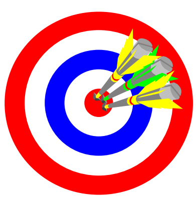
Chapter 11

Areas of Plane Figures

Objectives



A. Use the terms defined in the chapter correctly.
B. Properly use and interpret the symbols for the terms and concepts in this chapter.
C. Appropriately apply the postulates and theorems in this chapter.
D. Correctly calculate the perimeters of various plane figures.
E. Correctly calculate the areas of various plane figures.
F. Correctly compare the areas of various plane figures.
G. Correctly calculate the areas of combined plane figures.
H. Correctly calculate linear and regional geometric probability.

Section 11-1

Areas of Rectangles

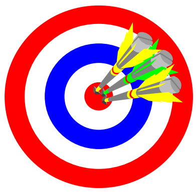
Homework Pages 426-427:

2-34 evens

Excluding 8, 12, 18, and 30

YOU MUST DRAW THE FIGURES!

Objectives



A. Understand the meaning of the phrase ‘area of a polygon’.
B. Understand and apply the area of a square postulate.
C. Understand and apply the area congruence postulate.
D. Understand and apply the area addition postulate.
E. Understand and apply the area of a rectangle theorem.
F. Apply the area theorems and postulates to real world problems.
G. Understand and apply the terms ‘altitude’, ‘base’, and ‘height’.

Reminder! Pattern Right Triangles

- Pattern right triangles can also be seen as RATIOS!
  - The Pythagorean Triples (based on lengths of sides)
    - 3x: 4x: 5x
    - 5x: 12x: 13x
    - 8x: 15x: 17x
    - 7x: 24x: 25x
  - The Special Right Triangles (based on angles)
    - 45-45-90
      - Based on sides → 1x: 1x, $\sqrt{2}x$
    - 30-60-90
      - Based on sides → 1x: $\sqrt{3}x$: 2x

What is LARGER?

- You have been cautioned not to refer to one figure being LARGER than another based simply on:
  - One side of a figure being longer than one side of a second figure
  - The measure of one angle of a figure being greater than the measure of an angle of a second figure
  - One figures ‘looking’ bigger than a second figure
- With the information in this chapter, you can finally say which figure is LARGER!
- In geometry, the larger figure has the greater area.
- To be ‘technically correct’, we should refer to the ‘area of a rectangular region’, not the ‘area of a rectangle’.

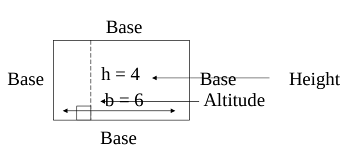
Key Measurement Terms

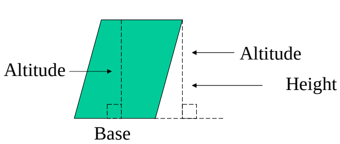
- Base →
  - Any side of a:
    - Triangle
    - Parallelogram
    - Rectangle
    - Rhombus
    - Square
  - or the parallel sides of a trapezoid.
  - The length of the base is indicated by a 'b'.
- Altitude → any segment perpendicular to a line containing a base from a point on the opposite side of the figure.
- Height → length of an altitude, indicated by an 'h'.

Key Measurement Terms

- Perimeter → The length of the 'outside' of a closed geometric figure.
- Perimeter of a Polygon → The sum of the lengths of the sides of the polygon.
- Circumference of a Circle → The perimeter of a circle

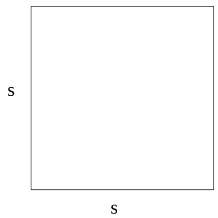
Terms in Action





★ Postulate 17

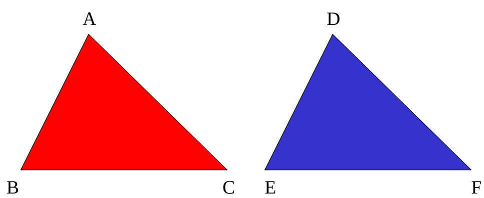
The area of a square is the square of the length of a side.



Area = s^2

★ Postulate 18

If two figures are congruent, then they have the same area.



Δ ABC ≅ Δ DEF

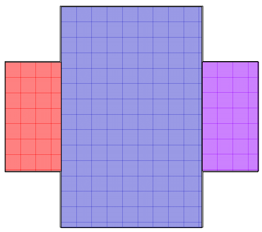
Area of Δ ABC = Area of Δ DEF

So …

How would you correctly identify one figure being larger or smaller than a second figure?

★ Postulate 19

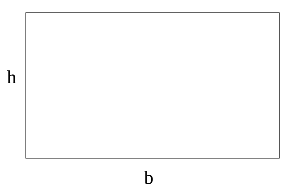
The area of a region is the sum of the areas of its non-overlapping parts.



Area of the cross = red area + blue area + purple area

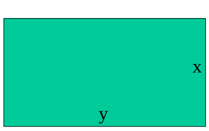
★ Theorem 11-1

The area of a rectangle equals the product of its base and its height.



Area = (b)(h)

Sample Problems



If x = 2 and y = 7, then perimeter = ?

Perimeter = 2 + 7 + 2 + 7 = 18

If x = 4 and y = 6, then Area = ?

Using Theorem 11-1, the area is the product of the base times height, 6 x 4 = 24 square units.

If the area = 70 square units and y = 4, then x = ?

