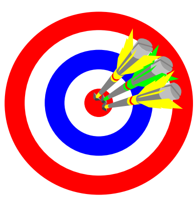
Chapter 9

Circles

Objectives



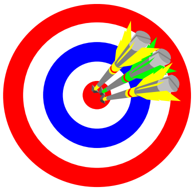
A. Recognize and apply terms relating to circles.
B. Properly use and interpret the symbols for the terms and concepts in this chapter.
C. Appropriately apply the postulates, theorems and corollaries in this chapter.
D. Recognize circumscribed and inscribed polygons.
E. Prove statements involving circumscribed and inscribed polygons.
F. Solve problems involving circumscribed and inscribed polygons.
G. Understand and apply theorems related to tangents, radii, arcs, chords, and central angles.

Section 9-1

Basic Terms: Tangents, Arcs and Chords

Homework Pages 330-331:
2-18 evens

Objectives

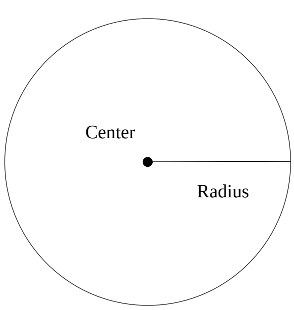


A. Understand and apply the terms circle, center, radius, chord, secant, diameter, tangent, point of tangency, and sphere.
B. Understand and apply the terms congruent circles, congruent spheres, concentric circles, and concentric spheres.
C. Understand and apply the terms inscribed in a circle and circumscribed about a polygon.
D. Correctly draw inscribed and circumscribed figures.

Circular Logic

- On a piece of paper, accurately draw a circle.
- What method did you use to make sure you drew a circle?
- ★ Circle → set of all coplanar points that are a given distance (radius) from a given point (center).
  - Basic Parts:
    - Radius → Distance from the center of a circle to any single point on the circle.
    - Center → Point that is equidistant from all points on the circle.
    - Indicated by symbol ⊙
      - ⊙P → Circle with center P
- Contrast a circle to a sphere:
  - Sphere → set of all points in space a given distance (radius) from a given point (center)

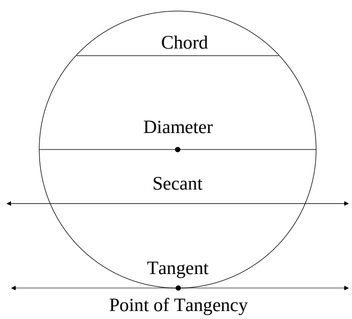
★Circle



Lines and Line Segments Related to Circles

- ⋆ Chord → segment whose endpoints lie on a circle.
- ⋆ Diameter → a chord that passes through the center.
- ⋆ Secant → line that contains a chord.
- ⋆ Tangent → line in the plane of a circle that intersects the circle at exactly one point.
- ⋆ Point of Tangency → the point of intersection between a circle and a tangent to the circle.

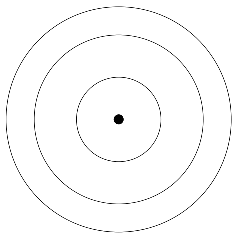
⋆ Segments & Lines



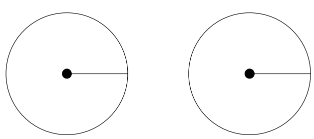
Circular Relationships

- Concentric circles → coplanar circles with the same center
- Concentric Spheres → spheres with the same center
- Congruent Circles → circles with congruent radii
- Congruent Spheres → spheres with congruent radii

Concentric Circles



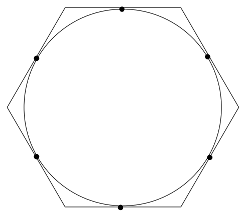
Congruent Circles



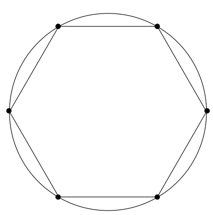
A Figure Within a Figure

- Circumscribed About A Polygon → all of the sides of the polygon are tangent to a circle
- Inscribed In a Circle → all of the vertices of the polygon lie on a circle

★ Circumscribed About A Polygon

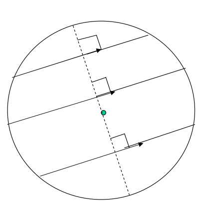


★ Inscribed In A Circle



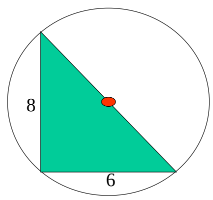
Sample Problems

1. Draw a circle and several parallel chords. What do you think is true of the midpoints of all such chords?



Sample Problems

3. Draw a right triangle inscribed in a circle. What do you know about the midpoint of the hypotenuse? Where is the center of the circle? If the legs of the right triangle are 6 and 8, find the radius of the circle.

