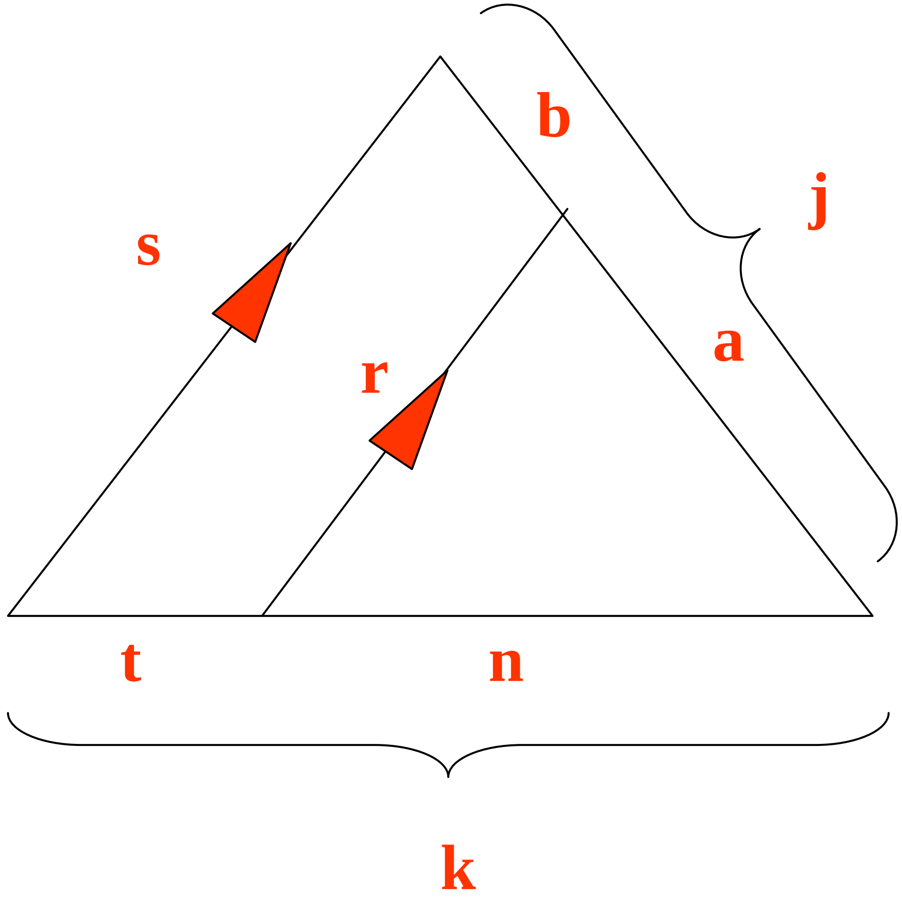
b. $\frac{j}{a} = \frac{s}{r}$

c. $\frac{a}{b} = \frac{n}{t}$

d. $\frac{t}{k} = \frac{a}{j}$

e. $\frac{r}{s} = \frac{n}{k}$

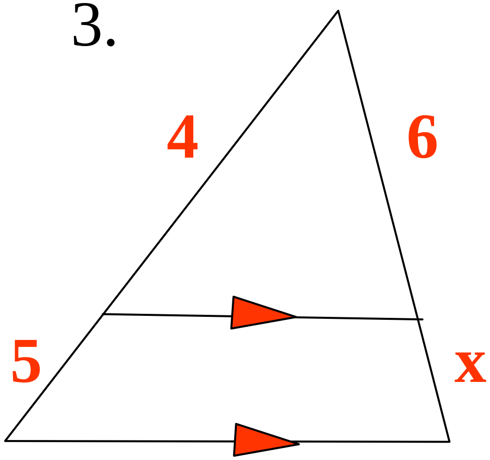
f. $\frac{b}{j} = \frac{t}{k}$



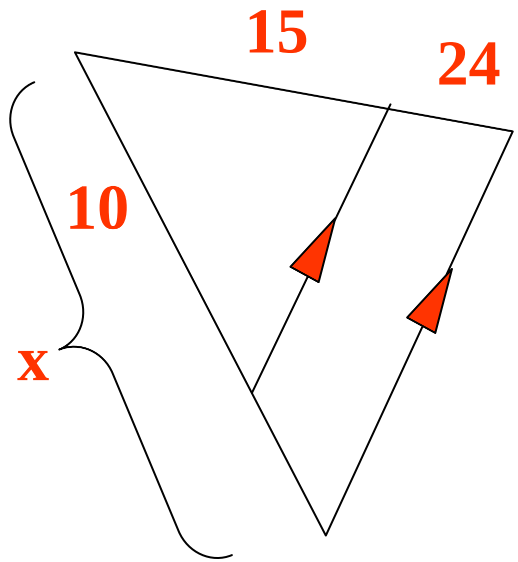
Sample Problems

Find the value of x.