Using Theorem 11-1, the area is the product of the base times height, 70 square units = 4x. x = 70/4 = 35/2

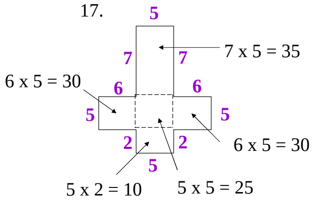
Sample Problems

	1	3	5	7
b	12	16	$4\sqrt{2}$	2x
h	5		$3\sqrt{2}$	x - 3
A		80		

	9	11	13	15
b	9	16	a + 3	x
h	4		a - 3	
A				$x^2 - 3x$
p		42		

Sample Problems

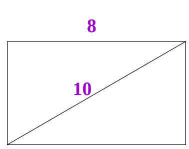
17.



By Postulate 19, Area = 35 + 30 + 25 + 10 + 30 = 130 sq units.

Sample Problems

19.



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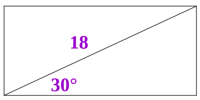

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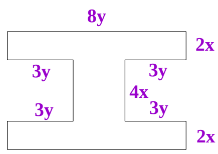
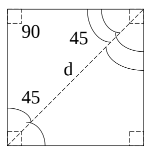


Sample Problems

25. Find the area of a square with diagonal of length d.



Definition of square:
- Parallelogram
- Congruent Sides
- 4 Right Angles

All squares are rhombi.
Diagonals of rhombi bisect angles.

Pattern Right Triangle → 45-45-90 $x, x, x\sqrt{2}$

$d = x\sqrt{2}$ $x = \frac{d}{\sqrt{2}}$ $Area = x \times x = \frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}} = \frac{d^2}{2} sq.units$

Sample Problems

27. A path 2 m wide surrounds a rectangular garden 20 m long and 12 m wide. Find the area of the path.
29. A room 28 ft long and 20 ft wide has walls 8 ft high.
    a. What is the total wall area?
    b. How many gallon cans of paint should be bought to paint the walls if 1 gal of paint covers 300 ft^2?
31. A rectangle having an area of 392m^2 is twice as long as it is wide. Find its dimensions.
35. You have 40 m of fencing to enclose a pen for your dog. If one side is x, what is the dimension, in terms of x, of the other side? Express the area in terms of x. Find the area of the pen if x = 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. What are the dimensions of the pen with the greatest area?

Section 11-2

Areas of Parallelograms,Triangles and Rhombuses

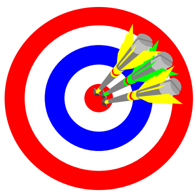
Homework Pages 431-433:

2-36 evens

Excluding 12, 16, 24, 32

YOU MUST DRAW THE FIGURES!

Objectives



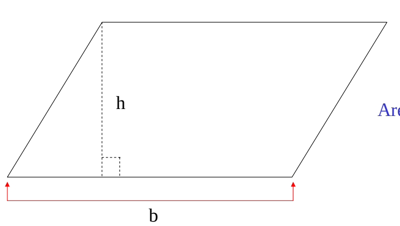
A. Understand and evaluate the area of a parallelogram.
B. Understand and evaluate the area of a triangle.
C. Understand and evaluate the area of a rhombus by using the lengths of the diagonals.

Reminder! Pattern Right Triangles

- Pattern right triangles can also be seen as RATIOS!
  - The Pythagorean Triples (based on lengths of sides)
    - 3x: 4x: 5x
    - 5x: 12x: 13x
    - 8x: 15x: 17x
    - 7x: 24x: 25x
  - The Special Right Triangles (based on angles)
    - 45-45-90
      - Based on sides → 1x: 1x, $\sqrt{2}x$
    - 30-60-90
      - Based on sides → 1x: $\sqrt{3}x$: 2x

★ Theorem 11-2

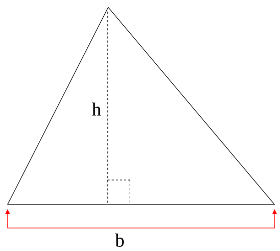
The area of a parallelogram equals the product of a base and the height to that base.



Area = (b)(h)

★ Theorem 11-3

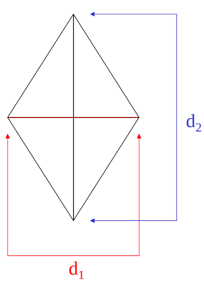
The area of a triangle equals half the product of a base and the height to that base.



Area = (½)(b)(h)

★ Theorem 11-4

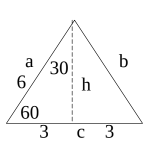
The area of a rhombus equals half the product of its diagonals.



Area = (½)(d_1)(d_2)

Sample Problems

12. Find the area of an equilateral triangle with perimeter = 18.



a = b = c

$a + b + c = 18$

$a + a + a = 18$ by substitution

$3a = 18 \quad or \quad a = 6$

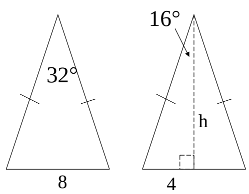
30-60-90 pattern right triangle

$x, x\sqrt{3}, 2x$

$x = 3 \quad h = 3\sqrt{3} \quad Area = \frac{1}{2} b \times h = \frac{1}{2}\left(6 \times 3\sqrt{3}\right) = 9\sqrt{3}$

Sample Problems

24. Find the area of an isosceles triangle with a 32 degree vertex angle and a base of 8 cm. (find to the nearest 10^{th})



$\tan 16° = \frac{4}{h}$

$(.2867) = \frac{4}{h}$

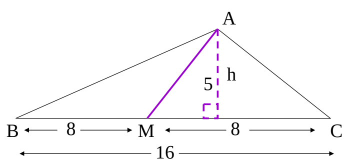
$h \approx \frac{4}{.2867} \approx 13.9519$

$Area = \frac{1}{2} b \times h = \left(\frac{1}{2}\right)(8)(13.9519)$

$\approx 55.8 cm^2$

Sample Problems

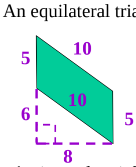
27. If AM is a median of Δ ABC, h = 5, and BC = 16 then find the areas of Δ ABC and Δ AMB.