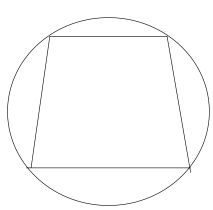


Sample Problems

5. The radii of two concentric circles are 15 and 7. A diameter AB of the larger circle intersects the smaller circle at C and D. Find two possible values for AC.

Sample Problems

7. Draw a circle with an inscribed trapezoid.



Sample Problems

Draw a circle and inscribe the polygon named.

9. a parallelogram

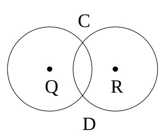
11. a quadrilateral PQRS with PR a diameter

Sample Problems

For each draw a ⊙O with radius 12. Then draw OA and OB to form an angle with the measurement given. Find AB.

13. m ∠ AOB = 180 15. m ∠ AOB = 120

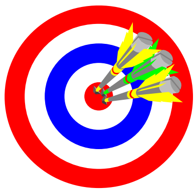
17. ⊙Q and ⊙R are congruent circles that intersect at C and D. CD is the common chord of the circles. What kind of quadrilateral is QDRC? Why? CD must be the perpendicular bisector of QR. Why? If QC = 17 and QR = 30, find CD.



Section 9-2

Tangents
Homework Pages 335-337:
2-18 evens
Excluding 14

Objectives

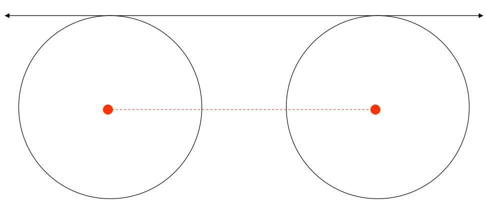


A. Understand and apply the terms "external common tangent" and "internal common tangent".
B. Understand and apply the terms "externally tangent circles" and "internally tangent circles".

C. Understand and apply theorems and corollaries dealing with the tangents of circles.

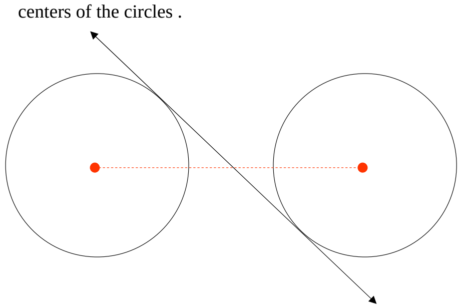
External Common Tangent

- External Common Tangent → a line that is tangent to two coplanar circles and doesn't intersect the segment joining the centers of the circles.



Internal Common Tangent

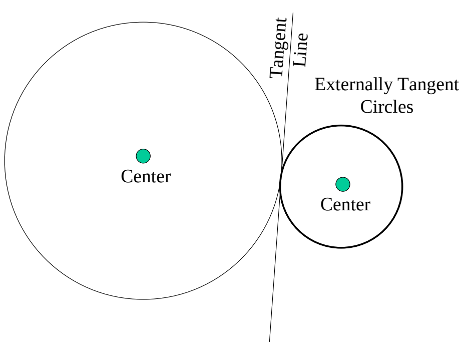
- Internal Common Tangent → a line that is tangent to two coplanar circles and intersects the segment joining the centers of the circles .



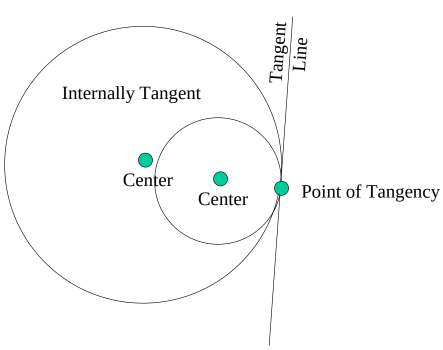
Tangent Circles

- Externally Tangent Circles → coplanar circles that are tangent to the same line at the same point and the centers are on opposite sides of the line.
- Internally Tangent Circles → coplanar circles that are tangent to the same line at the same point and the centers are on the same side of the line.