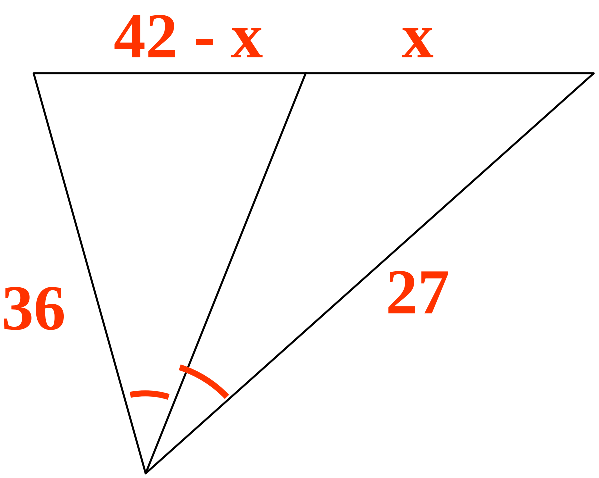
3.



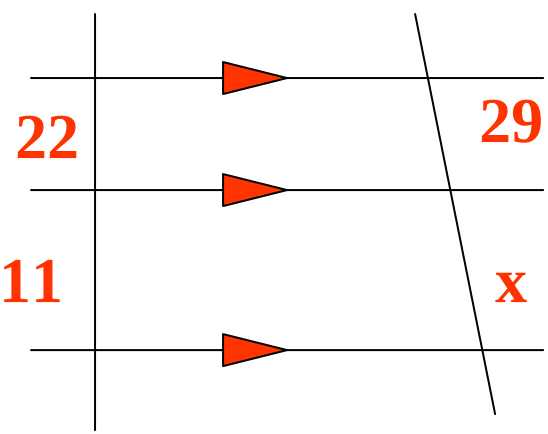
5.



7.

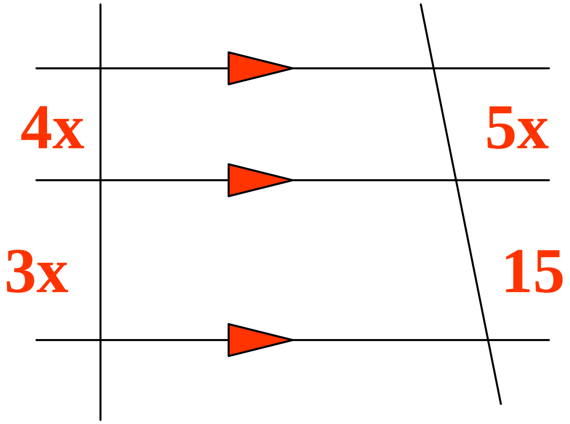


9.



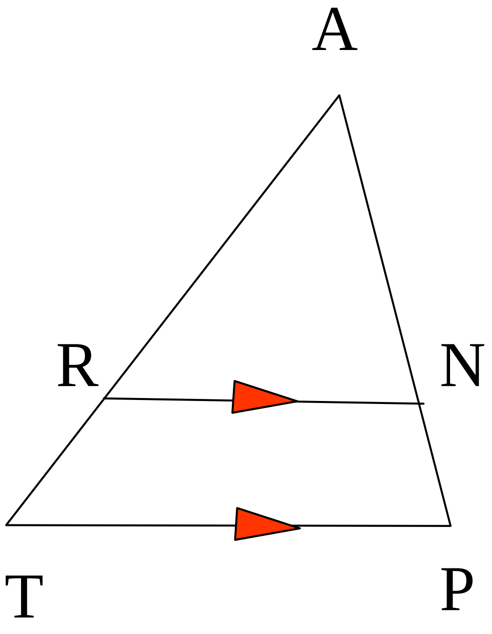
Sample Problems

11.

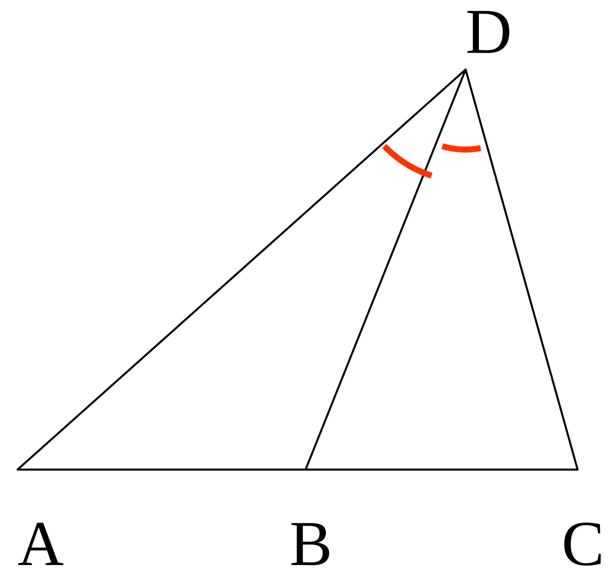


Fill in as many spaces as possible in the table below.

	AR	RT	AT	AN	NP	AP	RN	TP
13.					6	16		
15.	12		20			30	15	
17.		8	16	6				



Sample Problems



Complete.

21. AC = 60, CD = 30,
    AD = 50, BC = ?

23. AB = 2X - 12, BC = x,
    CD = x + 5, AD = 2x - 4, AC = ?

Draw a new picture.

25. The lengths of the sides of Δ ABC are BC = 12, CA = 13, and AB = 14. If M is the midpoint of CA, and P is the point where CA is cut by the bisector of ∠ B, find MP .

Chapter 7

Similarity

Review

Homework Page 279:

2-16 evens