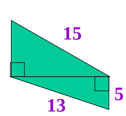


$Area\ \Delta ABC = \frac{1}{2} b \times h = \frac{1}{2}(16)(5) = 40\ units^2$

$Area\ \Delta AMB = \frac{1}{2} b \times h = \frac{1}{2}(8)(5) = 20\ units^2$

Sample Problems

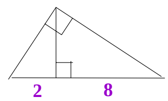
1. A triangle with base 5.2 and corresponding height 11.5.
3. A parallelogram with base $3\sqrt{2}$ and corresponding height $2\sqrt{2}$
5. An equilateral triangle with sides 8.
7. 

9. 

11. An isosceles right triangle with hypotenuse 8.
13. A parallelogram with a 45° angle and sides 6 and 10.
15. A 30°-60°-90° triangle with hypotenuse 10.
17. A rhombus with perimeter 68 and one diagonal 30.

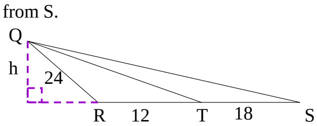
Sample Problems

19. A square inscribed in a circle with radius r.
25. Find the areas of all three triangles.



Sample Problems

29a. Find the ratio of the areas of Δ QRT and Δ QTS.
29b. If the area of Δ QRS is 240, find the length of the altitude from S.

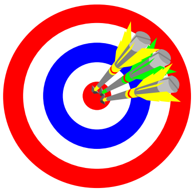


33. Find the area of an equilateral triangle with side 7.
35. The area of a rhombus is 100. Find the length of the two diagonals if one is twice as long as the other.

Section 11-3

Areas of Trapezoids
Homework Pages 436-437:
2-26 evens
Excluding 8, 14

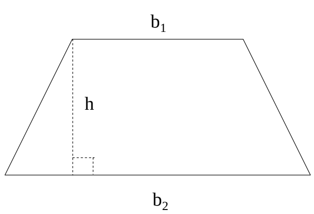
Objectives



A. Correctly identify the bases and height of a trapezoid.
B. Understand and apply the theorem of the area of a trapezoid.

★ Theorem 11-5

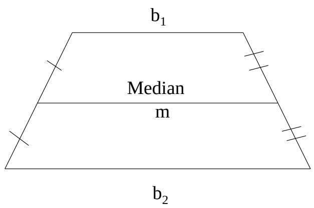
The area of a trapezoid equals half the product of the height and the sum of the bases.



Area = (½)(h)(b_1 + b_2)

Do you remember?

- What is the median of a trapezoid?
- What is the relationship between the lengths of the bases of a trapezoid and its median?



$$m = \frac{1}{2}(b_1 + b_2)$$

Sample Problems

trapezoid	1.
b_1	12
b_2	8
h	7
A	
m	

$Area = \frac{1}{2}h(b_1 + b_2) =$

$\frac{1}{2}(7)(12+8) = 70\ units^2$

$Median = \frac{1}{2}(b_1 + b_2) =$

$\frac{1}{2}(12+8) = 10\ units$

Sample Problems

trapezoid	8.
b_1	
b_2	3k
h	5k
A	$45k^2$
m	

$Area = \frac{1}{2}h(b_1 + b_2)$

$\frac{1}{2}(5k)(b_1 + 3k) = 45k^2\ units^2$

$(5k)(b_1 + 3k) = 90k^2$

$(b_1 + 3k) = 18k$

$b_1 = 15k$

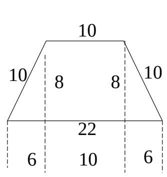
$m = \frac{1}{2}(b_1 + b_2)$

$m = \frac{1}{2}(15k + 3k)$

$m = 9k$

Sample Problems

17. An isosceles trapezoid with legs 10 and bases 10 and 22.



6, x, 10 right triangle.
x = 8.

By Theorem 11-5:

$A = \frac{1}{2}h(b_1 + b_2) =$

$\frac{1}{2}(8)(10+22) = 128\ units^2$

By area addition: $A = \frac{1}{2}(6)(8) + (10)(8) + \frac{1}{2}(6)(8) = 128\ units^2$

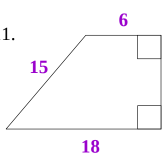
Sample Problems

trapezoid	3.	5.	7.
b_1		27 $3\frac{1}{6}$	
b_2		9 $4\frac{1}{3}$	3k
h		$1\frac{3}{5}$	5k
A		90	$45k^2$
m			

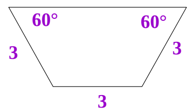
Sample Problems

9. A trapezoid has area 54 and height 6. How long is its median?

11.



13.



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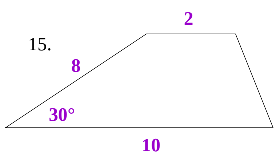


Sample Problems

23. An isosceles trapezoid has bases 12 and 28. The area is 300. Find the height and the perimeter.

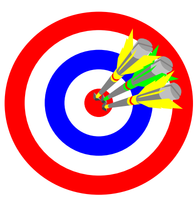
25. ABCDEF is a regular hexagon with side 12. Find the areas of the three regions formed by diagonals AC and AD.