Externally Tangent Circles

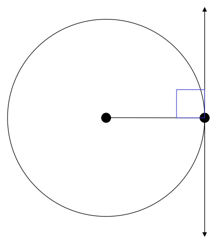


Internally Tangent Circles



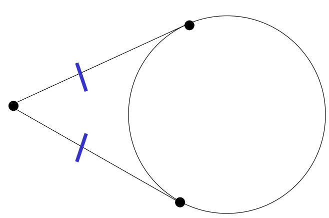
★ Theorem 9-1

If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency



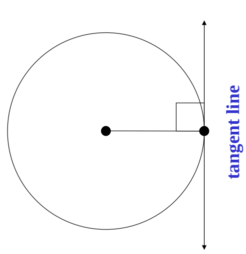
★ Theorem 9-1 Corollary 1

Tangents to a circle from a point are congruent.



Theorem 9-2

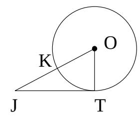
If a line in a plane of a circle is perpendicular to a radius at its outer endpoint, then the line is tangent to the circle.



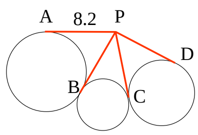
Sample Problems

JT is tangent to ⊙O at T.

1. If OT = 6 and JO = 10, JT = ?
3. If m TOJ = 60 and OT = 6, JO = ?

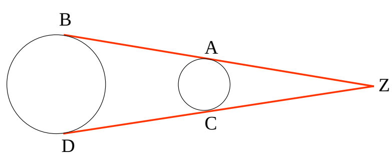


5. The diagram shows tangent lines and circles. Find PD.



Sample Problems

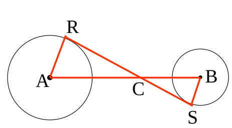
7. What do you think is true of the common external tangents AB and CD? Prove it. Will the results to this question still be true if the circles are congruent?



Sample Problems

9. Draw ⊙O with perpendicular radii OX and OY. Draw tangents to the circle at X and Y. If the tangents meet at Z, what kind of figure is OXZY? Explain. If OX = 5, find OZ.

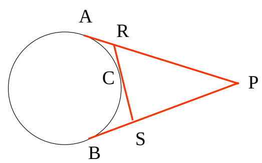
11. Given: RS is a common internal tangent to ⊙A and ⊙B. Explain why $\frac{AC}{BC} = \frac{RC}{SC}$



Sample Problems

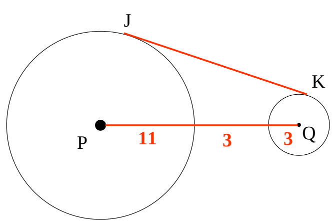
13. State the theorem which would describe the relationship between the planes tangent to a sphere at either end of a diameter.

15. PA, PB, and RS are tangents. Explain why PR + RS + SP = PA + PB



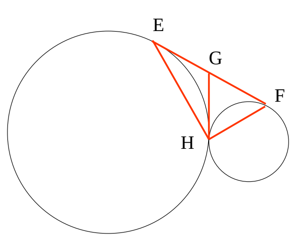
Sample Problems

17. JK is tangent to ⊙P and ⊙Q. JK = ?



Sample Problems

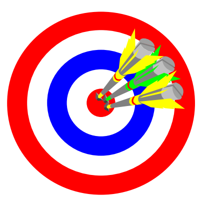
19. Given two tangent circles; EF is a common external tangent. Prove something about G. Prove something about ∠ EHF.



Section 9-3

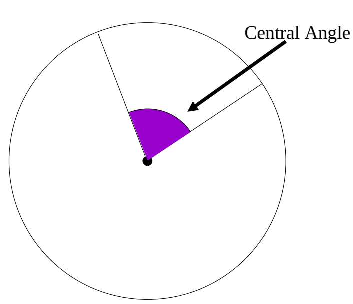
Arcs and Central Angles
Homework Pages 341-342:
2-18 evens
Excluding 12

Objectives



A. Understand and apply the term "central angle".
B. Understand and apply the terms "major arc", "minor arc", "adjacent arcs", "congruent arcs", and "intercept arc".
C. Understand and utilize the Arc Addition Postulate.
D. Understand and apply the theorem of congruent minor arcs.

