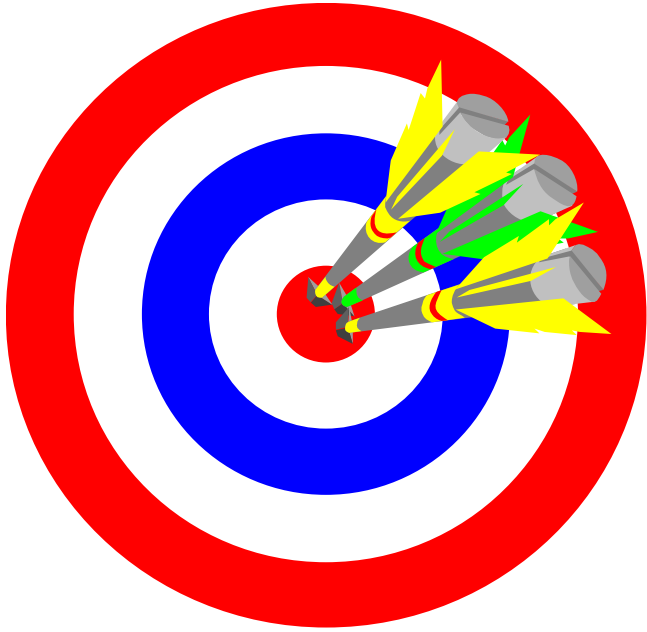


# Chapter Thirteen

## Coordinate Geometry

# Objectives



- A. Use the terms defined in the chapter correctly.
- B. Properly use and interpret the symbols for the terms and concepts in this chapter.
- C. Appropriately apply the postulates and theorems in this chapter.
- D. Understand and apply the distance and midpoint formulas.
- E. Calculate and use the slopes of lines.
- F. Perform the basics of vector mathematics.
- G. Graph linear equations
- H. Write the equations of straight line graphs
- I. Organize and write a coordinate proof.

# Section 13-1

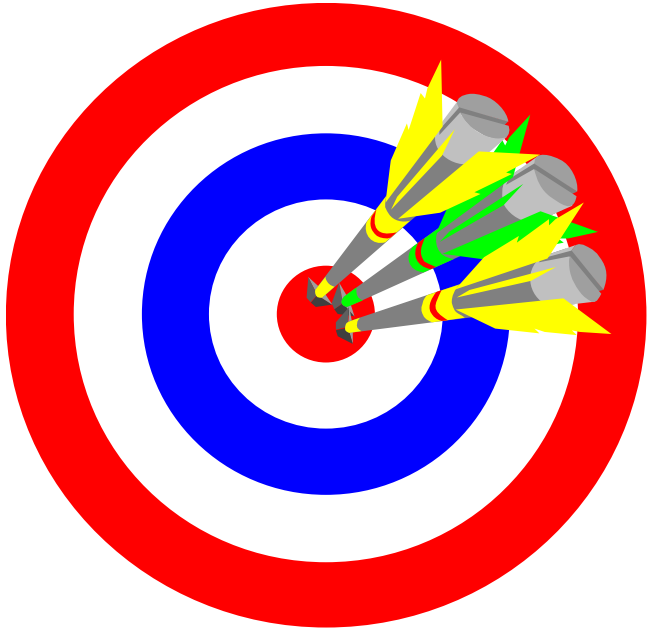
The Distance Formula

Homework Pages 526-527:

2-24 evens

Also #28, #36

# Objectives

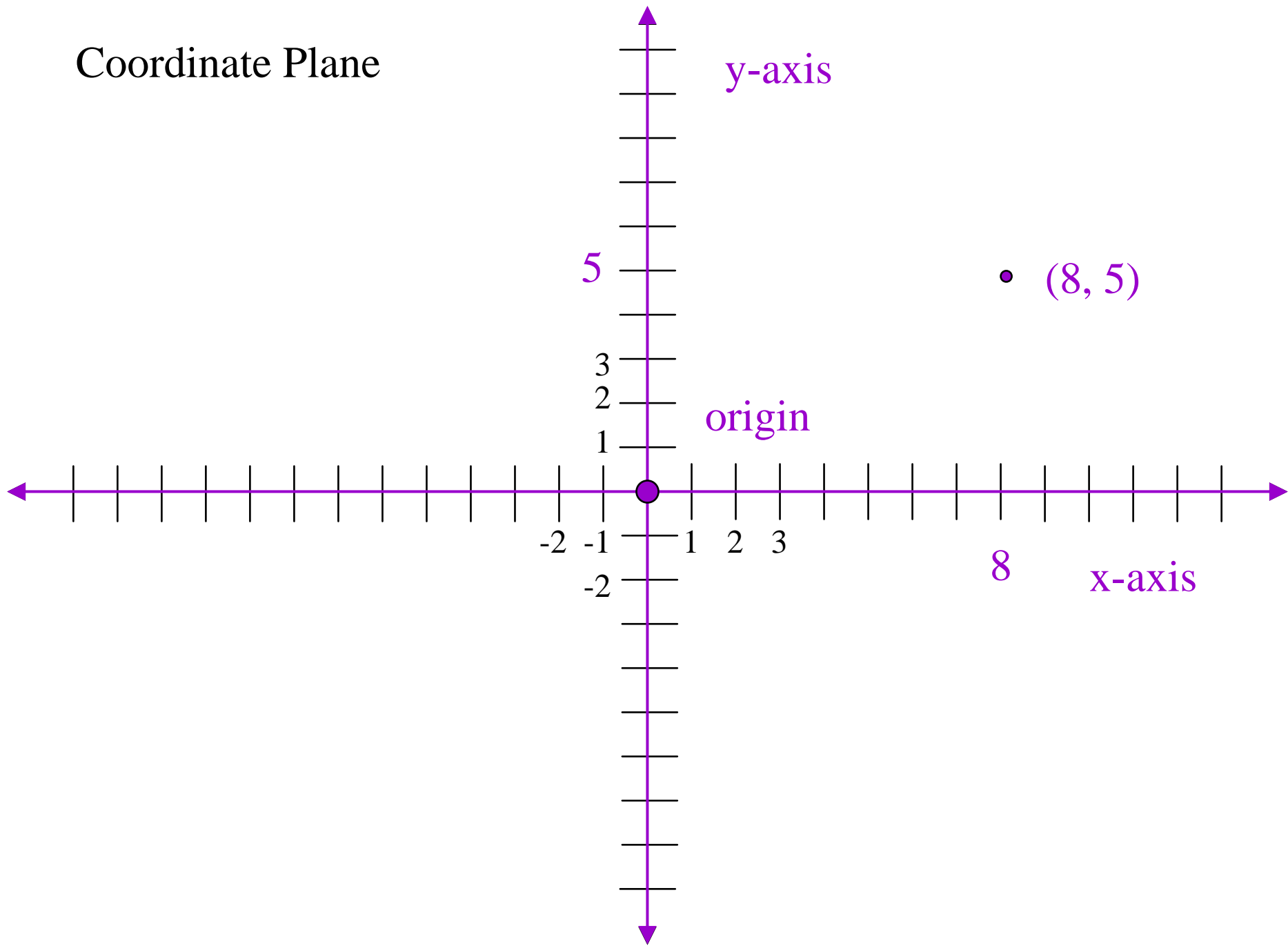


- A. Understand and apply the terms ‘origin’, ‘axes’, ‘quadrants’, and ‘coordinate plane’.
- B. Properly graph points, lines, and circles on a coordinate plane.
- C. Derive and utilize the distance formula.
- D. Understand the components of the equation of a circle.
- E. Graph a circle in a coordinate plane based on its equation.

# The Coordinate Plane

- **coordinate plane:** plane formed by the intersection of a horizontal and a vertical real number line, called the **coordinate axes**, where every point in the plane can be represented by an ordered pair of real numbers, called its **coordinates**.
- **x-axis:** the horizontal coordinate axis
- **y-axis:** the vertical coordinate axis
- **origin:** intersection of the coordinate axes

# Coordinate Plane



y-axis

5

(8, 5)

origin

3

2

1

-2

-1

1

2

3

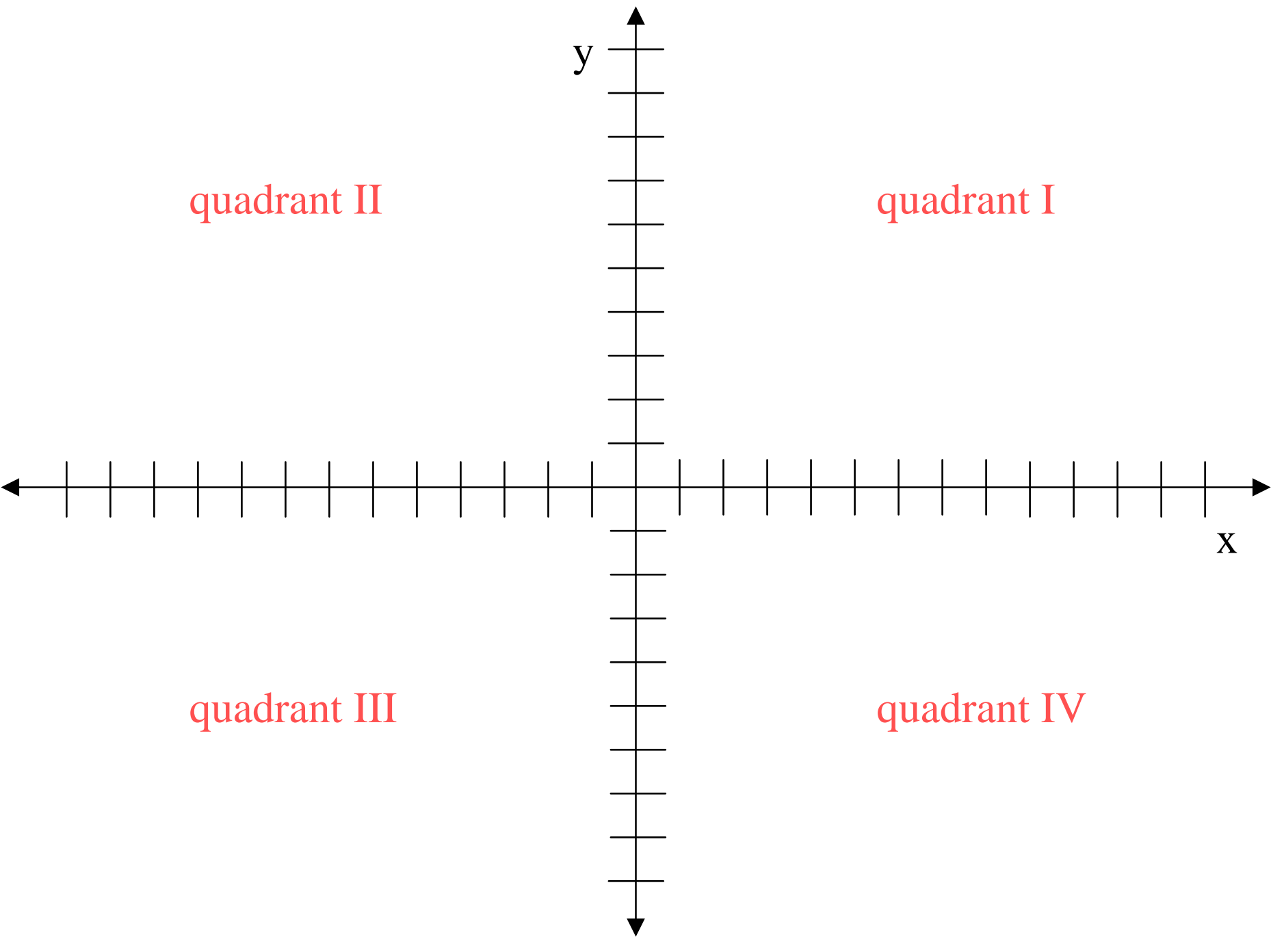
-2

8

x-axis

## The Coordinate Plane - Quadrants

- ★ quadrants: four regions of the plane created by the two axes.
  - quadrant I: both  $x$  &  $y$  are positive
  - quadrant II:  $x$  is negative;  $y$  is positive
  - quadrant III: both  $x$  &  $y$  are negative
  - quadrant IV:  $x$  is positive;  $y$  is negative