27. A trapezoid with area 100 has bases 5 and 15. Find the areas of the two triangles formed by extending the legs until they intersect.

Section 11-4

Areas of Regular Polygons
Homework Pages 443-444:
2-20 evens

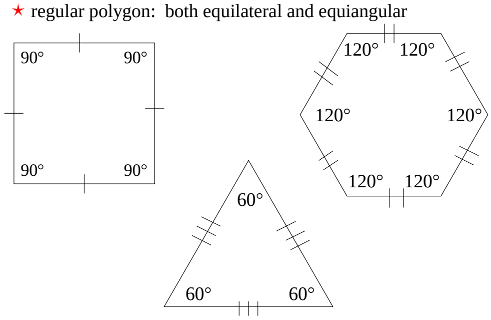
Objectives



A. Correctly identify and apply the terms ‘center of a regular polygon’, ‘radius of a regular polygon’, ‘central angle of a regular polygon’, and ‘apothem of a regular polygon’.
B. Understand and apply the theorem regarding the area of a regular polygon.

Regular Reminder

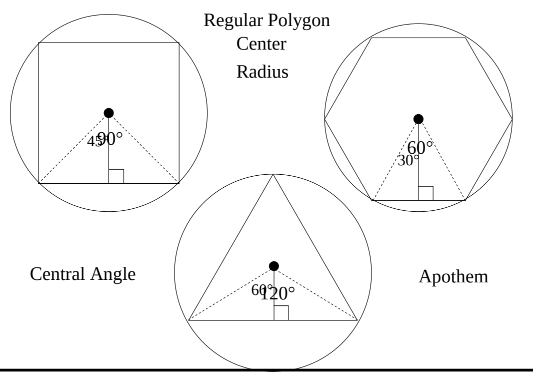
★ regular polygon: both equilateral and equiangular



Parts of a Regular Polygon

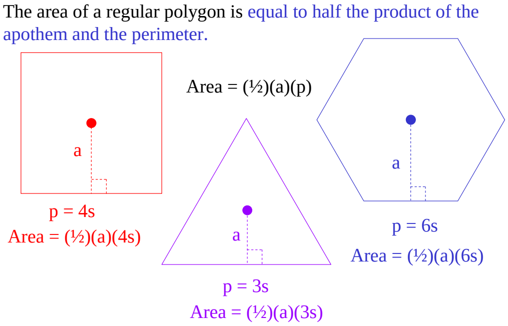
- ★ center of a regular polygon: center of the circumscribed circle.
- ★ radius of a regular polygon: distance from the center to a vertex.
- ★ central angle of a regular polygon: the angle formed by two radii drawn to consecutive vertices.
- ★ apothem of a regular polygon: distance from the center to a side.

★ Parts of a Regular Polygon



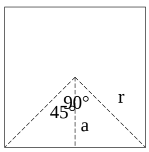
★ Theorem 11-6

The area of a regular polygon is equal to half the product of the apothem and the perimeter.



Sample Problems

	r	a	A
1.	$8\sqrt{2}$		



$45° - 45° - 90°$

$x, x, x\sqrt{2}$

$?, ?, 8\sqrt{2}$

$x = a = 8$

$s = 2(8) = 16$

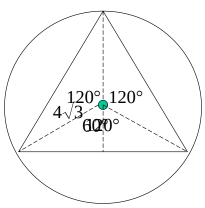
$p = 16 \times 4 = 64$

$Area = \frac{1}{2}ap =$

$\frac{1}{2}(8)(64) = 256\ units^2$

Sample Problems

13. Find the area of an equilateral triangle with radius $4\sqrt{3}$



30-60-90 Right Triangle

$x, x\sqrt{3}, 2x$

$2x = 4\sqrt{3}$

$x = 2\sqrt{3}$

$2\sqrt{3}, 6, 4\sqrt{3}$

$Apothem = 2\sqrt{3}$

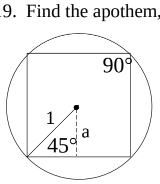
$Side = 2(6) = 12$

$Perimeter = 3s = 3(12) = 36$

$Area = \frac{1}{2}ap = \frac{1}{2}\left(\frac{2\sqrt{3}}{1}\right)\left(\frac{36}{1}\right) = 36\sqrt{3}\ units^2$

Sample Problems

19. Find the apothem, perimeter and area of the inscribed polygon.



$45 - 45 - 90$

$x, x, x\sqrt{2}$

$1 = x\sqrt{2}$

$x = \frac{\sqrt{2}}{2} = a$

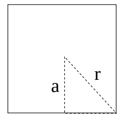
$s = \frac{\sqrt{2}}{2} \times 2 = \sqrt{2}$

$p = 4s = 4\sqrt{2}$

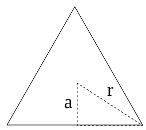
$$A = \frac{1}{2}ap = \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{4\sqrt{2}}{1}\right) = \frac{8}{4} = 2\ units^2$$

Sample Problems

	r	a	A
3.			49

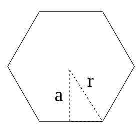


	r	a	p	A
5.	6			
7.			12	



Sample Problems

	r	a	p	A
9.	4			
11.		6		

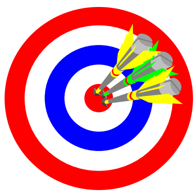


15. A regular hexagon with perimeter 72.
21. Find the perimeter and the area of a regular dodecagon inscribed in a circle with a radius of 1.