★ Central Angle → an angle whose vertex lies on the center of a circle.



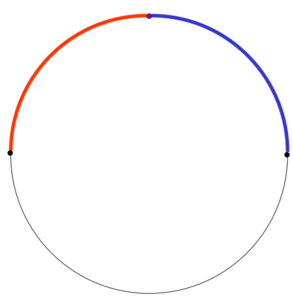
Not Noah's 'Arc'

- ★ Arc → an unbroken part of a circle.

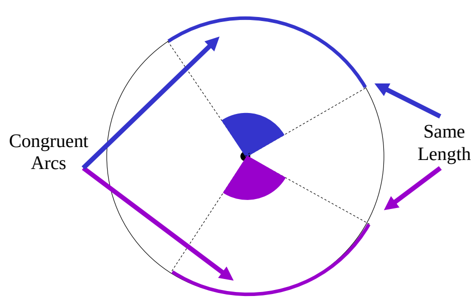
- Types of arcs:
  - Major arc
  - Minor arc
  - Adjacent arcs
  - Congruent arcs
  - Intercept arc
- The symbol for the measurement of an arc is:

$m\overset{\frown}{AB} \Rightarrow$ measurement of arc AB

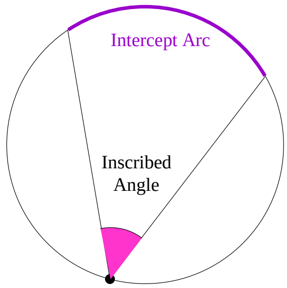
Adjacent Arcs → arcs of the same circle that have exactly one point in common .



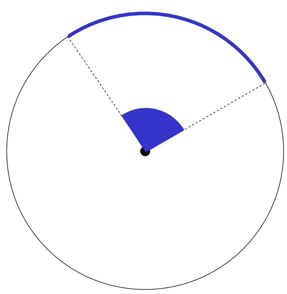
Congruent Arcs → arcs in the same circle or congruent circles that have the same measurement.



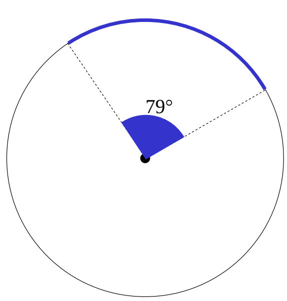
★ Intercepted Arc → the arc between the sides of an inscribed angle



Minor Arc → an unbroken part of a circle that measures less the 180 degrees.

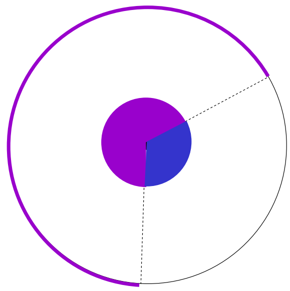


Measure of a minor arc = measure of its central angle.

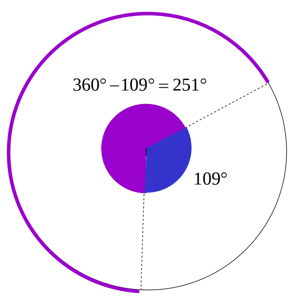


Measure of a minor arc = 79°

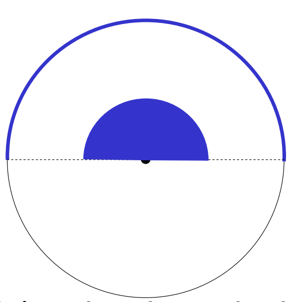
Major Arc → an unbroken part of a circle that measures more than 180 degrees.



Measure of major arc = 360 - measure of the minor arc



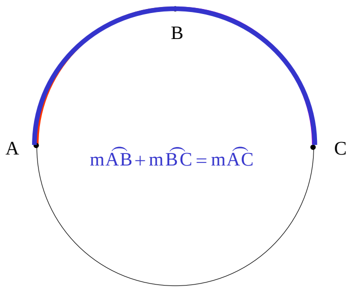
Semicircle → an unbroken part of a circle that measures exactly 180 degrees.



Semicircle → arcs whose endpoints are the endpoints of a diameter.

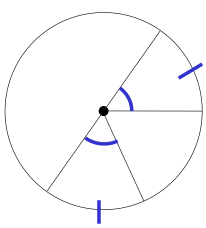
★ Postulate 16

The measure of the arc formed by two adjacent arcs is the sum of the measures of these two arcs.