quadrant II

quadrant I

quadrant III

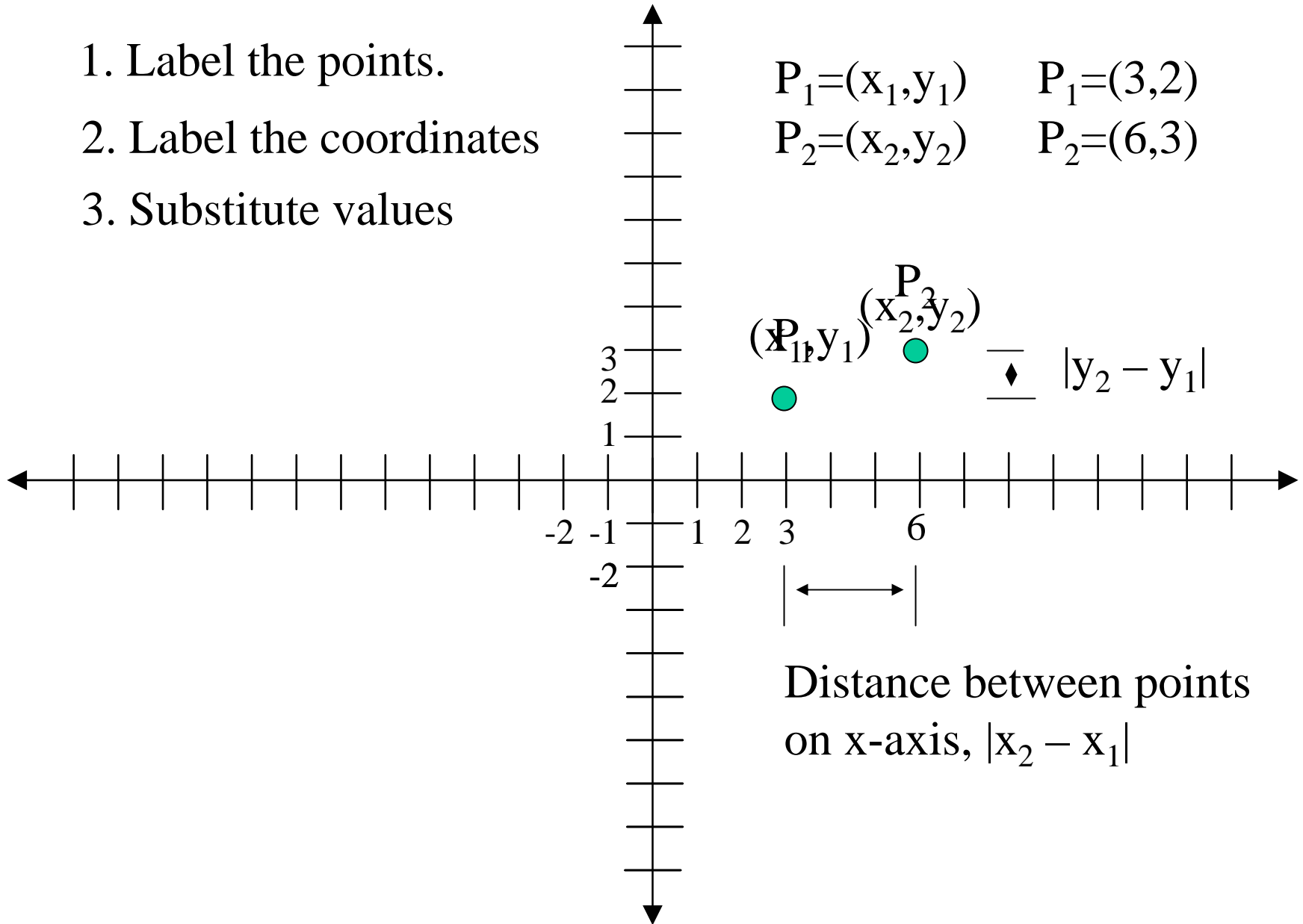
quadrant IV

y

x

# Building the Distance Formula

1. Label the points.
2. Label the coordinates
3. Substitute values



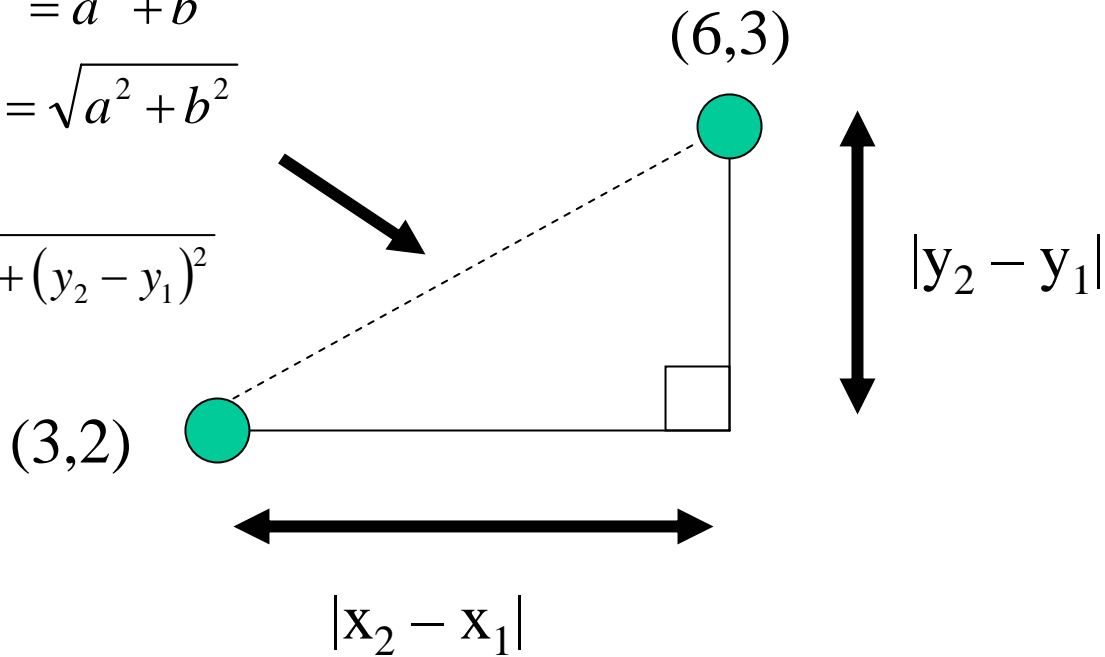
Distance between points  
on x-axis,  $|x_2 - x_1|$

# Building the Distance Formula

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

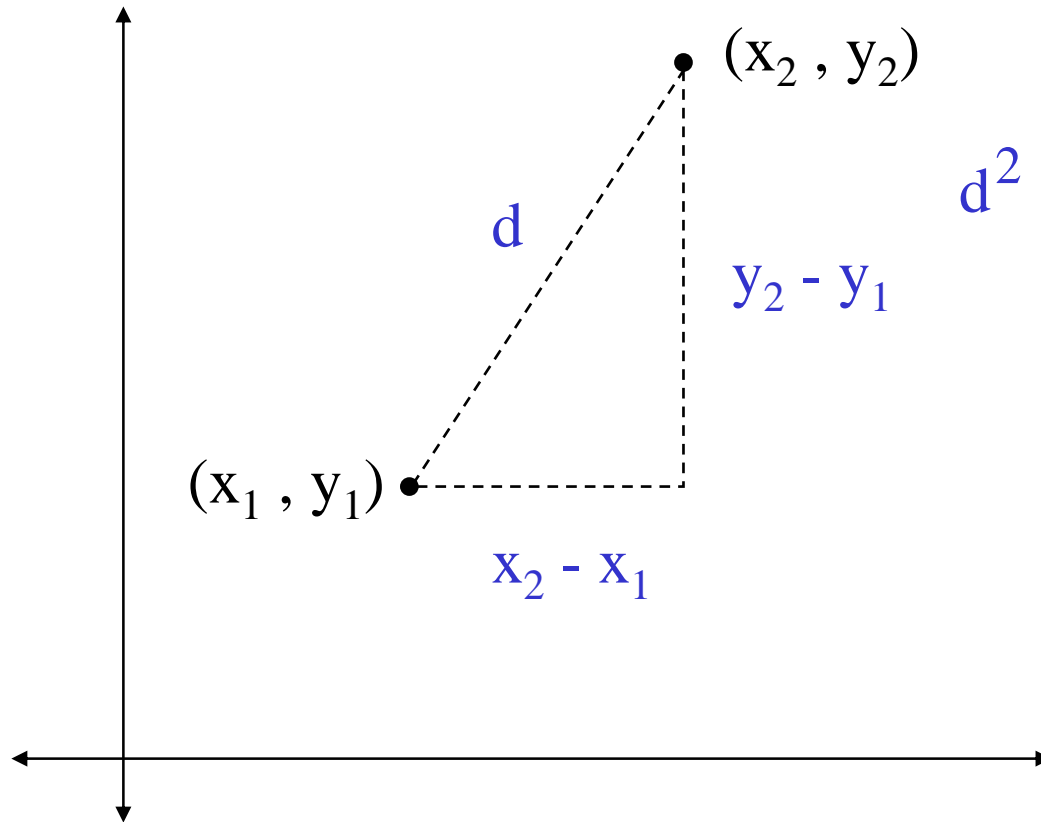
$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



## ★ Theorem 13-1 (Distance Formula)

The distance  $d$  between points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given

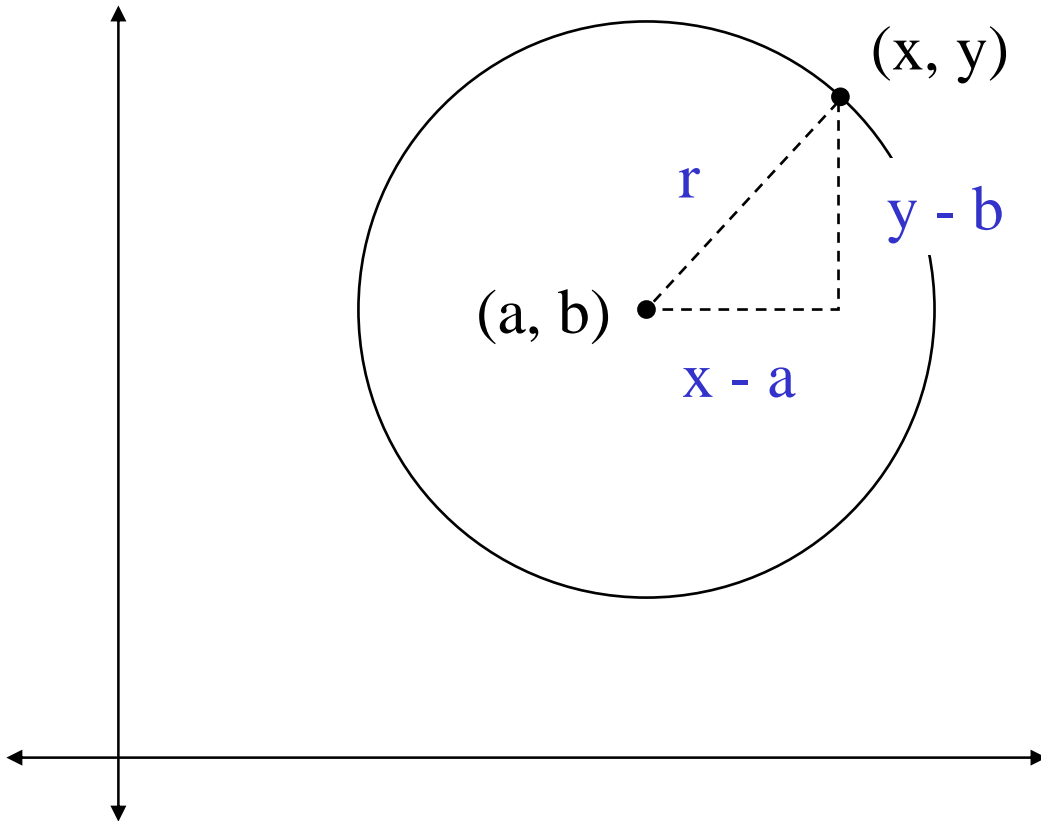
by:  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

★ Theorem 13-2 (Standard Form for a Circle)

An equation of a circle with center  $(a, b)$  and radius,  $r$ ,  
is  $(x - a)^2 + (y - b)^2 = r^2$ .



## Sample Problems

Find the distance between the points.

1.  $(-2, -3)$  &  $(-2, 4)$

3.  $(3, -4)$  &  $(-1, -4)$

5.  $(-6, -2)$  &  $(-7, -5)$

7.  $(-8, 6)$  &  $(0, 0)$

9.  $(5, 4)$  &  $(1, -2)$

11.  $(-2, 3)$  &  $(3, -2)$

Given the points A, B & C. Find AB, BC & AC. Are A, B & C collinear? If so, which point is in the middle?

13. A(0, 3) B(-2, 1) C(3, 6)

15. A(-5, 6) B(0, 2) C(3, 0)

## Sample Problems

Find the center and the radius of each circle.

17.  $(x + 3)^2 + y^2 = 49$

19.  $(x - j)^2 + (y + 14)^2 = 17$

Write an equation of the circle with the given center and radius.

21.  $C(3, 0) r = 8$

23.  $C(-4, -7) r = 5$

# Section 13-2

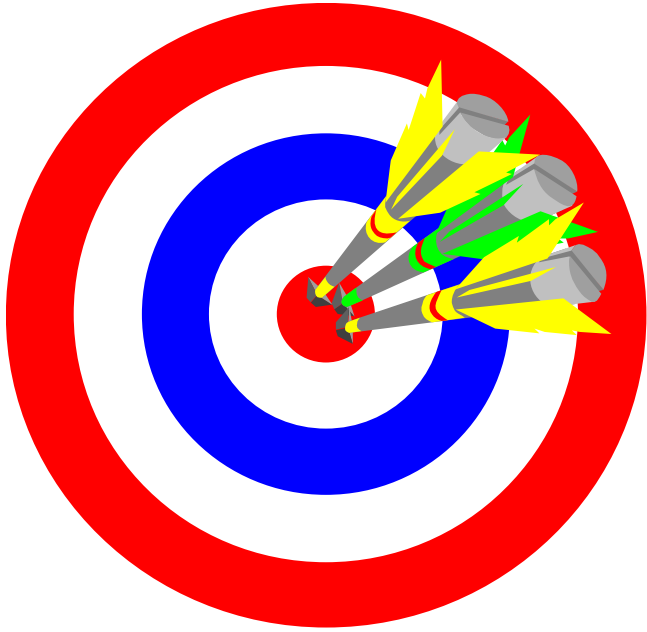
Slope of a Line

Homework Pages 532-533:

2-24 evens

Excluding 10

# Objectives



- A. Understand the terms ‘linear equation’ and ‘slope of a line’.
- B. Understand and identify lines with positive, negative, zero, and undefined slopes.
- C. Calculate the slopes of various lines.
- D. Use the slope of a line to graph linear equations.

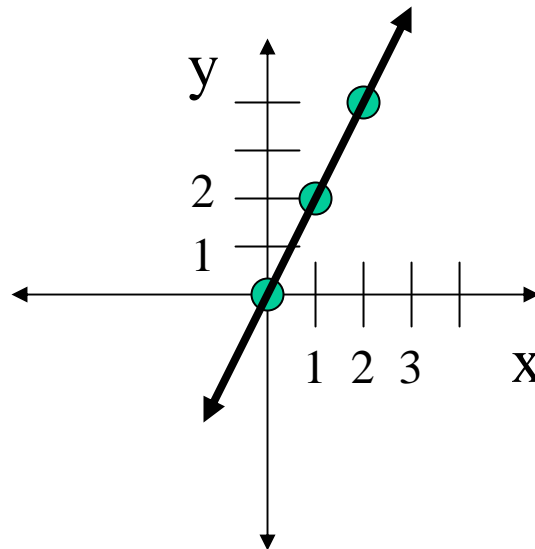
## ★ Linear Equation

- A linear equation is any equation where the graph of the solution set is a line.
- Example:  $y = 2x$

SOME solutions!

If $x =$	Then $y =$ ( $2x$ )
0	0
1	2
2	4

Plot the points.



Draw the line.

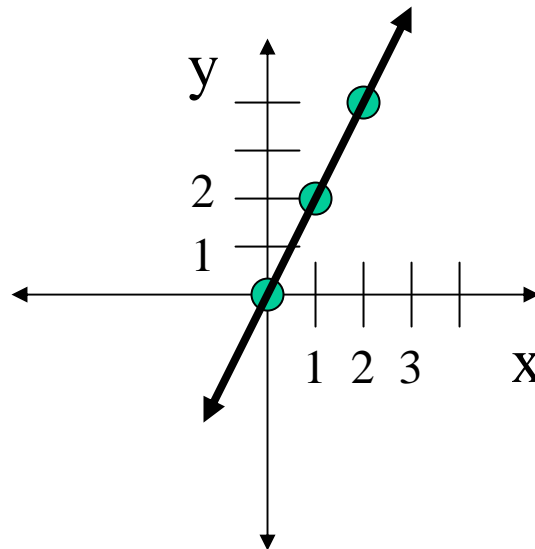
## ★ Linear Equation

- NOTE! All values of  $x$  and  $y$  that satisfy the equation  $y = 2x$  form a coordinate  $(x, y)$  that is ON the line.
- NOTE! All coordinates  $(x, y)$  on the line make the equation  $y = 2x$  true!
- Therefore, the GRAPH represents ALL of the solutions to the equation!

SOME solutions!

If $x =$	Then $y =$ ( $2x$ )
0	0
1	2
2	4

Plot the points.