Section 11-5

Circumferences and Areas of Circles
Homework Pages 448-450:
2-30 evens
Excluding 4, 10, 14, 24, 28

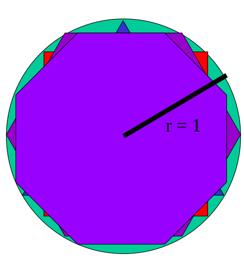
Objectives



A. Understand and apply the term 'circumference of a circle'.
B. Apply the equations for circumference and area of a circle to real world problems.
C. Understand the relationship between the diameter of a circle and its circumference.
D. Derive the approximate value for pi.
E. Identify multiple approximations for pi.

- circle:
  - area: the limit of the sequence of numbers representing the areas of the inscribed regular polygons as the number of sides goes to infinity.
    - ★Area $=\pi r^2$
  - circumference: the limit of the sequence of numbers representing the perimeters of the inscribed regular polygons as the number of sides goes to infinity.
    - ★ Circumference = $2\pi r$ or πd

Area of a Circle



Area ≈ 1.3

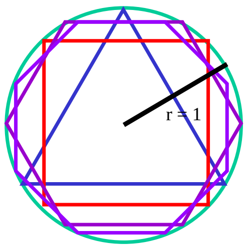
Area = 2

Area ≈ 2.60

Area ≈ 2.83

Area ≈ $\pi r^2 \approx 3.14$

Circumference of a Circle



Perimeter ≈ 5.20

Perimeter ≈ 5.66

Perimeter = 6

Perimeter ≈ 6.12

Circumference = 2 π r
Circumference ≈ 6.28

Approximations of π

You can approximate π as:

- 3.14
- 3.1416
- 3.14159
- $\frac{22}{7}$

Sample Problems

	3.
r	$\frac{5}{2}$
C	
A	

In terms of π

$$C = 2\pi r = \left(\frac{2}{1}\right)(\pi)\left(\frac{5}{2}\right) = 5\pi$$

Since the radius is given as a fraction, use

$$\pi \approx \tfrac{22}{7}$$

$$C = 2\pi r \approx \left(\frac{2}{1}\right)\left(\frac{22}{7}\right)\left(\frac{5}{2}\right) \approx$$

$$\left(\frac{2}{2}\right)\left(\frac{22}{7}\right)\left(\frac{5}{1}\right) \approx (1)\left(\frac{22}{7}\right)\left(\frac{5}{1}\right) \approx \frac{110}{7}$$

In terms of π

$$A = \pi r^2 = (\pi)\left(\frac{5}{2}\right)^2 = \frac{25\pi}{4}u^2$$

In terms of a fraction

$$A = \pi r^2 \approx \left(\frac{22}{7}\right)\left(\frac{5}{2}\right)^2 \approx \frac{(22)25}{(7)4} \approx \frac{275}{14}u^2$$

Sample Problems

11. A basketball rim has a diameter 18 in. Find the circumference and the area enclosed by the rim.
Use $\pi \approx 3.14$

$$C = \pi d \approx (3.14)(18) \approx 57 \; inches$$

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 \approx (3.14)\left(\frac{18}{2}\right)^2$$

$$\approx (3.14)(9)^2 \approx (3.14)(81) \approx 254 \; inches^2$$

Sample Problems

	1.	5.	7.
r	7		
C		20π	
A			25π

9. Find the circumference and the area of a circle when the diameter is (a) 42 (b) 14k. $\pi \approx \frac{22}{7}$

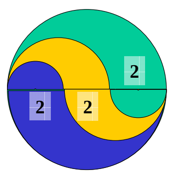
Sample Problems

13. If 6oz of dough are needed to make an 8 in pizza, how much dough is needed to make a 16 in pizza of the same thickness?
15. A school's wrestling mat is a square with 40 ft sides. A circle 28 ft in diameter is painted on the mat. No wrestling is allowed outside the circle. Find the area of the mat not used during wrestling. $\pi \approx \frac{22}{7}$
17. Which is a better buy, a 10 in pizza costing $5 or a 15 in pizza costing $9?
21. The tires of a racing bike are approximately 70 cm in diameter. (a) How far does a racer travel in 5 min if the wheels are turning at a speed of 3 revolutions per second? (b) How many revolutions does the wheel make in a 22 km race? Use $\pi \approx \frac{22}{7}$

Sample Problems

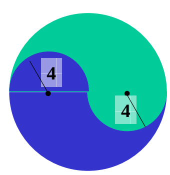
23. A target consists of four concentric circles with radii 1, 2, 3, and 4. (a) Find the area of the bull's eye and of each ring of the target. (b) Find the area of the nth ring if the target contains n rings and a bull's eye.

25. Find the area of each shaded region.



Sample Problems

27. Find the area of the blue region.

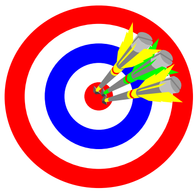


29. Draw a square and its inscribed and circumscribed circles. Find the ratio of the areas of these two circles.

Section 11-6

Arc Lengths and Areas of Sectors
Homework Pages 453-455:
2-24 evens
Excluding 4, 8, 20

Objectives



A. Identify the sector of a circle.
B. Calculate the sector area of a circle based on arc length and radius.
C. Calculate the radius of a circle based on the arc length and sector area.
D. Calculate the arc length of a circle based on the sector area and radius.
E. Apply the sector area formulas to real world problems.

- sector of a circle: region of circle defined by two radii and the arc connecting the outer endpoints.
  - ★ arc length equals the product of the circumference and the measurement of the arc divided by 360
  - ★ sector area equals the product of the area and the measurement of the arc divided by 360

★ NOTE!