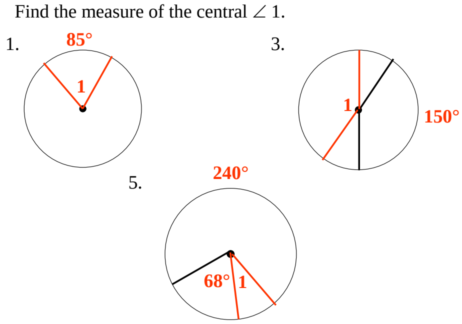
★ Theorem 9-3

In the same circle or congruent circles, two minor arcs are congruent if and only if their central angles are congruent.



Sample Problems

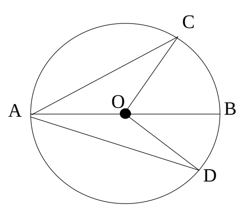
Find the measure of the central ∠ 1.



Sample Problems

7. At 11 o'clock the hands of a clock form an angle of ?
9. Draw a circle. Place points A, B and C on it in such positions that $m\overset{\frown}{AB} + m\overset{\frown}{BC} \neq m\overset{\frown}{AC}$

Sample Problems



OC, OB, and OA are all radii.
So OC = OB = OA

If $m\angle COB = 42$, then $m\angle COA = 138$.

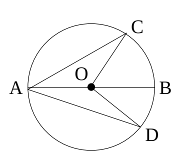
Since $OC = OA$, then ΔAOC is isosceles.

Since ΔAOC is isosceles, $m\angle ACO = m\angle CAO$.

$m\angle AOC + m\angle CAO + m\angle ACO = 180$
$138 + (2 \times m\angle CAO) = 180$
$(2 \times m\angle CAO) = 42$
$m\angle CAO = 21$

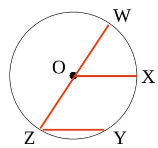
Sample Problems

$m\overset{\frown}{CB}$	70	60	66	60	p
$m\overset{\frown}{BD}$	30	28	?	?	q
$m\angle COD$	?	?	100	?	?
$m\angle CAD$	?	?	?	52	?



Sample Problems

15. Given: WZ is a diameter of ⊙O;
$m\overset{\frown}{WX} = m\overset{\frown}{XY} = n$
Prove: $m\angle Z = n$



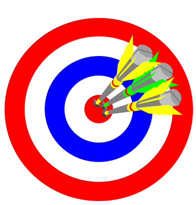
The latitude of a city is given. Find the radius of this circle of latitude.

17. Milwaukee, Wisconsin; 43°N
19. Sydney, Australia; 34°S

Section 9-4

Arcs and Chords
Homework Pages 347-348:
2-22 evens
Excluding 10

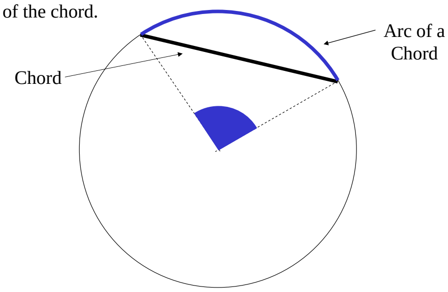
Objectives



A. Understand the term ‘arc of a chord’.
B. Understand and apply theorems relating arcs and chords to circles.
C. Use the theorems related to arcs and chords to solve problems involving circles.

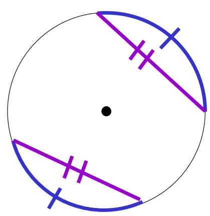
Arc of a Chord

- Arc of a Chord → the minor arc created by the endpoints of the chord.



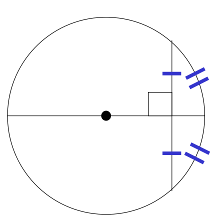
★ Theorem 9-4

In the same circle or congruent circles:
(1) Congruent arcs have congruent chords.
(2) Congruent chords have congruent arcs.



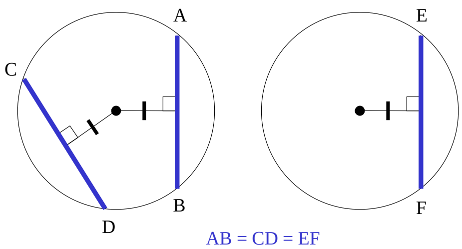
★ Theorem 9-5

A diameter that is perpendicular to a chord bisects the chord and its arc.