## Slopes of Lines

$$\star \text{ slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

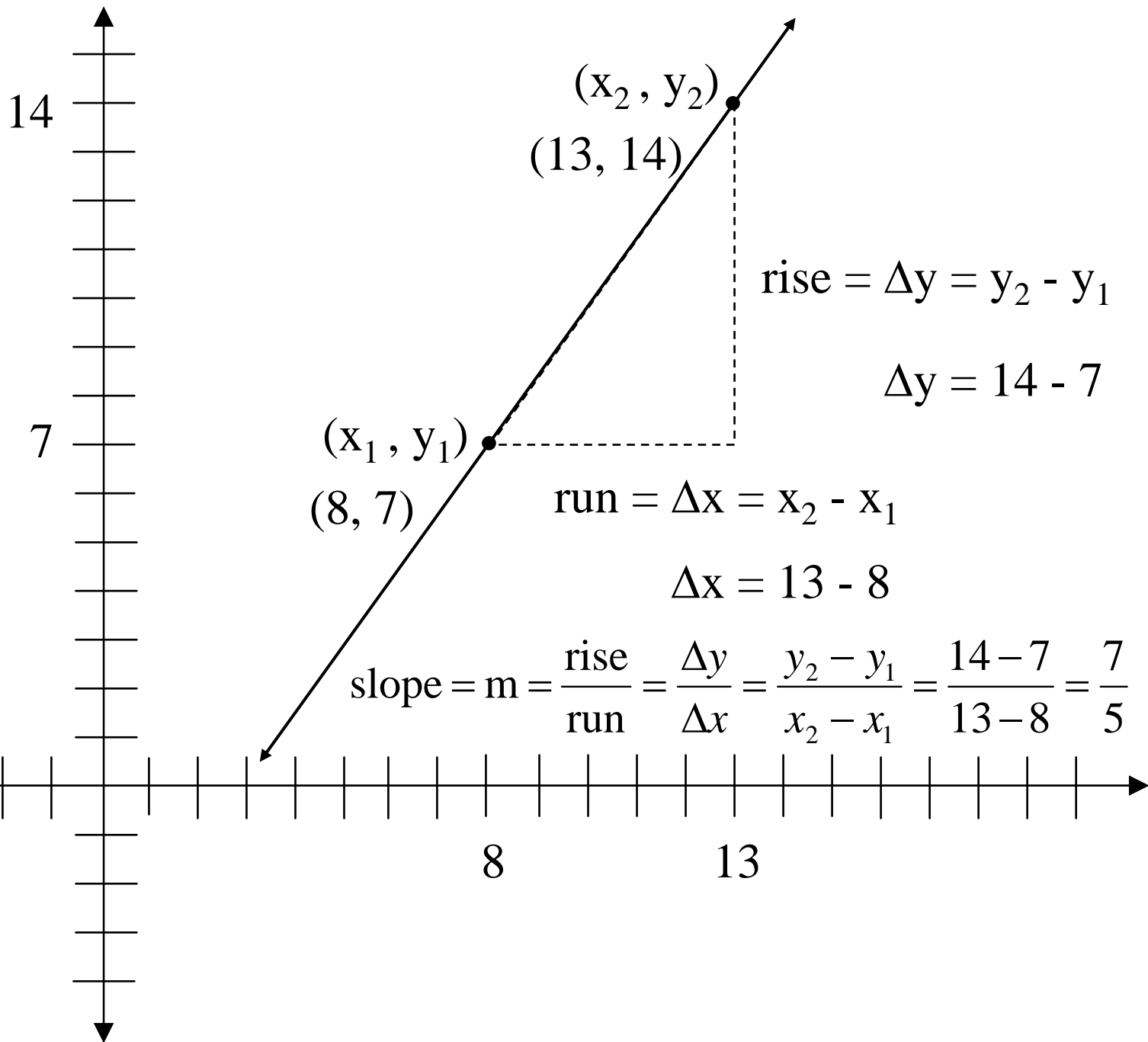
★ lines with positive slope: rise to the right

★ lines with negative slope: rise to the left

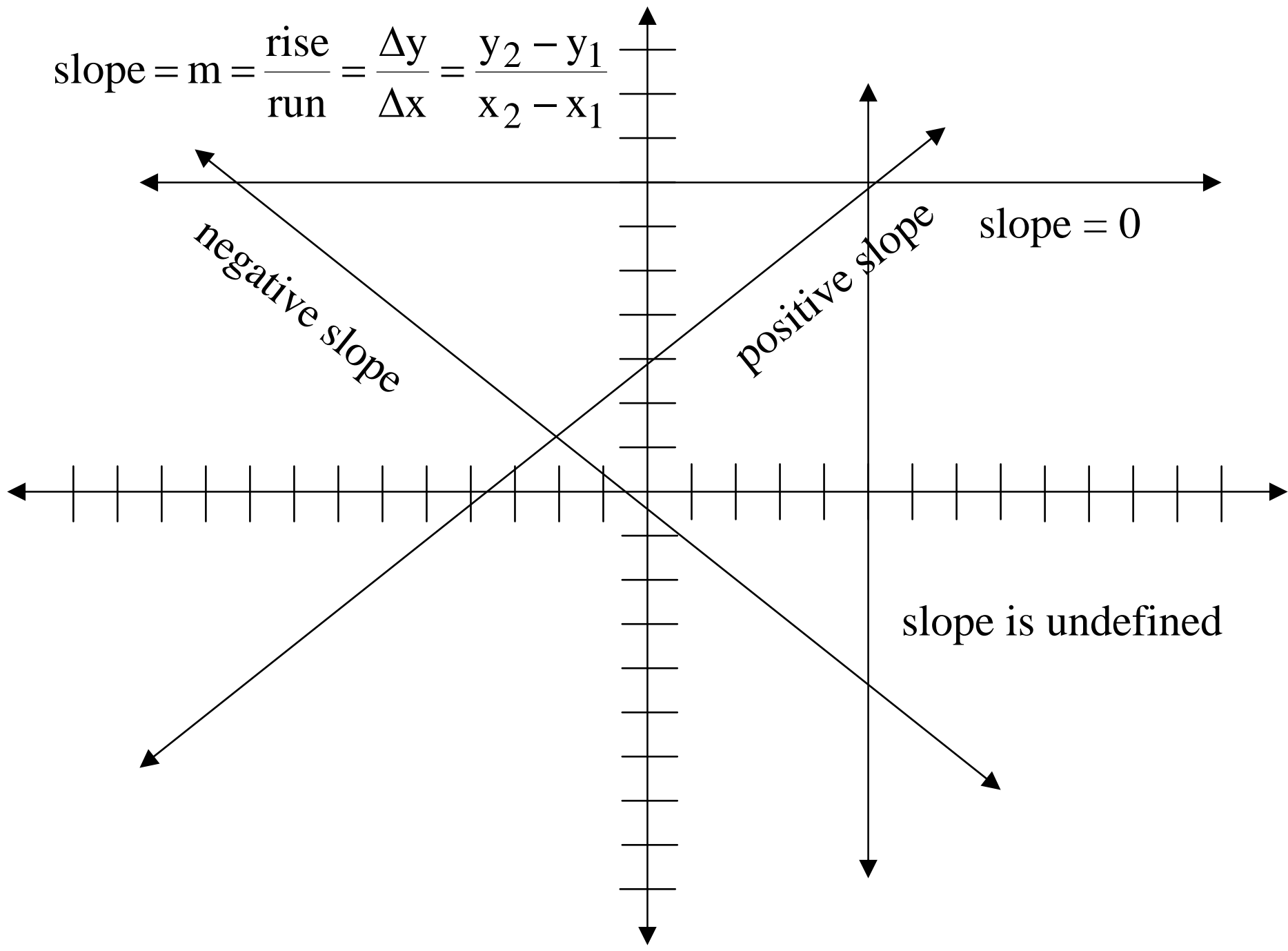
• steeper line: has a slope with a greater absolute value.

★ the slope of a horizontal line: is zero

★ the slope of a vertical line: is undefined



$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



## Sample Problems

Find the slope of the line through the given points.

7.  $(7, 2)$  &  $(2, 7)$

Label the points:  $P_1 = (7, 2)$   $P_2 = (2, 7)$

Label the coordinates of the points:

$$x_1 = 7, y_1 = 2, x_2 = 2, y_2 = 7$$

Write the formula!      slope =  $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Fill in the blanks  $\rightarrow$       slope =  $\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 2}{2 - 7} = \frac{5}{-5} = -\frac{1}{1} = -1$

## Sample Problems

Find the *SLOPE* and the length of AB

15. A(0, - 9) & B(8, - 3)

Label the points:  $P_1 = A = (0, -9)$   $P_2 = B = (8, -3)$

Label the coordinates of the points:

$$x_1 = 0, y_1 = -9, x_2 = 8, y_2 = -3$$

Write the formula!      slope =  $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Fill in the blanks  $\rightarrow$       slope =  $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - -9}{8 - 0} = \frac{6}{8} = \frac{3}{4}$

## Sample Problems

Find the slope and the ***LENGTH*** of AB

15. A(0, -9) & B(8, -3)

Label the points:  $P_1 = A = (0, -9)$   $P_2 = B = (8, -3)$

Label the coordinates of the points:

$$x_1 = 0, y_1 = -9, x_2 = 8, y_2 = -3$$

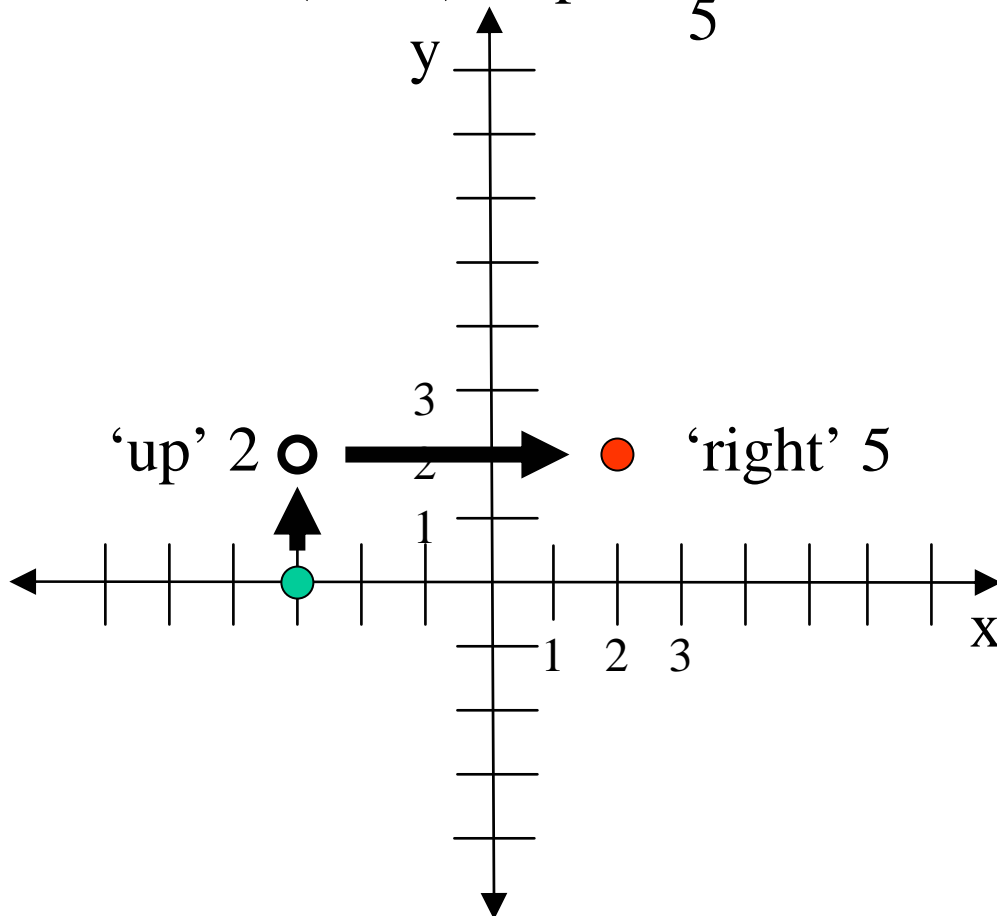
Write the formula!      distance =  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Fill in the blanks →      distance =  $d = \sqrt{(8 - 0)^2 + (-3 - -9)^2} =$   
 $\sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$

## Sample Problems

Find one point to the left and to the *right* of the given point on the same line.

17.  $P(-3, 0)$  slope =  $\frac{2}{5}$



Plot the point.

If numerator of slope is positive, go up y-units.  
(if numerator negative, go down y-units)

If denominator of slope is positive, go right x-units.  
(if denominator negative, go left x-units)

New point (2, 2)

## Sample Problems

Find one point to the *left* and to the right of the given point on the same line.

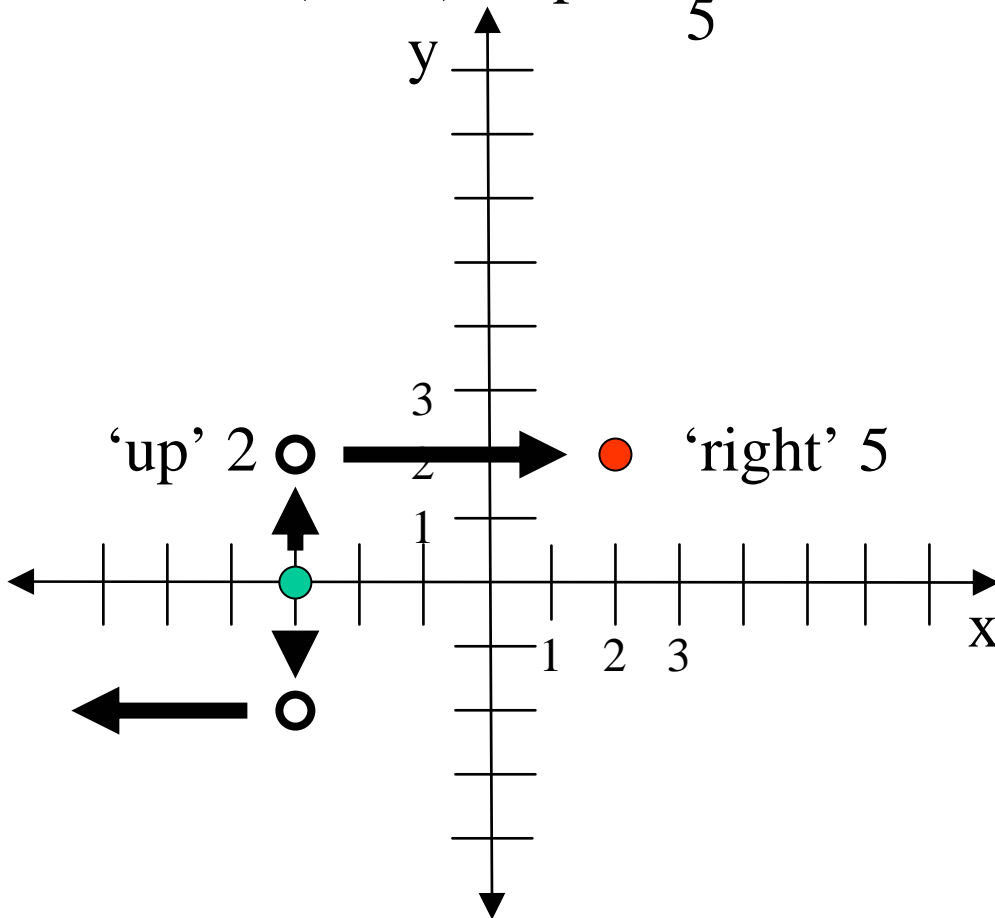
17.  $P(-3, 0)$  slope =  $\frac{2}{5}$

$$\frac{2}{5} = \frac{-2}{-5} ??$$

If numerator of slope is negative, go down y-units.