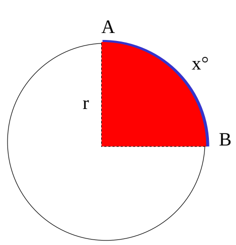
The MEASURE of the arc is the distance around the circle.

The measure of the arc equals the measure of the central angle and is expressed in ***degrees***.

The LENGTH of the arc is the LINEAR distance of the arc.

The length is expressed in ***linear measures***, such as feet or centimeters.

★ Arc Length & Sector Area



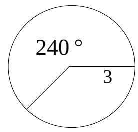
If $m\overset{\frown}{AB} = x$, then the length of $\overset{\frown}{AB} = 2\pi r\left(\frac{x}{360}\right)$

If $m\overset{\frown}{AB} = x$, then the area of sector$AOB = \pi r^2\left(\frac{x}{360}\right)$

Sample Problems

Make a sketch, find the arc length of AB, find the sector area of AOB.

	4.
m ∠ AOB	240°
radius	3



$m\ \overset{\frown}{AB} = 240°$

$Length\ \overset{\frown}{AB} = \frac{x}{360°} \bullet 2\pi r =$

$\frac{240°}{360°} \bullet 2\pi(3) = 4\pi$

Area of sector $AOB = \frac{x}{360°} \bullet \pi r^2 =$

$\frac{240°}{360°} \bullet \pi(3)^2 = 6\pi\ units^2$

Does it make sense?

- Look at the previous problem. Does it make sense?
  - If there are 360 degrees in a circle, then does it make sense that the length of a 240 degree arc would be 2/3 of the circumference of the circle?

$\frac{240°}{360°} = \frac{2}{3}$

Circumference of circle with radius $= 3$

$C = 2\pi r = 2(3)\pi = 6\pi$

$m\ \overset{\frown}{AB} = 240°$

$Length\ \overset{\frown}{AB} = \frac{x}{360°} \bullet 2\pi r =$

$\frac{240°}{360°} \bullet 2\pi(3) = 4\pi$

$$\frac{4\pi}{6\pi} = \frac{2}{3}$$

Does it make sense?

- Look at the previous problem. Does it make sense?
  - If there are 360 degrees in a circle, then does it make sense that the sector area of a sector defined by a 240 degree arc would be 2/3 of the area of the circle?

$\frac{240°}{360°} = \frac{2}{3}$

Area of circle with radius $= 3$

$A = \pi r^2 = \pi(3)^2 = 9\pi$

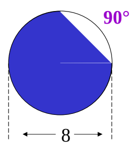
Area of sector $AOB = \frac{x}{360°} \bullet \pi r^2 =$

$\frac{240°}{360°} \bullet \pi(3)^2 = 6\pi\ units^2$

$$\frac{6\pi\ u^2}{9\pi\ u^2} = \frac{2}{3}$$

Sample Problems – find the area of the shaded region

15.



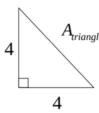
Find the area of the circle.

$$A_{circle} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = 16\pi\ units^2$$

Find the area of the sector.

$$A_{sector} = \frac{x}{360}\pi r^2 = \frac{90}{360}\pi 4^2 = 4\pi\ units^2$$

Find the area of the triangle.



$$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8\ units^2$$

$$A_{region} = A_{circle} - A_{sector} + A_{triangle}$$

$$A_{region} = 16\pi - 4\pi + 8 = (12\pi + 8)u^2$$

Sample Problems

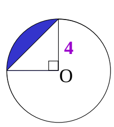
Make a sketch, find the arc length of AB, find the sector area of AOB.

	1.	3.	5.	7.	9.
m ∠ AOB	30	120	180	40	108
radius	12	3	1.5	$\frac{9}{2}$	$5\sqrt{2}$

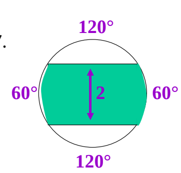
11. The area of a sector AOB is 10π and m ∠ AOB = 100. Find the radius of circle O.

Sample Problems

13.

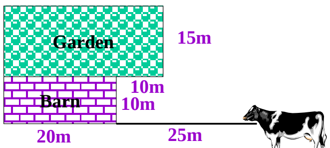


17.



Sample Problems

19. A rectangle with length 16 cm and width 12 cm is inscribed in a circle. Find the area of the region inside the circle but outside the rectangle.
23. A cow is tied by a 25 m rope to the corner of a barn. A fence keeps the cow out of the garden. Find the grazing area.

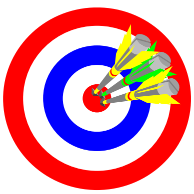


25. Two circles have radii 6 cm and their centers are 6 cm apart. Find the area of the region common to both circles.

Section 11-7

Ratios of Areas
Homework Pages 458-460:
2-28 evens
Excluding 8, 14, 22, 26

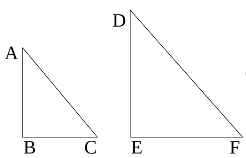
Objectives



A. Compare the areas of triangles that share similar components.
B. Utilize the scale factor of similar polygons to determine the ratios of the perimeters of the similar polygons.
C. Utilize the scale factor of similar polygons to determine the ratios of the areas of the similar polygons.
D. Calculate perimeters and areas of similar polygons based on their scale factor.

Do You Remember?

- What is a 'scale factor'?
- The scale factor refers to similar polygons.
- For similar polygons, the scale factor is the ratio of the lengths of two **CORRESPONDING** sides.