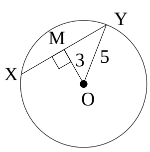
★ Theorem 9-6

In the same circle or congruent circles:
(1) Chords equally distant from the center (or centers) are congruent.
(2) Congruent chords are equally distant from the center (or centers).



AB = CD = EF

Sample Problems



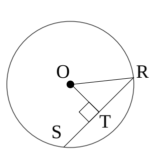
1. XY = ?

ΔOMY is a pattern right triangle.
Therefore, $MY = 4$.

A diameter that is perpendicular to a chord bisects the chord and the arc.

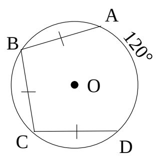
XY = 2MY = 2(4) = 8

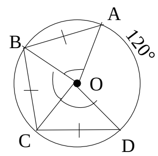
Sample Problems



3. OT = 9, RS = 18
OR = ?

Sample Problems

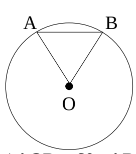




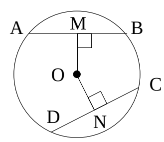
5. $m\overset{\frown}{BC} = ?$

$$\frac{360° - 120°}{3} = 80°$$

Sample Problems



7. m ∠ AOB = 60; AB = 24
OA = ?



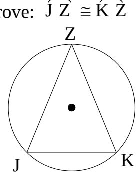
9. AB = 18; OM = 12
ON = 10; CD = ?

Sample Problems

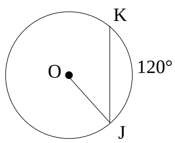
11. Sketch a circle O with radius 10 and chord XY, 8. How far is the chord from O?

13. Sketch a circle P with radius 5 and chord AB that is 2 cm from P. Find the length of AB.

15. Given: ∠ J ≅ ∠ K
Prove: $\overset{\frown}{JZ} \cong \overset{\frown}{KZ}$



Sample Problems



17. OJ = 10, JK = ?

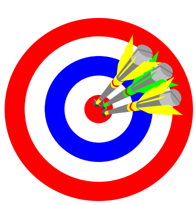
19. A plane 5 cm from the center of a sphere intersects the sphere in a circle with diameter 24 cm. Find the diameter of the sphere.

21. Use trigonometry to find the measure of the arc cut off by a chord 12 cm long in a circle of radius 10 cm.

Section 9-5

Inscribed Angles
Homework Pages 354-356:
2-24 evens

Objectives



A. Understand and apply the term "inscribed angle".

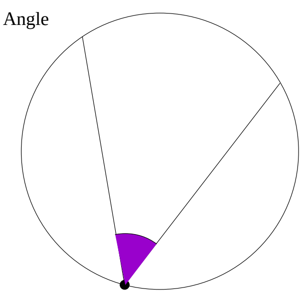
B. Understand and apply the theorems and corollaries associated with inscribed angles of circles.

C. Use the theorems and corollaries associated with inscribed angles to solve problems involving circles.

★ Inscribed Angle → an angle whose vertex lies on a circle an whose sides contain chords of the circle.

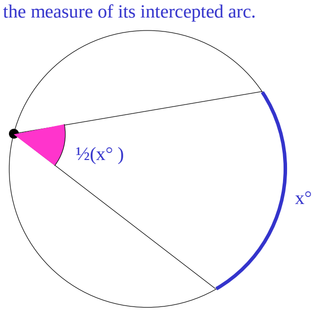
★ Angles

Inscribed Angle



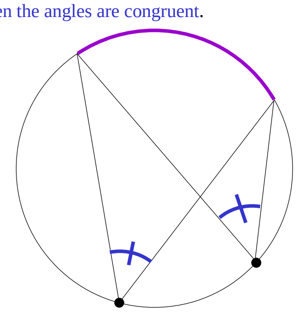
★ Theorem 9-7

The measure of an inscribed angle is equal to half the measure of its intercepted arc.



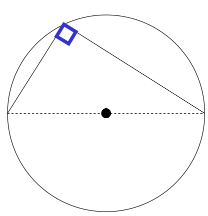
★ Theorem 9-7 Corollary 1

If two inscribed angles intercept the same arc,
then the angles are congruent.