If denominator of slope is negative, go left x-units.

New point  $(-8, -2)$



## Sample Problems

Find the slope of the line through the given points.

3.  $(1, 2)$  &  $(3, 4)$

5.  $(1, 2)$  &  $(-2, 5)$

9.  $(6, -6)$  &  $(-6, -6)$

11.  $(-4, -3)$  &  $(-6, -6)$

Find the slope and the length of AB

13. A $(-3, -2)$  & B $(7, -6)$

## Sample Problems

Find one point to the left and to the right of the given point on the same line.

17.  $P(-3, 0)$  slope =  $\frac{2}{5}$

19.  $P(0, -5)$  slope =  $-\frac{1}{4}$

Show point P, Q and R are collinear by showing PQ and QR have the same slope.

21.  $P(-8, 6)$   $Q(-5, 5)$   $R(4, 2)$

Complete.

23. A line with slope  $\frac{3}{4}$  passes through points  $(2, 3)$  &  $(10, ?)$

# Section 13-3

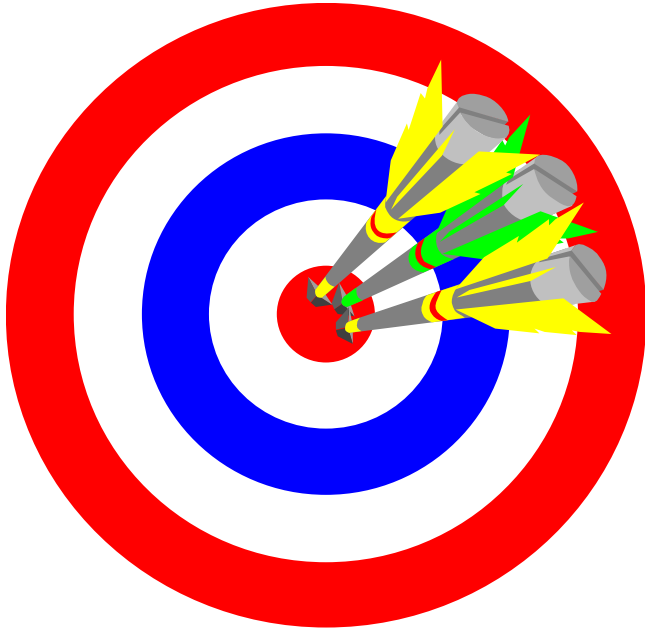
Parallel and Perpendicular Lines

Homework Pages 537-538:

2-20 evens

Excluding 14

# Objectives



- A. Understand the relationship between the geometric and algebraic definitions of parallel and perpendicular lines.
- B. Determine if 2 lines are parallel, perpendicular, or neither.
- C. Find the slopes of lines parallel or perpendicular to a given line.

## Geometric versus Algebraic ‘speak’

The number 1 lesson from this chapter is to show the tight linkage between geometry and algebra.

One key to understanding this ‘linkage’ is to understand the different ‘dialects’ we speak in the mathematical language.

Relate this to the many ‘dialects’ within the United States.

The vast majority of citizens of the United States speak an English ‘dialect’. In other words, the roots of our language are the same whether we are from the deep south or from the industrial northeast. However, the words we use and the way we pronounce them are different, thus creating the ‘dialect’. We mean the same thing, but say the things differently.

## Geometric versus Algebraic ‘speak’ - continued

•When we speak ‘geometrically’, we use terms such as:

- Points
- Lines
- Planes
- Angles
- Polygons
- Solids

•When we speak ‘algebraically’, we use terms such as:

- Numbers
- Variables
- Equations
- Sets
- Inequalities

Both the geometric and algebraic ‘dialects’ are a part of the language of mathematics.

## Parallel Lines

From Chapter 3, the *geometric* definition of parallel lines is:

Two coplanar lines that do not intersect.

★ Theorem 13-3 give the *algebraic* definition of parallel lines:

Two non-vertical lines are parallel if and only if their slopes are equal.

In other words, if  $m_1=m_2$ , then line 1 is parallel to line 2.

# Perpendicular Lines

From Chapter 3, the *geometric* definition of perpendicular lines is:

Two coplanar lines that intersect at right angles.

★ Theorem 13-4 give the *algebraic* definition of perpendicular lines:

Two non-vertical lines are perpendicular if and only if the product of their slopes is  $(- 1)$ .

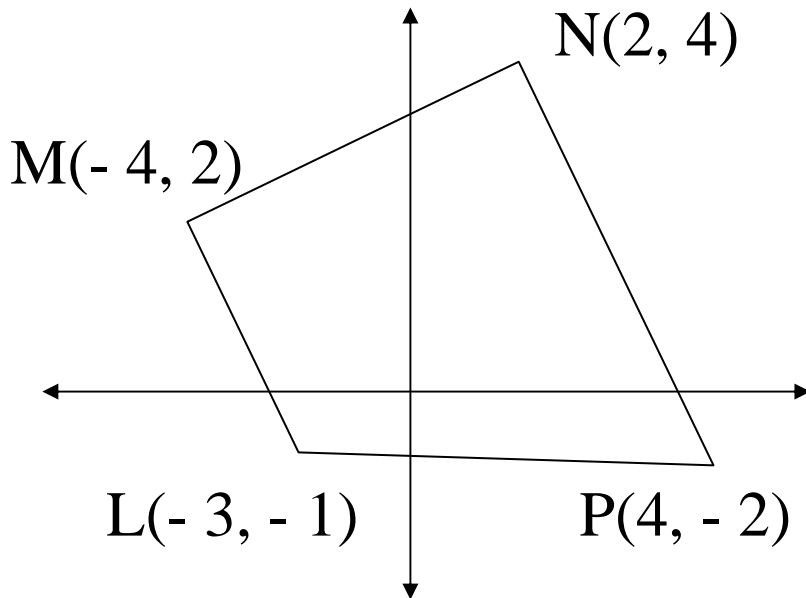
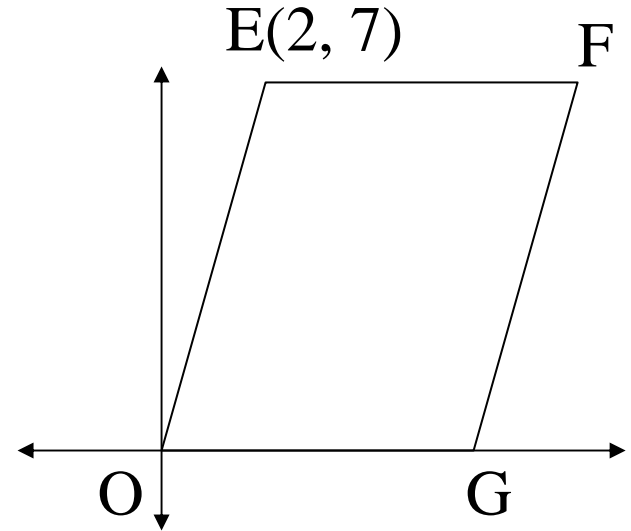
In other words, if  $(m_1 \times m_2 = -1)$ , then line 1 is perpendicular to line 2.

## Sample Problems

1. Find the slope of (a) AB (b) any line parallel to AB (c) any line perpendicular to AB.  $A(-2, 0)$  &  $B(4, 4)$

3. OEFG is a parallelogram.

What is the slope of each side?



5. What is the slope of LM & PN? Why are they parallel? What is the slope of MN & LP? Why are they not parallel? What kind of quadrilateral is LMNP?

## Sample Problems

7. Find the slope of each side and each altitude of  $\triangle ABC$ .  
A(0, 0) B(7, 3) C(2, - 5)
9. Identify the legs of right  $\triangle RST$ . R(- 3, - 4) S(2, 2) T(14, - 8)
11. Given parallelogram ABCD, A(- 6, - 4) B(4, 2) C(6, 8)  
D(- 4, 2). Show that opposite sides are parallel and opposite  
sides are congruent.
13. R(- 4, 5) S(- 1, 9) T(7, 3) U(4, - 1) Show that RSTU is a  
rectangle. Show that the diagonals are congruent.
- Decide what type of quadrilateral HIJK is, then tell why.
15. H(0, 0) I(5, 0) J(7, 9) K(1, 9)
17. H(7, 5) I(8, 3) J(0, - 1) K(- 1, 1)
19. Point N(3, - 4) is on the circle  $x^2 + y^2 = 25$ . Find the slope  
of the line tangent to the circle at N.

# Section 13-4

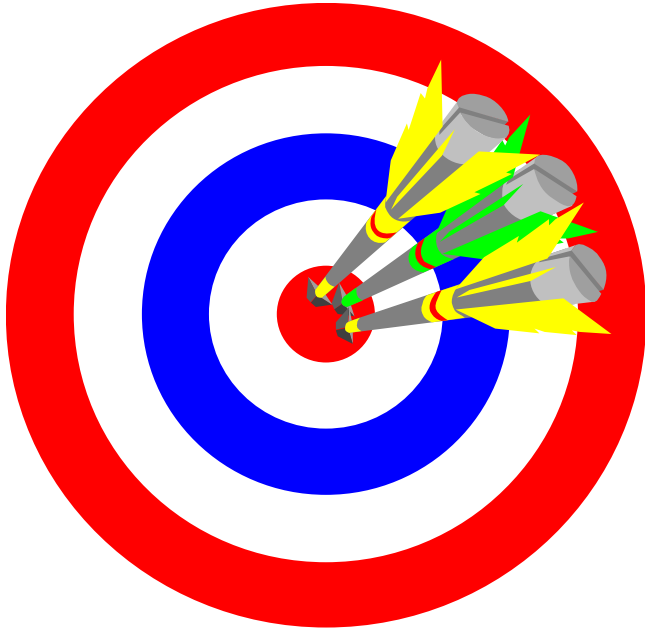
## Vectors

Homework Pages 541-542:

2-30 evens

Excluding 8, 16

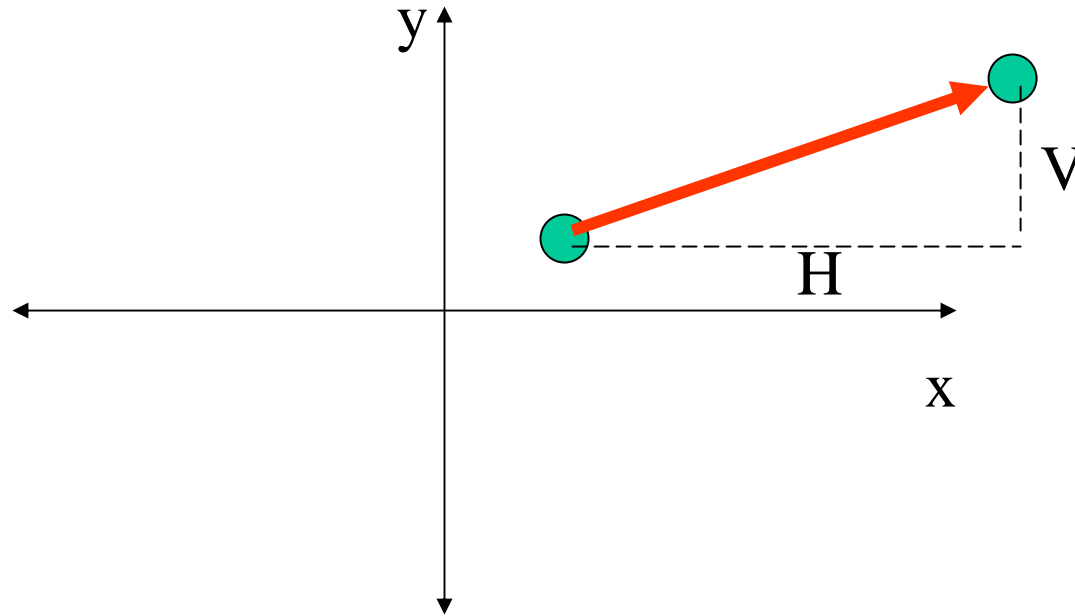
# Objectives



- A. Understand and utilize the terms ‘vector’, ‘magnitude’, and ‘direction’.
- B. Identify and calculate the horizontal and vertical components of a vector.
- C. Recognize and identify equal, parallel, and perpendicular vectors.
- D. Calculate the magnitude and slope of a vector.
- E. Perform vector addition, vector subtraction, and scalar multiplication.

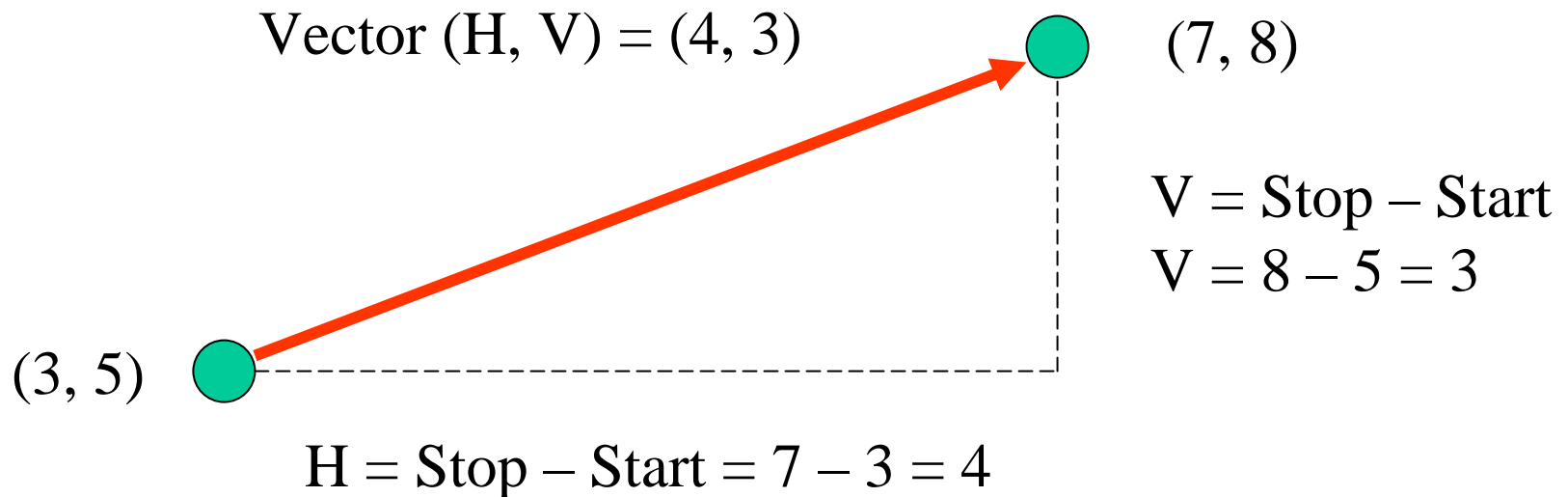
# Vectors

- ★ vector: a line segment with both magnitude and direction, written as an ordered pair of numbers  $(H, V)$  where  $H$  is the horizontal component and  $V$  is the vertical component.