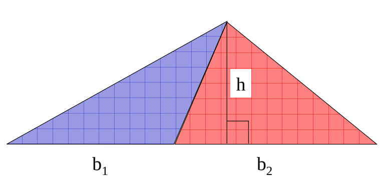
If $\Delta ABC \sim \Delta DEF$,
$BC = 5$, and $EF = 10$

Then scale factor $= 5:10 = 1:2$

Comparing the Areas of Triangles

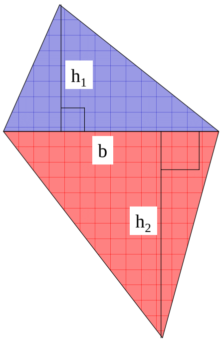
★ Comparing Areas of Triangles
- If two triangles have equal heights, then the ratio of their areas equals the ratio of their bases.
- If two triangles have equal bases, then the ratio of their areas equals the ratio of their heights.
- If two triangles are similar, then the ratio of their areas is equal to the square of their scale factor.

★ Triangles With the Same Height



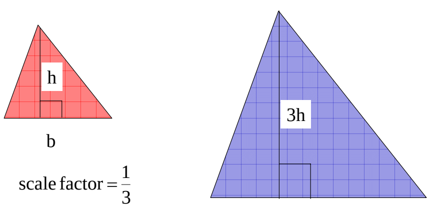
$$\frac{A_1}{A_2} = \frac{\left(\frac{1}{2}\right)(h)(b_1)}{\left(\frac{1}{2}\right)(h)(b_2)} = \frac{b_1}{b_2}$$

★ Triangles With the Same Base



$$\frac{A_1}{A_2} = \frac{\left(\frac{1}{2}\right)(b)(h_1)}{\left(\frac{1}{2}\right)(b)(h_2)} = \frac{h_1}{h_2}$$

★ Similar Triangles

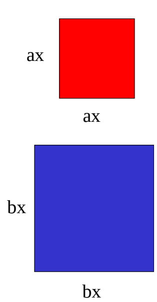


$$\text{scale factor} = \frac{1}{3}$$

$$\frac{A_1}{A_2} = \frac{\left(\frac{1}{2}\right)(b)(h)}{\left(\frac{1}{2}\right)(3b)(3h)} = \frac{1}{(3)(3)} = \frac{1}{3^2} = \left(\frac{1}{3}\right)^2$$

★ Theorem 11-7

If the scale factor of two similar figures is a : b, then
(1) the ratio of their perimeters is a : b
(2) the ratio of their areas is $a^2 : b^2$



$$\text{scale factor} = \frac{ax}{bx} = \frac{a}{b}$$

$$\frac{\text{perimeter red}}{\text{perimeter blue}} = \frac{ax + ax + ax + ax}{bx + bx + bx + bx} = \frac{4ax}{4bx} = \frac{a}{b}$$

$$\frac{\text{area red}}{\text{area blue}} = \frac{(ax)(ax)}{(bx)(bx)} = \frac{a^2x^2}{b^2x^2} = \frac{a^2}{b^2}$$

Sample Problems

similar figures	1.
scale factor	1:4
ratio of perimeters	1:4
ratio of areas	1:16

Since the scale factor is 1:4, then a = 1, b =4.

The ratio of the perimeters is equivalent to the scale factor.

The ratio of the areas is equal to the squares of the elements of the scale factor.

Sample Problems

similar figures	8.
scale factor	$\sqrt{2}:1$
ratio of perimeters	$\sqrt{2}:1$
ratio of areas	2:1

Since the ratio of the areas is 2:1, then

$a^2 = 2$ or $a = \sqrt{2}$

$b^2 = 1$ or $b = 1$

Sample Problems

15. A quadrilateral with sides 8 cm, 9 cm, 6 cm, and 5 cm has an area of 45 square centimeters. Find the area of a similar quadrilateral whose longest side is 15 cm.

LOGIC → Since the longest side of the first quadrilateral is 9 cm, and the quadrilaterals are similar, then the longest side of the second quadrilateral must CORRESPOND to the side of length 9 cm.

Scale factor → 9:15 or 3:5 a = 3, b = 5

$$\text{Ratio of areas} = a^2 : b^2 = 3^2 : 5^2 = 9 : 25$$

$$\frac{9}{25} = \frac{45}{x}$$

$$x = 125 cm^2$$

Sample Problems

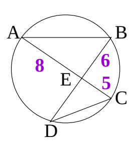
similar figures	3.	5.	7.
scale factor	r:2s		
ratio of perimeters		3:13	
ratio of areas			9:64

9. On a map of California, 1 cm corresponds to 50 km. Find the ratio of the map's area to the actual area of California.

11. L, M and N are the midpoints of the sides of Δ ABC. Find the ratio of the perimeters and the ratio of the areas of Δ LMN & Δ ABC.

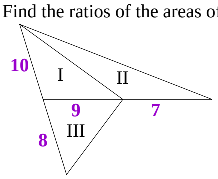
Sample Problems

13.

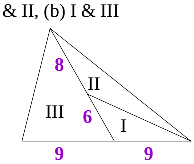


Find the ratios of the areas of (a) I & II, (b) I & III

17.

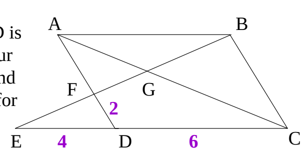


19.



Sample Problems

21. In the diagram, ABCD is a parallelogram. Name four pairs of similar triangles and give the ratio of the areas for each pair.