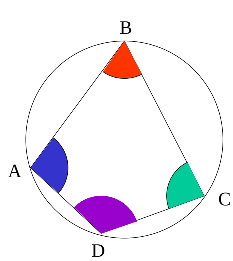
★ Theorem 9-7 Corollary 2

An angle inscribed in a semicircle is a right angle.



★ Theorem 9-7 Corollary 3

If a quadrilateral is inscribed in a circle,
then its opposite angles are supplementary.

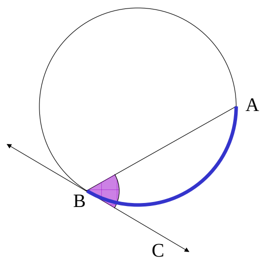


$m\angle A + m\angle C = 180$

$m\angle B + m\angle D = 180$

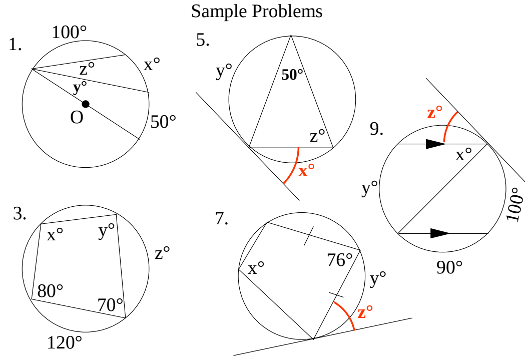
★ Theorem 9-8

The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.



$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AB}$$

Sample Problems



Sample Problems

17. Draw an inscribed quadrilateral ABCD and its diagonals intersecting at E. Name two pairs of similar triangles.

ABCD is an inscribed quadrilateral.

19. $m\angle A = x$, $m\angle B = 2x$, $m\angle C = x + 20$. Find x and $m\angle D$.

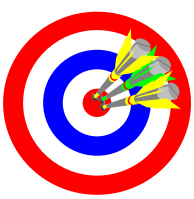
21. $m\angle D = 75$, $m\overset{\frown}{AB} = x^2$, $m\overset{\frown}{BC} = 5x$ and $m\overset{\frown}{CD} = 6x$. Find x and $m\angle A$.

23. Equilateral Δ ABC is inscribed in a circle. P and Q are midpoints of arcs BC amd CA respectively. What kind of figure is quadrilateral AQPB? Why?

Section 9-6

Other Angles
Homework Pages 359-360:
2-26 evens

Objectives

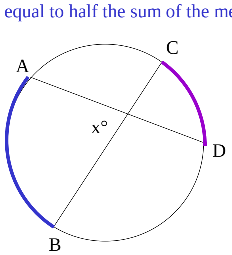


A. Understand and apply the theorem relating to two chords intersecting inside of a circle.
B. Understand and apply the theorem relating two secants, two tangents or a secant and a tangent of a circle.

C. Use these theorems to solve problems relating to circles.

★ Theorem 9-9

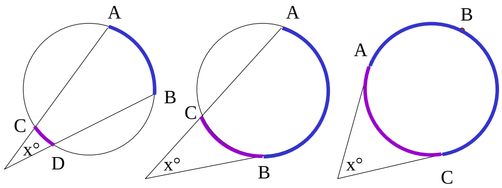
The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.



$$x = \frac{m\overset{\frown}{AB} + m\overset{\frown}{CD}}{2}$$

★ Theorem 9-10

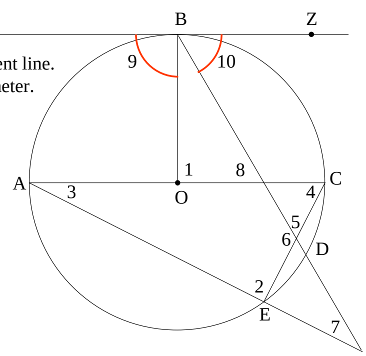
The measure of the angle formed by two secant, two tangents or a secant and a tangent drawn from a point outside a circle is equal to half the difference of the measures of the intercepted arcs.