# Vector Components

- ★ horizontal component: the distance traveled left or right from the starting to the ending point, found by taking  $X_{\text{stop}} - X_{\text{start}}$
- ★ vertical component: the distance traveled up or down from the starting point to the ending point, found by taking  $Y_{\text{stop}} - Y_{\text{start}}$



## Vector Terms

- Equal Vectors  $\rightarrow$  vectors with the same magnitude and direction
  - Note that equal vectors may or may not be collinear
- Magnitude of a Vector  $\rightarrow$  The length of the vector.
  - Found by using the distance formula

- dot product:  $(H_1, V_1) \bullet (H_2, V_2) = H_1H_2 + V_1V_2$
- ★ vector addition:  $(H_1, V_1) + (H_2, V_2) = (H_1 + H_2, V_1 + V_2)$
- ★ vector subtraction:  $(H_1, V_1) - (H_2, V_2) = (H_1 - H_2, V_1 - V_2)$
- ★ scalar multiplication:  $k(H_1, V_1) = (kH_1, kV_1)$
- ★ magnitude:  $|AB| = \sqrt{H^2 + V^2}$
- ★ slope:  $\frac{V}{H}$

## Sample Problems

Find  $\vec{AB}$  and  $|\vec{AB}|$

3.  $A(6, 1)$   $B(4, 3)$

‘Order’ of vector dictates that Point A is the start point and Point B is the stop point.

$$H = x_{\text{stop}} - x_{\text{start}} = 4 - 6 = -2$$

$$V = y_{\text{stop}} - y_{\text{start}} = 3 - 1 = 2$$

$$\vec{AB} = (H, V) = (-2, 2)$$

$$|\vec{AB}| = \textit{magnitude} = \sqrt{(4-6)^2 + (3-1)^2} = \sqrt{4+4} = 2\sqrt{2}$$

## Sample Problems

Perform the scalar multiplication.

11.  $3(4, -1)$

The scalar  $k$  is 3. The horizontal component  $H$  is 4.  
The vertical component  $V$  is -1.

$$k(H, V) = (kH, kV) = 3(4, -1) = (3 \times 4, 3 \times -1) = (12, -3)$$

## Sample Problems

Find the vector sum.

$$21. (3, -5) + (4, 5)$$

$$u_1 = (H_1, V_1) = (3, -5)$$

$$u_2 = (H_2, V_2) = (4, 5)$$

$$\begin{aligned} u_3 = u_1 + u_2 &= (H_1, V_1) + (H_2, V_2) = \\ &(H_1 + H_2, V_1 + V_2) = (3 + 4, -5 + 5) = (7, 0) \end{aligned}$$

$$u_3 = u_1 - u_2 ?$$

$$\begin{aligned} u_3 = u_1 - u_2 &= (H_1, V_1) - (H_2, V_2) = \\ &(H_1 - H_2, V_1 - V_2) = (3 - 4, -5 - 5) = (-1, -10) \end{aligned}$$

## Sample Problems

Find  $\overrightarrow{AB}$  and  $|\overrightarrow{AB}|$

1.  $A(1, 1) B(5, 4)$

3.  $A(6, 1) B(4, 3)$

5.  $A(3, 5) B(-1, 7)$

7.  $A(0, 0) B(5, -9)$

9.  $A(-1, -1) B(-4, -7)$

Perform the scalar multiplication.

11.  $3(4, -1)$

13.  $\frac{1}{3}(6, 9)$

15.  $-\frac{1}{2}(6, -4)$

## Sample Problems

17. The vectors  $(8, 6)$  and  $(12, k)$  are parallel. Find  $k$ .

19. The vectors  $(8, k)$  and  $(9, 6)$  are perpendicular. Find  $k$ .

Find the vector sum.

21.  $(3, -5) + (4, 5)$

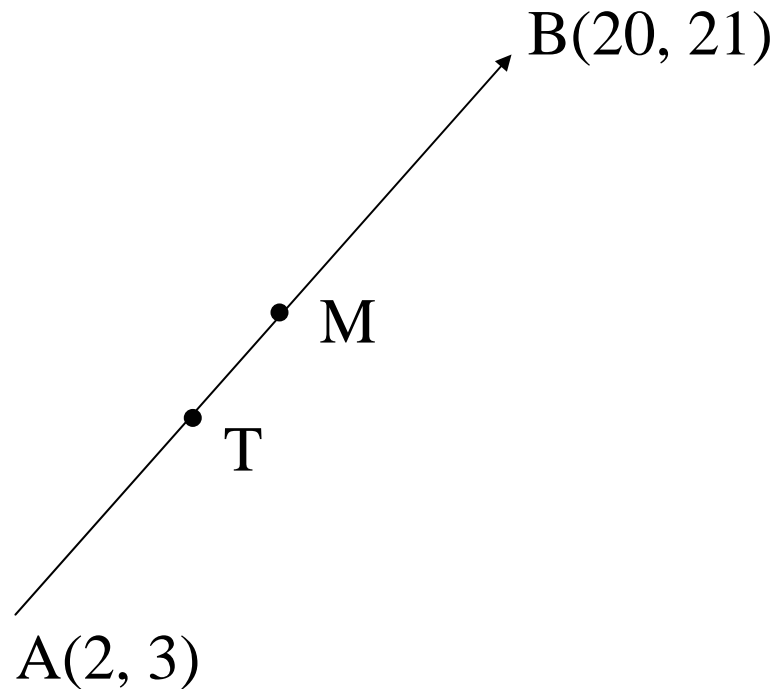
23.  $(-3, -3) + (7, 7)$

25.  $(7, 2) + 3(-1, 0)$

27. An object,  $K$ , is being pulled by two forces  $\overrightarrow{KX} = (-1, 5)$  and  $\overrightarrow{KY} = (7, 3)$ . What single force has the same effect as the two forces acting together? What is the magnitude of this force?

## Sample Problems

29. M is the midpoint of AB. T is the trisector of AB.  
A(2, 3) B(20, 21). Find  $\overrightarrow{AB}$ ,  $\overrightarrow{AM}$  and  $\overrightarrow{AT}$ . Find the  
coordinates of M & T.

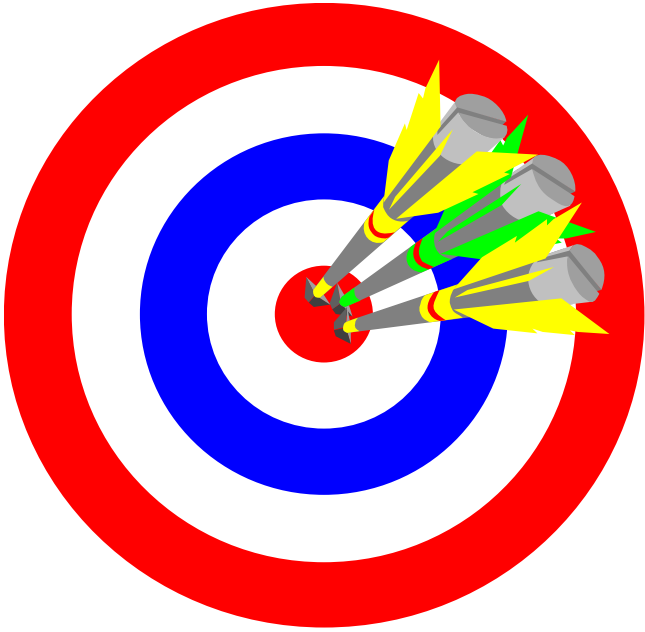


# Section 13-5

The Midpoint Formula  
Homework Pages 545-546:  
2-20 evens  
Excluding 6, 18

# Objectives

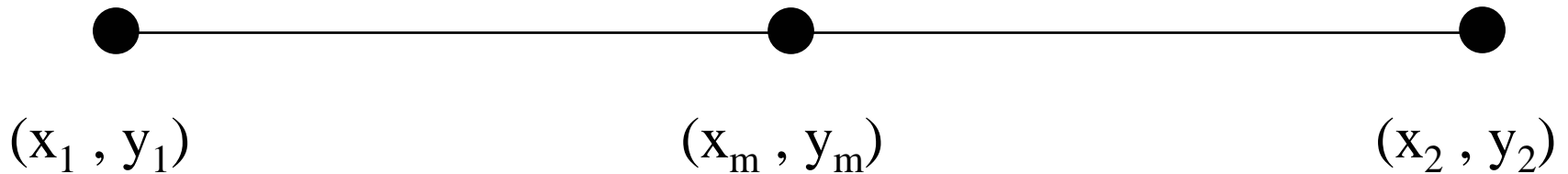
- A. Understand and utilize the midpoint formula.



★ Theorem 13-5 (Midpoint Formula)

The midpoint  $(x_m, y_m)$  of a segment that joins the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is the point

$$(x_m, y_m) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



## Sample Problems

Find the coordinates of the midpoint.

3.  $(6, -7)$  &  $(-6, 3)$

$$\text{midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$P_1 = (6, -7) \quad P_2 = (-6, 3)$$

$$x_1 = 6, y_1 = -7 \quad x_2 = -6, y_2 = 3$$

$$\text{midpoint} = \left( \frac{6 + (-6)}{2}, \frac{-7 + 3}{2} \right) = (0, -2)$$

## Sample Problems

Find the length, slope and midpoint of PQ.

7.  $P(3, -8)$   $Q(-5, 2)$

Label the points:  $P_1 = P = (3, -8)$ ,  $P_2 = Q = (-5, 2)$

Label the coordinates of the points:

$$x_1 = 3, y_1 = -8, x_2 = -5, y_2 = 2$$

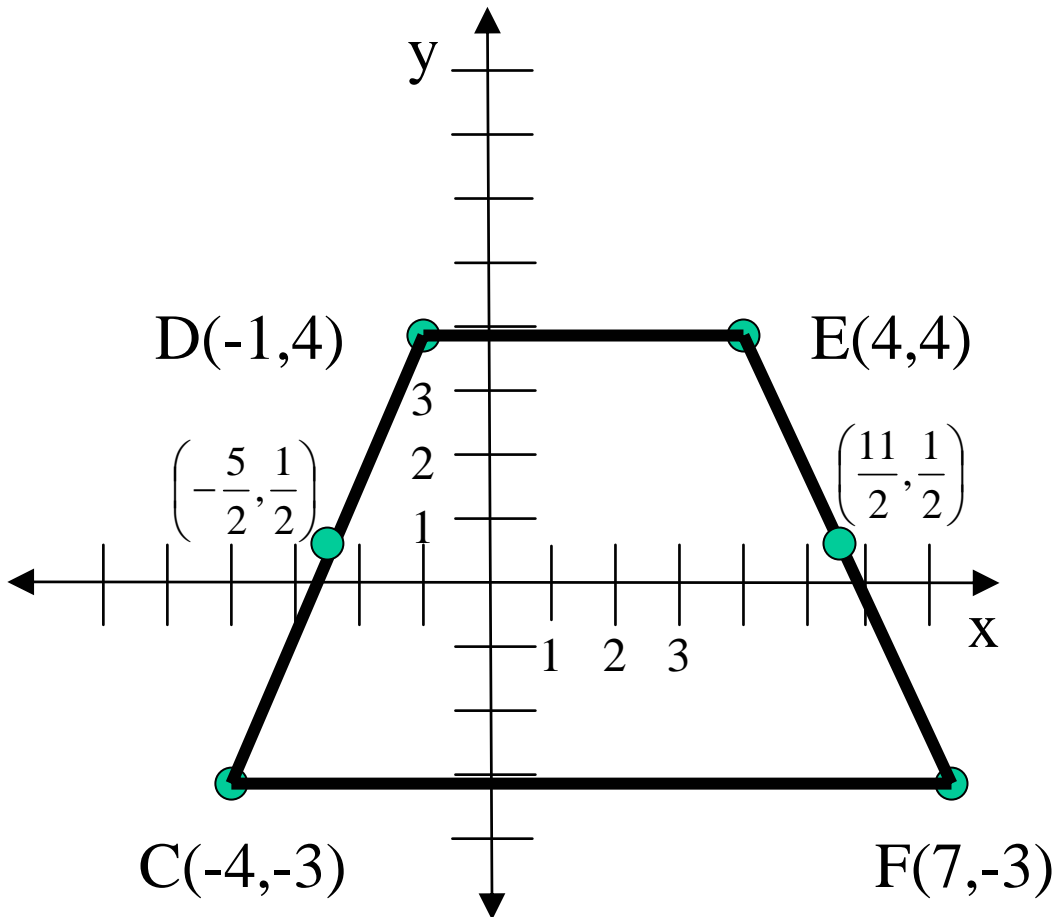
$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5 - 3)^2 + (2 - -8)^2} = \sqrt{64 + 100} = 2\sqrt{41}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - -8}{-5 - 3} = \frac{10}{-8} = -\frac{5}{4}$$

$$\text{midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + -5}{2}, \frac{-8 + 2}{2} \right) = (-1, -3)$$

## Sample Problems

15. Find the midpoints of the legs and then the length of the median of the trapezoid CDEF with vertices C(-4, -3), D(-1, 4), E(4, 4) & F(7, -3) .