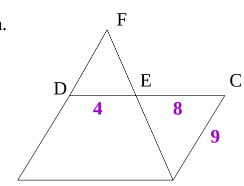


23. ABCD is a parallelogram.

$\frac{\text{area of } \Delta\, DEF}{\text{area of } \Delta\, ABF}$

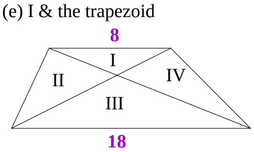
$\frac{\text{area of } \Delta\, DEF}{\text{area of } \Delta\, CEB}$

$\frac{\text{area of } \Delta\, DEF}{\text{area of trap } DEBA}$

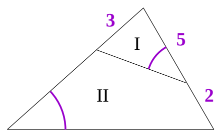


Sample Problems

25. Find the ratio of the areas
(a) I & III (b) I & II
(c) I & IV (d) II & IV
(e) I & the trapezoid



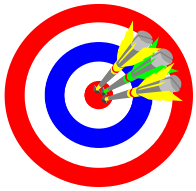
27. Find the ratio of the areas of region I and II.



Section 11-8

Geometric Probability
Homework Pages 463-464:
2-16 evens
Excluding 10, 12

Objectives



A. Understand the classical meaning of probability.
B. Understand and apply the concept of linear probability.
C. Understand and apply the concept of regional (area) probability.

"Classical" Probability

- Classical probability depends on:
  - Knowing all possible outcomes to an experiment.
  - Each possible outcome having the same chance of occurring.
  - Knowing the number of outcomes that describe the 'success' of an event.
- If I flip a coin:
  - The possible outcomes are 'heads' or 'tails'.
  - If the coin in 'fair', then it is equally likely that a 'head' will appear as a 'tail'.
  - A successful event might be flipping the coin and having it come up tails.

"Classical" Probability

The probability of an event occurring is:

$$\frac{\text{Number of 'successful' outcomes}}{\text{Number of 'possible' outcomes}}$$

In the coin experiment, a successful outcome would be the appearance of a tail when the coin is flipped.
The possible outcomes would be a 'head' or a 'tail'.

$$\text{Probability (Tails)} = \frac{\text{\# of successes}}{\text{Total \# outcomes}} = \frac{1}{2}$$

The probability of drawing a heart from a standard deck of cards:

$$\text{Probability (Heart)} = \frac{\text{\# of successes}}{\text{Total \# outcomes}} = \frac{13}{52} = \frac{1}{4}$$

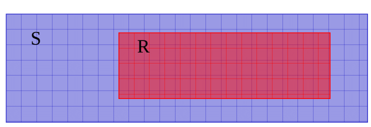
★ geometric probability:
- linear probability: the chances of a randomly chosen point being on a given segment. Found by dividing the length of the target segment by the length of the total segment.
- regional probability: the chances of a randomly chosen point being within the boundary of a given region. Found by dividing the area of the target region by the total area of the object.

★ Linear Geometric Probability

Probability that point P is on $\overline{AC} = \frac{AC}{AB}$

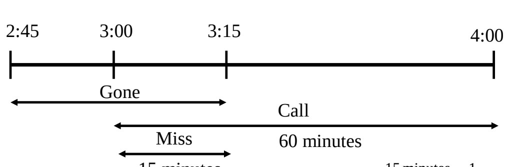
★ Regional Geometric Probability

Probability that a point P is in region R $= \frac{\text{area of R}}{\text{area of S}}$



Sample Problems

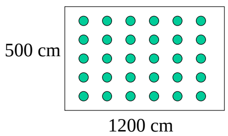
3. A friend promises to call you at home sometime between 3 PM and 4 PM. At 2:45 PM you must leave your house unexpectedly for half an hour. What is the probability that you miss the first call?



$\text{Probability (Miss)} = \frac{15 \text{ minutes}}{60 \text{ minutes}} = \frac{1}{4}$

Sample Problems

7. A dart is thrown at a board 12 m long and 5 m wide. Attached to the board are 30 balloons, each with radius 10 cm. Assuming each balloon lies entirely on the board, find the probability that a dart that hits the board will also hit a balloon.



$$A_{balloon} = \pi r^2 = \pi(10)^2 = 100\pi\ cm^2$$

$$A_{30\ balloons} = (30)(100\pi\ units^2) = 3000\pi\ cm^2$$

$$A_{board} = bh = (1200)(500) = 60{,}000\ cm^2$$

$$\text{PR (balloon hit)} = \frac{3000\pi\ cm^2}{60{,}000\ cm^2} = \frac{\pi}{200}$$

Sample Problems

1. If M is the midpoint of AB and Q is the midpoint of MB. If a point on AB is picked at random, what is the probability that the point is on MQ?
5. A circular dartboard has a 40 cm diameter. Its bull's eye has a diameter 8 cm. (a) If an amateur throws a dart and hits the board, what is the probability that the dart hits the bull's eye? (b) After many throws, 75 darts have hit the target. Estimate the number hitting the bull's eye.
9. At a carnival game, you can toss a quarter on a large table that has been divided into squares 30 mm on a side. If the coin comes to rest without touching any line, you win. Otherwise you lose your coin. What are your chances of winning on one toss if a quarter has a radius of 12 mm.

Sample Problems

11. A piece of wire 6 in long is cut into two pieces at a random point. What is the probability that both pieces of wire will be at least 1 in long?
13. Darts are thrown at a 1 m square which contains an irregular red region. Of 100 darts thrown, 80 hit the square. Of these, 10 hit the red region. Estimate the area of the region.
15. Find the radius (r) of a coin used in the carnival game described in problem #9 if the the probability of winning is 0.25 and the formula for calculating this probability is

$$\text{probability of winning} = \left(\frac{30-2r}{30}\right)^2$$

Chapter 11

Areas of Plane Figures

Review

Homework Page 471: 2-18 evens