$x = \frac{m\widehat{AB} - m\widehat{CD}}{2}$ $x = \frac{m\widehat{AB} - m\widehat{BC}}{2}$ $x = \frac{m\widehat{ABC} - m\widehat{AC}}{2}$

Sample Problems

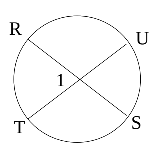
BZ is a tangent line.
AC is a diameter.
$m\widehat{BC} = 90$
$m\widehat{CD} = 30$
$m\widehat{DE} = 20$



Sample Problems

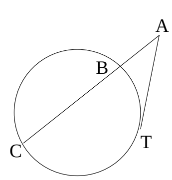
Complete.

11. If arc RT = 80 and arc US = 40, then m ∠ 1 = ?
13. If m ∠ 1 = 50 and arc RT = 70, then arc US = ?



AT is a tangent.

15. If arc CT = 110 and arc BT = 50, then m ∠ A = ?
17. If m ∠ A = 35 and arc CT = 110, then arc BT = ?

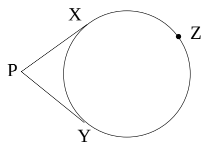


Sample Problems

PX and PY are tangents.

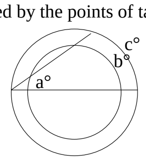
19. If arc XY = 90, then m ∠ P = ?

21. If m ∠ P = 65, then arc XY = ?



23. A quadrilateral circumscribed about a circle has angles 80, 90, 94 and 96. Find the measures of the four non-overlapping arcs determined by the points of tangency.

27. Write an equation involving a, b and c.



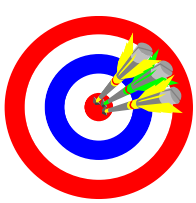
Section 9-7

Circles and Lengths of Segments

Homework Pages 364-366:

2-26 evens

Objectives

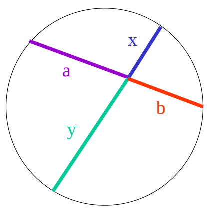


A. Understand and apply theorems relating the product of segments of chords, secants, and tangents of a circle.

B. Use these theorems to solve problems involving circles.

★ Theorem 9-11

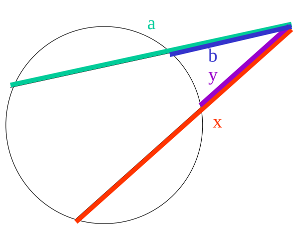
When two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord.



(a) (b) = (x) (y)

★ Theorem 9-12

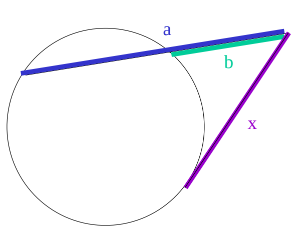
When two secants are drawn to a circle from an external point, the product of one secant segment and its external segment equals the product of the other secant segment and its external segment.



(a) (b) = (x) (y)

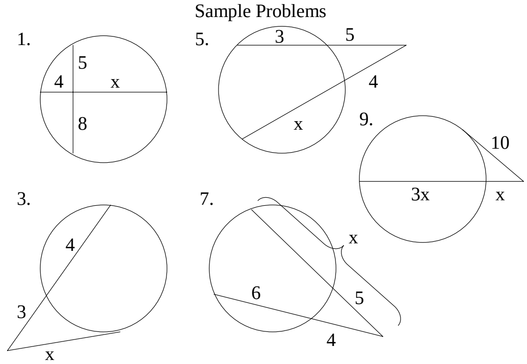
★ Theorem 9-13

When a secant segment and a tangent segment are drawn to a circle from an external point, the product of the secant segment and its external segment equals the square of the tangent segment



(a)(b)= x^2

Sample Problems

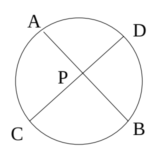


Sample Problems

Chords AB and CD intersect at P.

13. AP = 6, BP = 8, CD = 16
    DP = ?

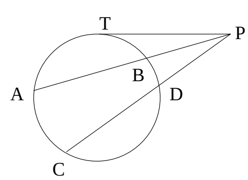
15. AB = 12, CP = 9, DP = 4
    BP = ?



PT is tangent to the circle.

17. PT = 6, PB = 3
    AB = ?

19. PD = 5, CD = 7,
    AB = 11, PB = ?



Sample Problems

23. A bridge over a river has the shape of a circular arc. The span of the bridge is 24 meters. The midpoint of the arc is 4 meters higher than the endpoints. What is the radius of the circle that contains the arc.

Chapter 9

Circles
Review
Homework Page 371:
2-16 evens