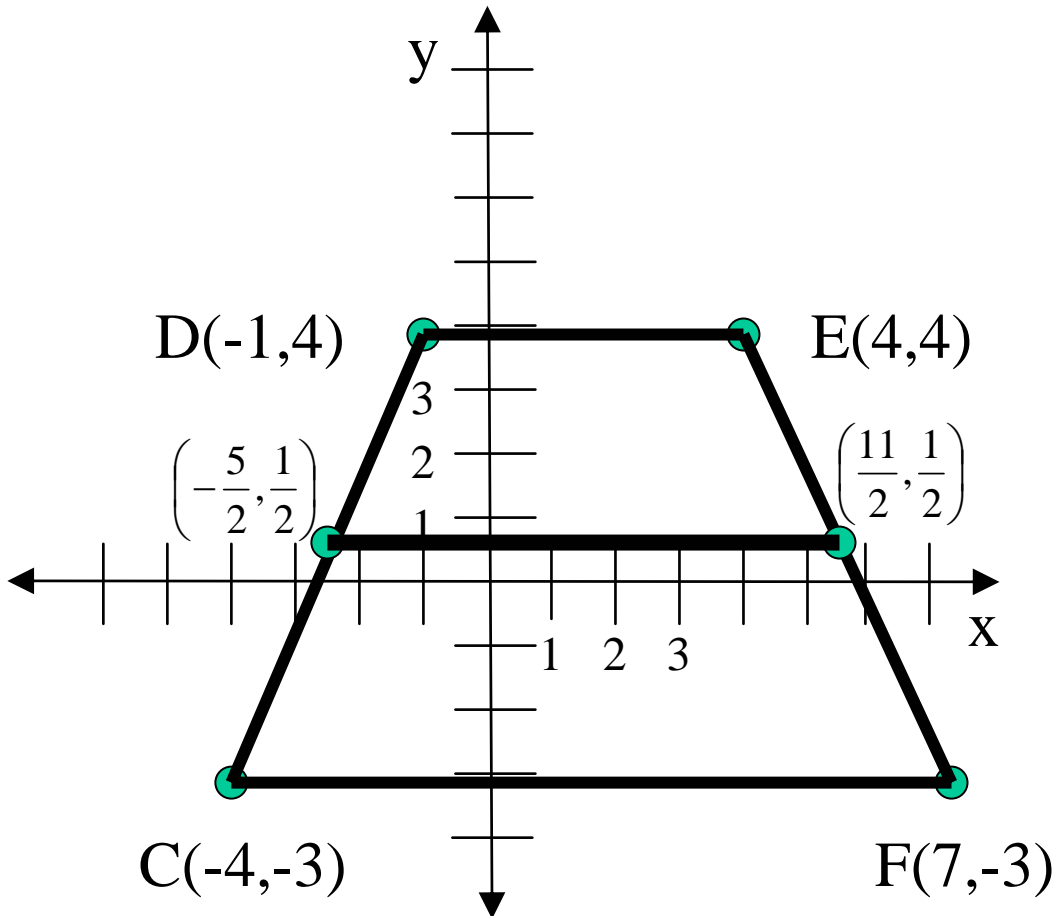


$$\begin{aligned} mid_{CD} &= \left( \frac{x_C + x_D}{2}, \frac{y_C + y_D}{2} \right) \\ &= \left( \frac{-4 + -1}{2}, \frac{-3 + 4}{2} \right) = \left( -\frac{5}{2}, \frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} mid_{EF} &= \left( \frac{x_E + x_F}{2}, \frac{y_E + y_F}{2} \right) \\ &= \left( \frac{4 + 7}{2}, \frac{4 + -3}{2} \right) = \left( \frac{11}{2}, \frac{1}{2} \right) \end{aligned}$$

## Sample Problems

15. Find the midpoints of the legs and then the length of the median of the trapezoid CDEF with vertices C(-4, -3), D(-1, 4), E(4, 4) & F(7, -3) .



$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{\left(\frac{11}{2} - -\frac{5}{2}\right)^2 + \left(\frac{1}{2} - \frac{1}{2}\right)^2} \\&= \sqrt{\left(\frac{16}{2}\right)^2 + (0)^2} = 8\end{aligned}$$

## Sample Problems

Find the coordinates of the midpoint.

1.  $(0, 2)$  &  $(6, 4)$
5.  $(2.3, 3.7)$  &  $(1.5, - 2.9)$

Find the length, slope and midpoint of PQ.

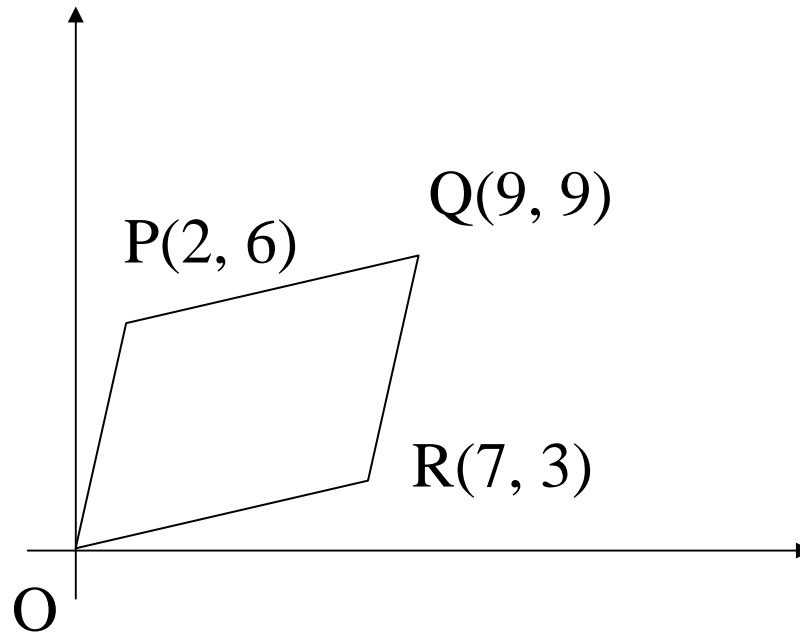
9.  $P(- 7, 11)$   $Q(1, - 4)$

M is the midpoint of AB. Find M.

11.  $A(1, - 3)$   $M(5, 1)$
13.  $A(0, 0)$  &  $B(8, 4)$ , show  $P(2, 6)$  is on the perpendicular bisector of AB.

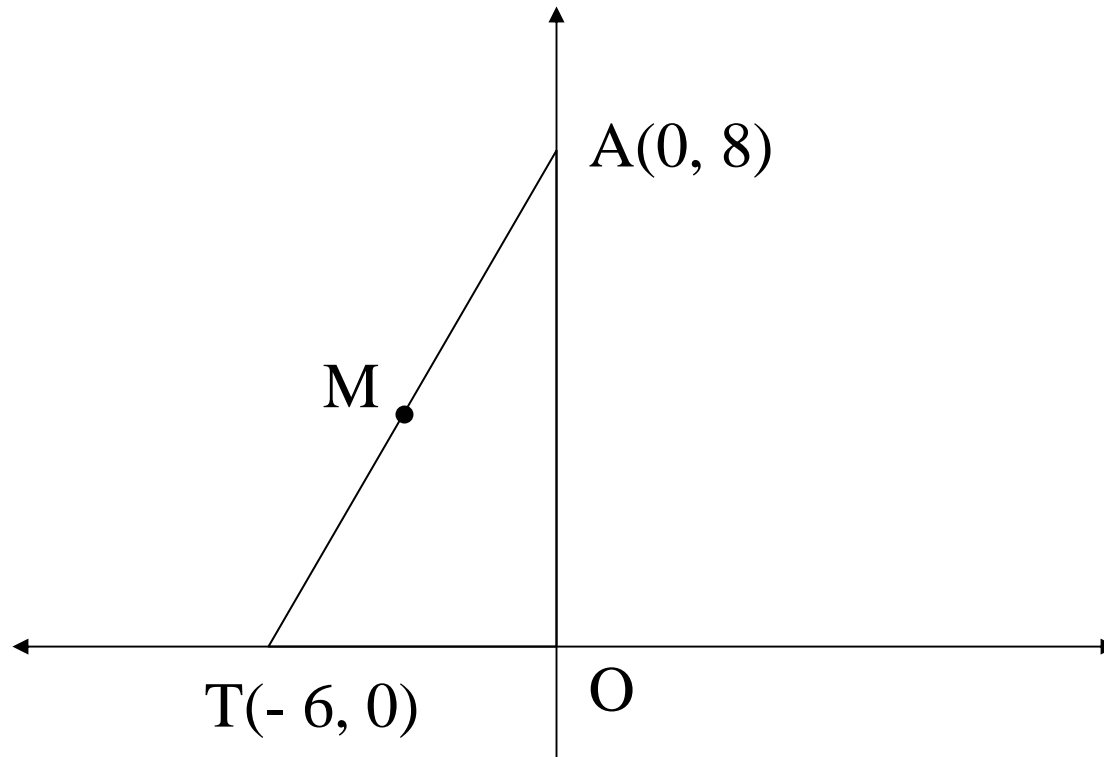
## Sample Problems

17. Show that  $OQ$  &  $PR$  have the same midpoint. What kind of quadrilateral is  $OPQR$ ? Show that the opposite sides are parallel. Show the opposite sides are congruent.



## Sample Problems

19. In right  $\triangle OAT$   $M$  is the midpoint of  $AT$ . What are the coordinates of  $M$ ? Find  $MA$ ,  $MT$  &  $MO$ . Find the equation of the circle that circumscribes the triangle.



# Section 13-6

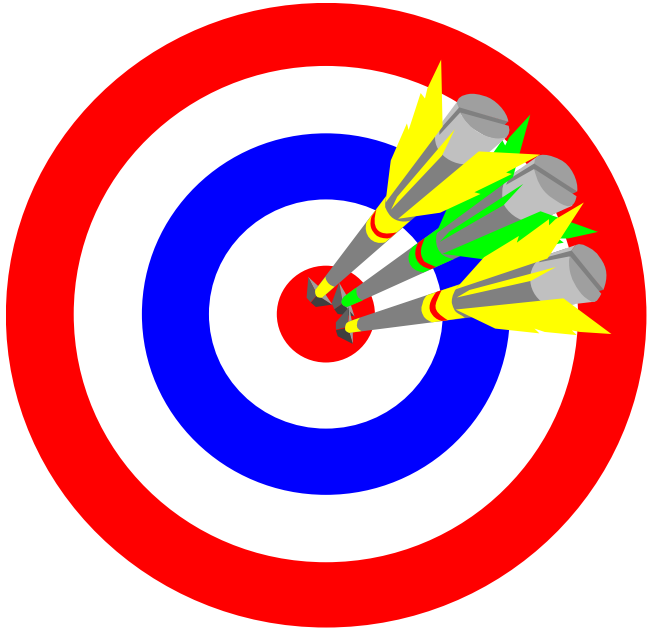
Graphing Linear Equations

Homework Pages 550-551:

2-34 evens

Excluding 6, 12, 18, 24, 30, 32

# Objectives



- A. Identify linear equations.
- B. Understand and utilize the standard form of a linear equation.
- C. Understand and utilize the slope-intercept form of a linear equation.
- D. Understand and apply multiple methods for solving systems of linear equations.

Remember!

A linear equation is any equation whose solution graph is a line.

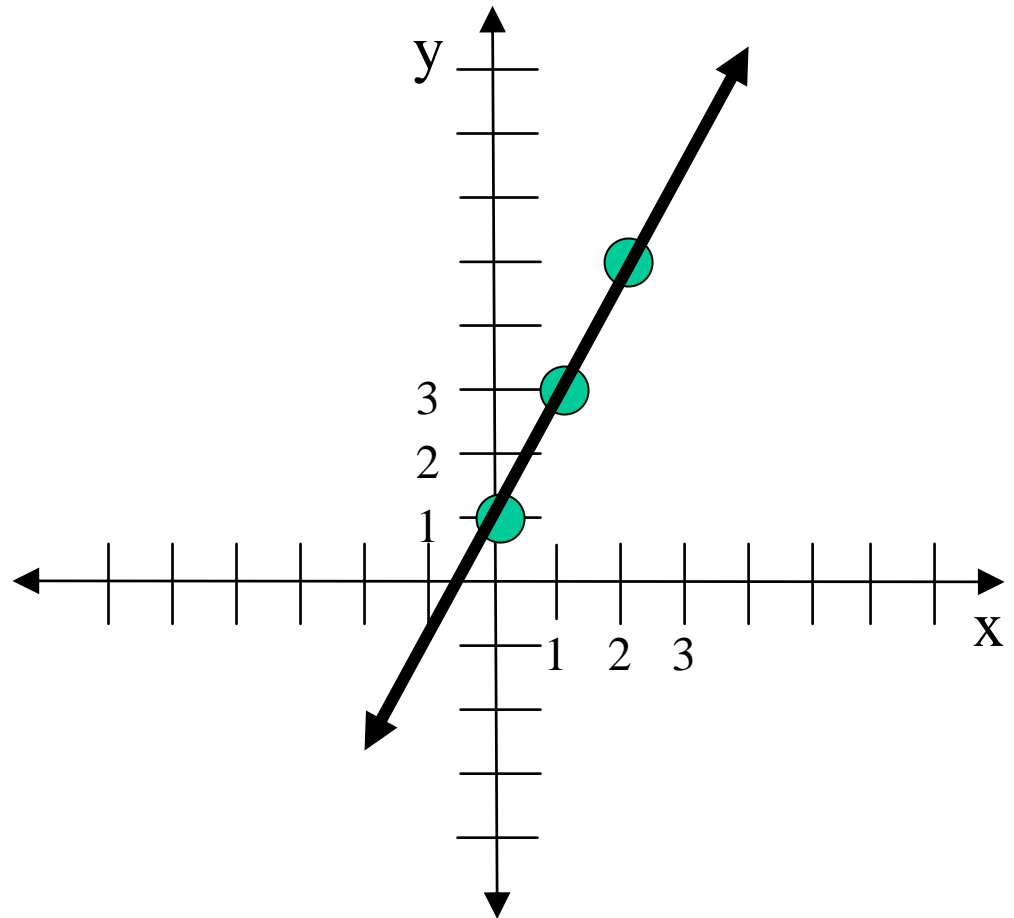
## ★ Methods for Graphing a Line:

### 1. Plot two or more points

- Remember → any two points are contained on one unique line.
- For graphing purposes, I recommend you plot at least 3 points

$$y = 2x + 1$$

If $x = ?$	Then $y =$
0	1
1	3
2	5



★ Theorem 13-6 (Standard Form of a Line)

The graph of any equation that can be written in form  $Ax + By = C$  where  $A$  and  $B$  are not both zero and  $A$ ,  $B$  and  $C$  are integers, is a line.

$$\text{slope} = -\frac{A}{B}$$

x - intercept is the point  $\left(\frac{C}{A}, 0\right)$

y - intercept is the point  $\left(0, \frac{C}{B}\right)$

★ Theorem 13-7 (Slope Intercept Form of a Line)

A line with the equation  $y = mx + b$  has slope  $m$  and  $y$ -intercept  $b$ .

## ★ Methods for Graphing a Line:

1. Plot two points
2. Plot one point (Y-intercept) and rise and run according to the slope.

$$y = 2x + 1$$

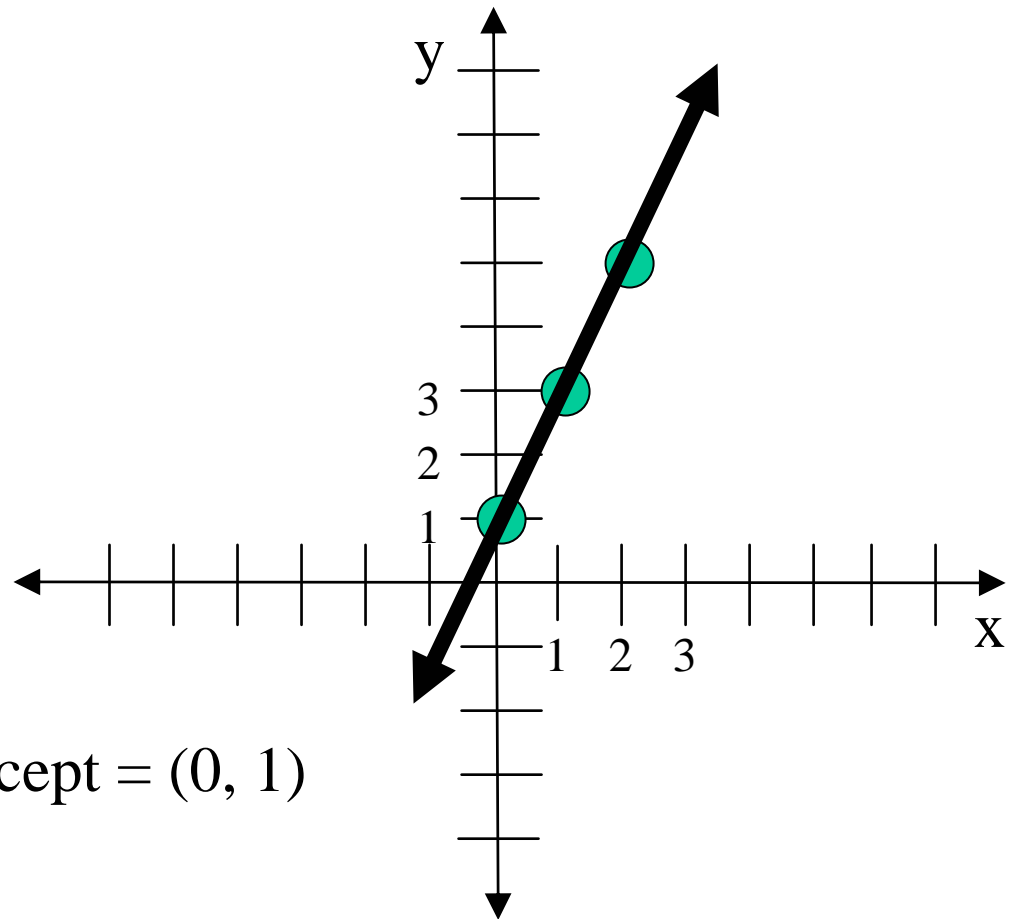
Slope-intercept  
form of a linear  
equation is  $\rightarrow$

$$Y = mX + B$$

$$m = 2 \text{ or } m = \frac{2}{1}$$

$$B = 1$$

Y-intercept = (0, 1)



## ★ Solving Systems of Linear Equations - Graphing

- Graph the two linear equations and identify the intersection point.
  - The point of intersection of the two lines gives the  $x$  and  $y$  coordinates that will make **BOTH** linear equations true.
  - Can be used on any system but your answer is only as accurate as your graph.

## ★ Solving Systems of Linear Equations - Graphing

$$y = x + 1$$

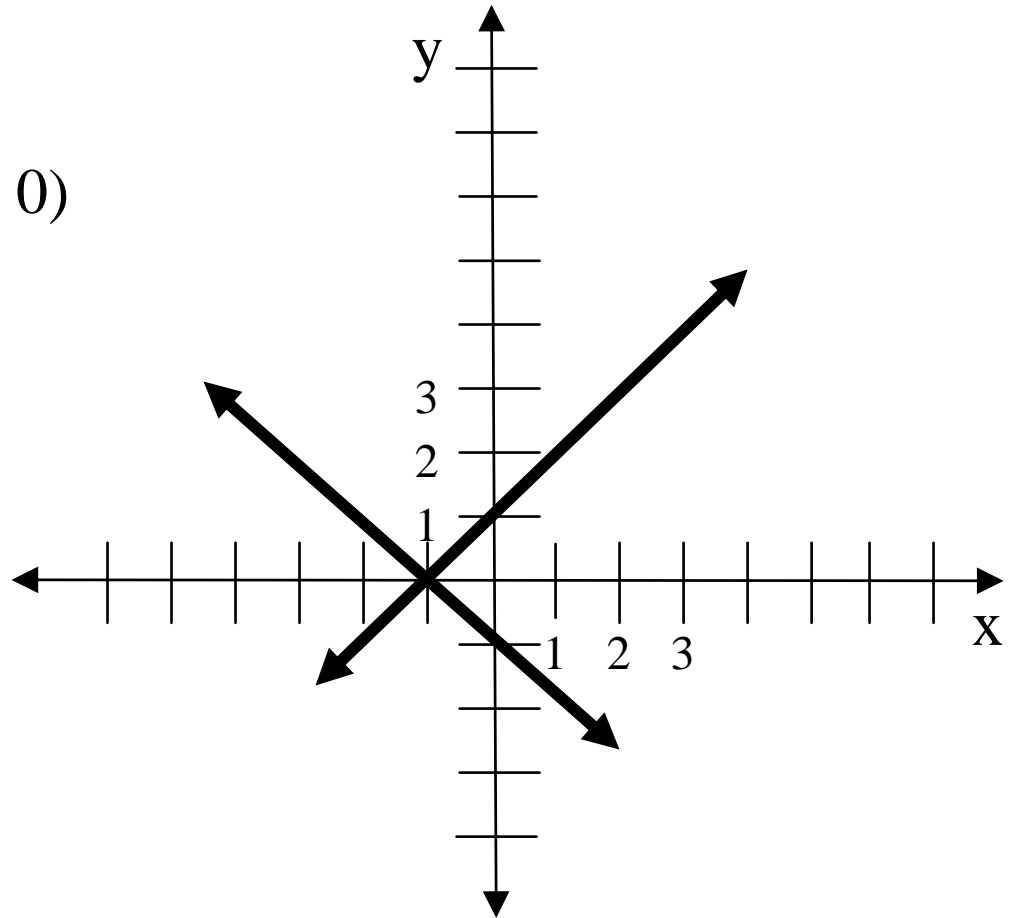
$$y = -x - 1$$

Point of intersection  $(-1, 0)$

CHECK your answer!

$$y = x + 1 = (-1) + 1 = 0$$

$$y = -x - 1 = -(-1) - 1 = 0$$



## ★ Solving Systems of Linear Equations – Isolate and Substitute

- Solve for (isolate)  $x$  or  $y$  in one equation.
- Substitute the expression from the step above into the second equation.
- Solve the second equation for the remaining variable.
- Substitute the answer from the step above into either original equation and solve for the other variable.
- Can be used on any system but works best when one coefficient is either 1 or - 1.

★ Solving Systems of Linear Equations –  
Isolate and Substitute

$$2x + y = 12$$

$$3x - 2y = -17$$

Solution (1, 10)

Isolate (solve for)  $y$  in the first equation  $\rightarrow y = 12 - 2x$

Using the result of the isolation, substitute for  $y$  in the second equation  $\rightarrow 3x - 2(12 - 2x) = -17$ .

$$\text{Solve for } x \rightarrow 3x - 24 + 4x = -17$$

$$7x = 7 \rightarrow x = 1$$

Substitute  $x$  value back into original equation  $\rightarrow$

$$2(1) + y = 12 \rightarrow y = 10$$

## ★ Solving Systems of Linear Equations – Linear Combination

- Addition: add the two equations to eliminate one of the variables. Works only if the coefficients of one of the variables are opposites.
- Subtraction: subtract the two equations to eliminate one of the variables. Works only if the coefficients of one of the variables are the same.
- Multiply one or both equations by a constant in order to create coefficients of the same variable that are either the same or opposites, then add/subtract.

## ★ Solving Systems of Linear Equations – Linear Combination

$$5x + 2y = 16$$

$$3x - 4y = -6$$

What to do?

Graphing would be complicated.

Isolating  $x$  or  $y$  would leave nasty fractions.

Adding or subtracting equations will not eliminate variable.

Multiply both sides of first equation by 2  $\rightarrow 10x + 4y = 32$

Now add the two equations:

$$10x + 4y = 32$$

$$3x - 4y = -6$$

-----

$$13x + 0y = 26$$

$$x = 2$$

$$5(2) + 2y = 16$$

$$2y = 6$$

$$y = 3$$

Solution (2, 3)

## Sample Problems

1. Graph  $y = mx$  if  $m = 2, -2, \frac{1}{2}, -\frac{1}{2}$

3. Graph  $y = \frac{1}{2}x + b$  for  $b = 0, 2, -2, -4$

5. Graph  $y = 0, y = 3, y = -3$

Find the  $x$  &  $y$  intercepts and the slope, then graph.

7.  $3x + y = -21$

9.  $3x + 2y = 12$

11.  $5x + 8y = 20$

## Sample Problems

Find the slope, x & y intercepts, then graph.

13.  $y = 2x - 3$

15.  $y = -4x$

17.  $y = -\frac{2}{3}x - 4$

19.  $4x + y = 10$

21.  $5x - 2y = 10$

23.  $x - 4y = 6$

## Sample Problems

Solve the system

25.  $x + y = 3$

$$x - y = -1$$

27.  $x + 2y = 10$

$$3x - 2y = 6$$

29.  $4x + 5y = -7$

$$2x - 3y = 13$$

# Section 13-7

Writing Linear Equations

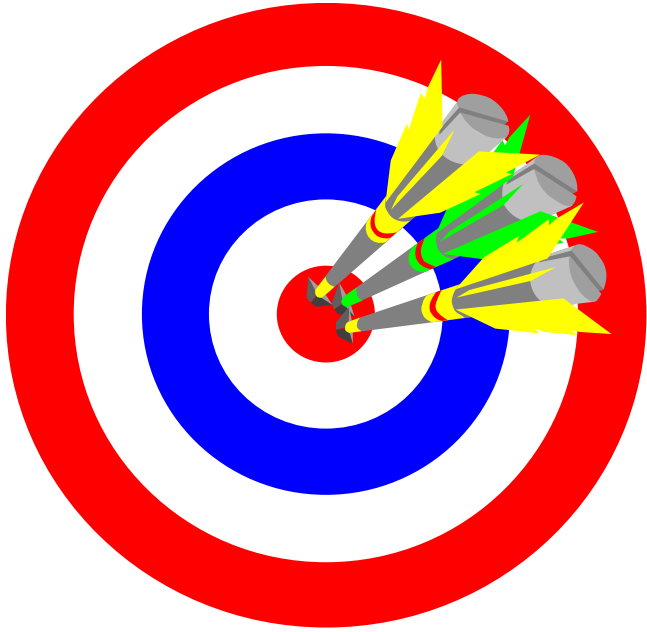
Homework Page 555:

2-28 evens

Excluding 6, 16

Final Answers must be in **STANDARD  
FORM** for a linear equation ( $Ax + By = C$ )

# Objectives



- A. Understand and utilize the point-slope form of a linear equation.
- B. Use various pieces of information about a line or linear equation to determine the standard, slope-intercept, and/or point-slope form of the linear equation.

★ Theorem 13-8 (Point Slope Form of a Line)

An equation of the line that passes through the point  $(x_1, y_1)$  and has slope  $m$  is  $y - y_1 = m(x - x_1)$ .

## ★ Writing an Equation of the Line

- Find the slope and one point on the line.
- Use the point-slope form of a line; put the slope in for  $m$  and the point in for  $(x_1, y_1)$ .
- Distribute and rearrange the equation until it is in standard form, with all coefficients being integers.

## Sample Problems

Write the equation of the line in standard form.

5. slope =  $-\frac{7}{5}$  y-intercept (0, 8)

Write the point-slope form of a linear equation:  $y - y_1 = m(x - x_1)$

Fill in the give point and slope:  $y - 8 = -\frac{7}{5}(x - 0)$

$$y - 8 = -\frac{7}{5}(x - 0)$$

Put in standard form ( $Ax + By = C$ ):

$$y - 8 = -\frac{7}{5}x$$

$$\frac{7}{5}x + 1y = 8$$

## Sample Problems

25. line through (5, 7) and parallel to the line  $y = 3x - 4$

The slope of the line  $y = 3x - 4$  is '3', so  $m = 3$ .

You are given the point (5, 7).

Use the point-slope formula:  $y - y_1 = m(x - x_1)$

$$y - 7 = 3(x - 5)$$

$$y - 7 = 3x - 15$$

$$3x - y = 8$$

## Sample Problems

Write the equation of the line in standard form.

1. slope = 2 y-intercept = (0, 5)

3. slope =  $\frac{1}{2}$  y-intercept (0, - 8)

5. slope =  $-\frac{7}{5}$  y-intercept (0, 8)

7. x-intercept (8, 0) y-intercept (0, 2)

9. x-intercept (- 8, 0) y-intercept (0, 4)

11. point (1, 2) slope = 5

13. point (- 3, 5) slope =  $\frac{1}{3}$

## Sample Problems

15. point  $(-4, 0)$  slope =  $-\frac{1}{2}$
17. line through  $(1, 1)$  &  $(4, 7)$
19. line through  $(-3, 1)$  &  $(3, 3)$
21. vertical line through  $(2, -5)$
23. line through  $(5, -3)$  and parallel to the line  $x = 4$
27. line through  $(-3, -2)$  and perpendicular to the line  $8x - 5y = 0$
29. perpendicular bisector of the segment joining  $(0, 0)$  &  $(10, 6)$
31. the line through  $(5, 5)$  that makes a  $45^\circ$  angle measured counterclockwise from the positive x axis

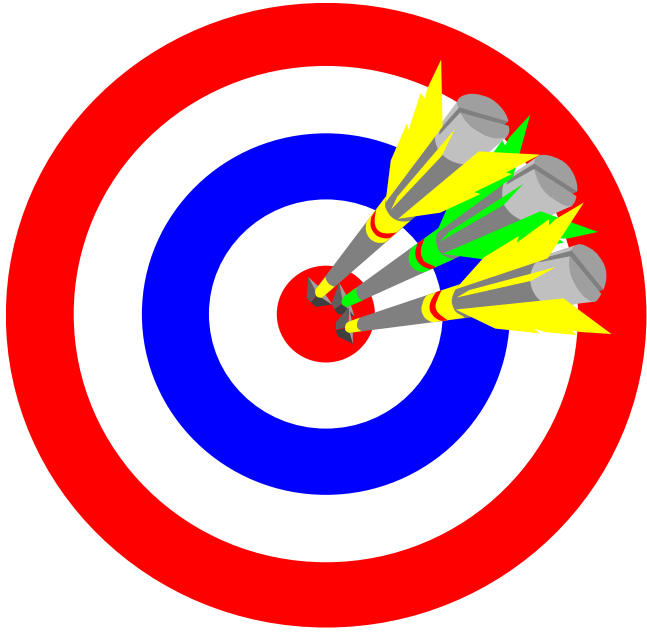
# Section 13-8

Organizing Coordinate Proofs

Homework Pages 558-559:

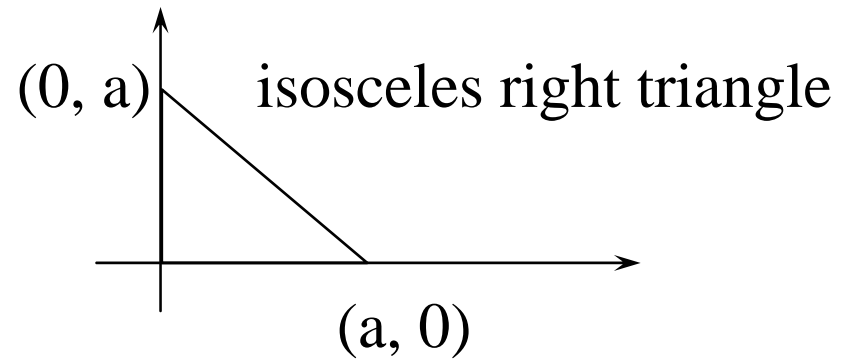
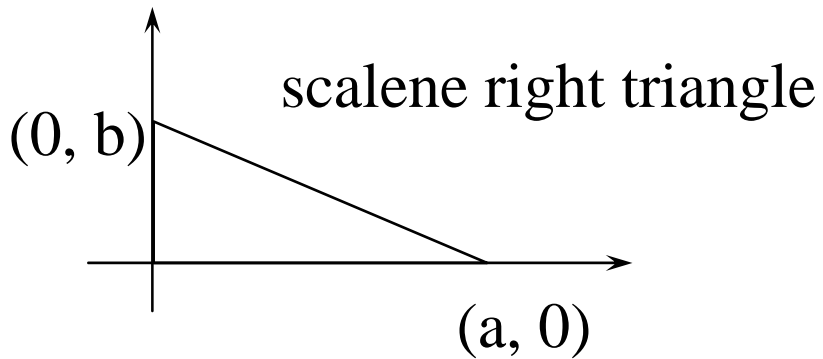
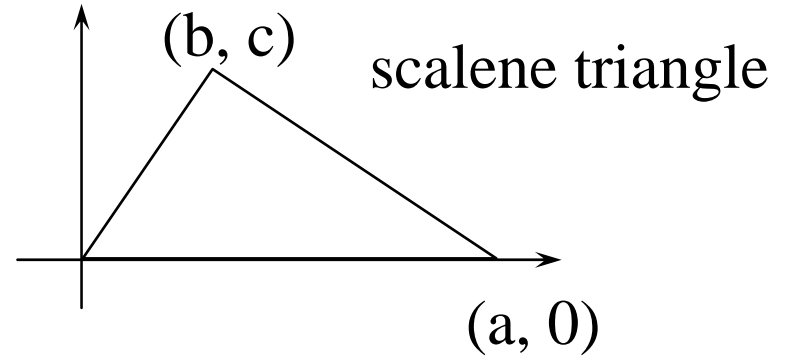
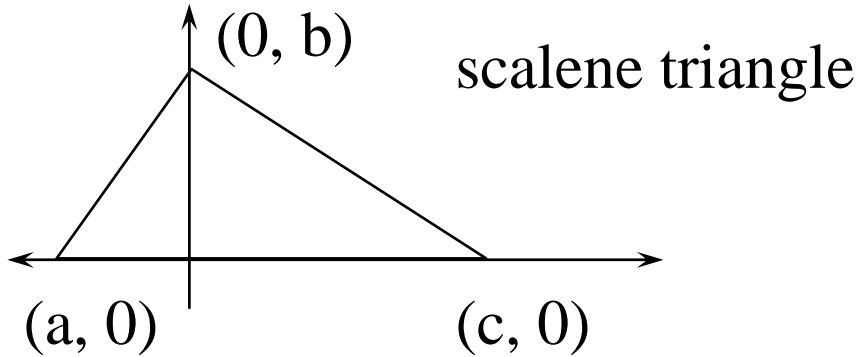
2-10 evens

# Objectives

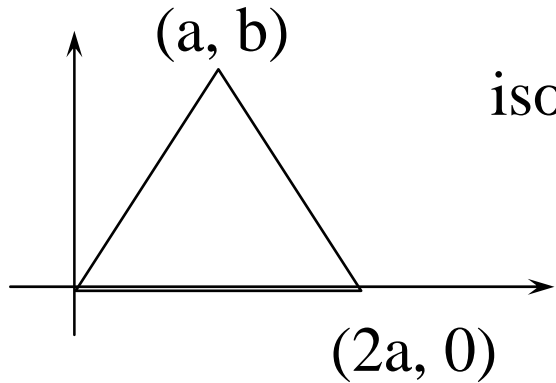


# Organizing Coordinate Proofs

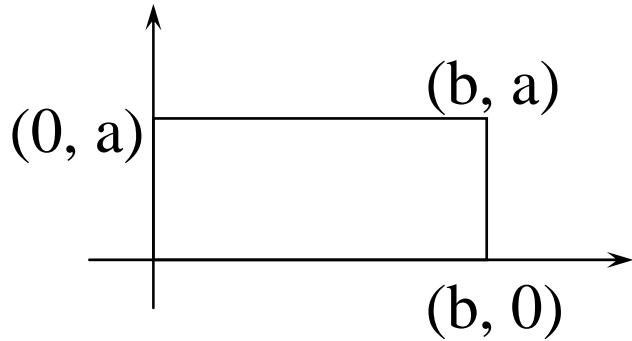
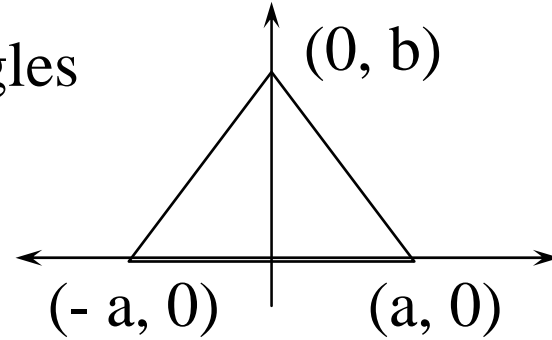
## Some Sample Diagrams



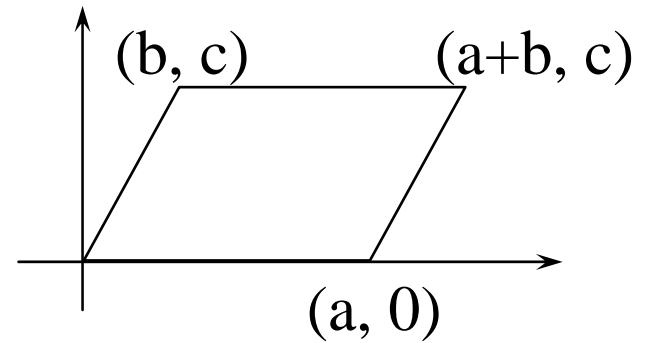
# Organizing Coordinate Proofs



isosceles triangles

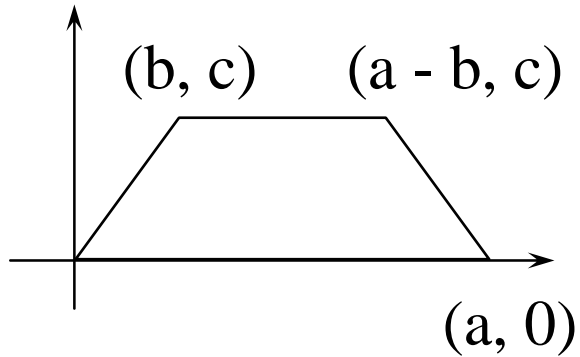


rectangle

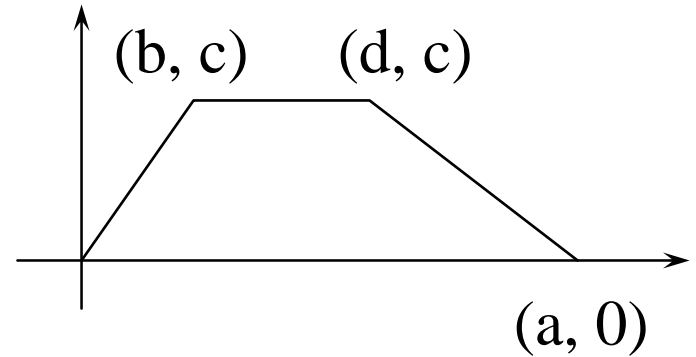


parallelogram

# Organizing Coordinate Proofs



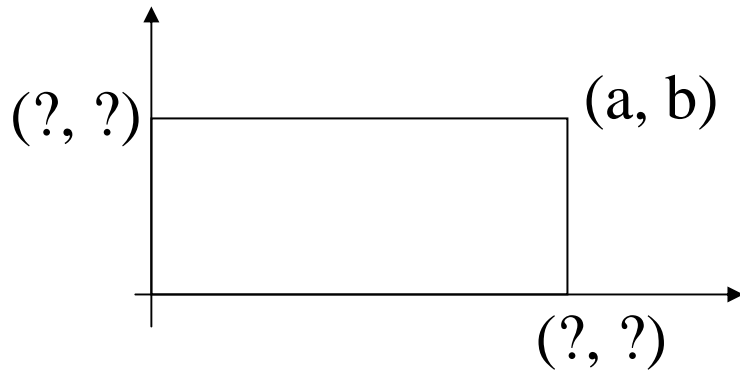
isosceles trapezoid



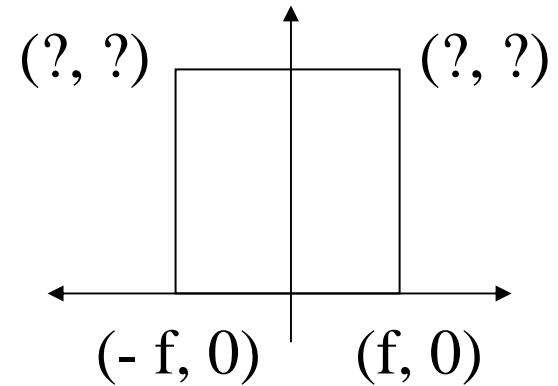
trapezoid

# Sample Problems

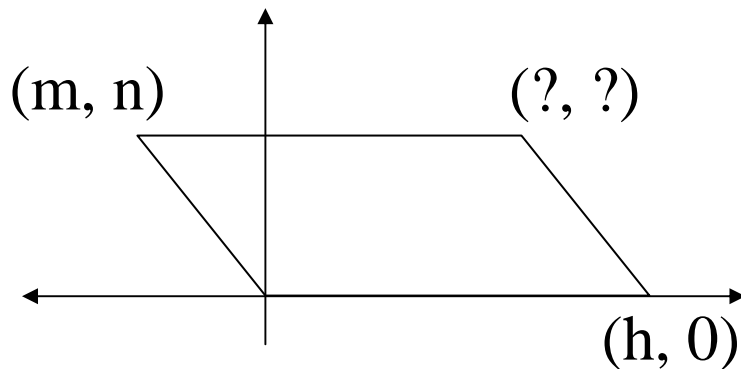
Supply the missing coordinates without using any new variables.



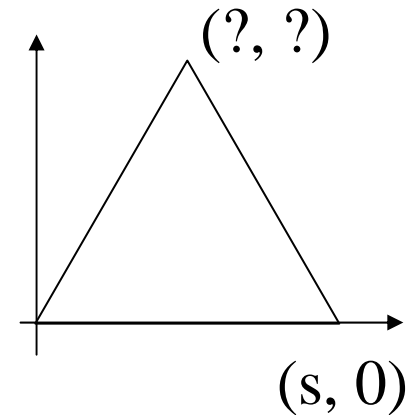
1. rectangle



3. square

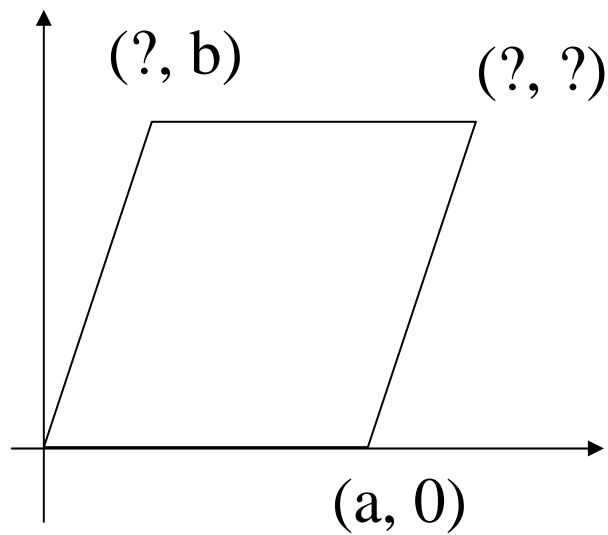


5. parallelogram



7. equilateral triangle

# Sample Problems



9. rhombus

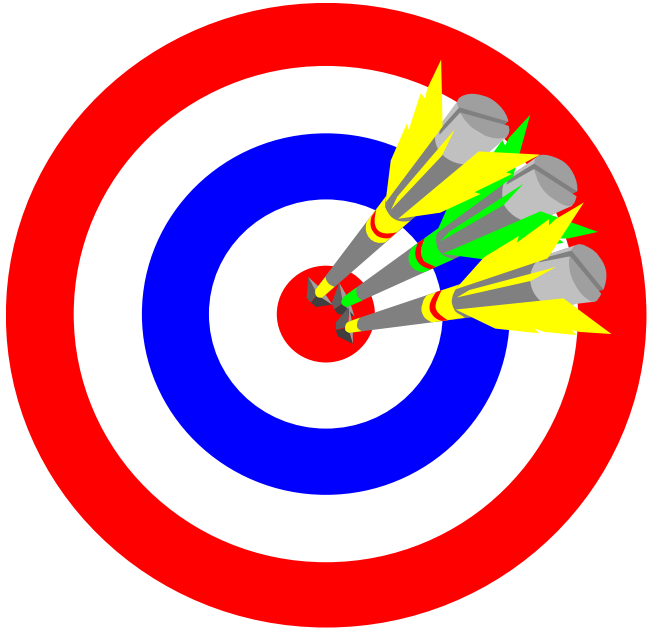
# Section 13-9

Coordinate Geometry Proofs

Homework Page 562:

2-10 evens

# Objectives



## Writing Coordinate Proofs

- To write a coordinate proof of a geometric theorem the first step is to place the diagram on a generic graph i.e. a graph without numbers.
- The second step is to label each vertex with variable coordinates. Each x-coordinate and each y-coordinate must be assigned a different variable unless a relationship has already been proven to exist.
  - To minimize the number of letters used to label the vertices, it is wise to place one vertex on the origin and as many other vertices as possible on the coordinate axes.

## Writing Coordinate Proofs

- The third step is to write the given and the prove statements as algebraic expressions, using the coordinates from the diagram.
- The fourth step is to create the body of the proof by using the rules of algebra to transform the given equation into the equation in the prove statement.
  - Most proofs can be accomplished by using the equations from theorems 13-1 to 13-7 along with the definitions of slope and vectors to write or transform your given statement.

## Sample Problems

1. the diagonals of a rectangle are congruent.
3. the diagonals of a rhombus are perpendicular.
5. the segments joining the midpoints of the diagonals of a trapezoid is parallel to the bases and has a length equal to half the difference of the lengths of the bases.
- 7.

# Chapter Thirteen

## Coordinate Geometry

### Review

Homework Page 568: 2-18 evens