Section 3-1

Open Sentences in Two Variables

Objectives

- solve for the range of a equation over a given domain
- complete ordered pairs for an equation
- solve for a coefficient or constant of an equation that will make an ordered pair a solution
- find integer solutions to an equation
- solve word problems involving two variable equations

Open Sentences in Two Variables

- open sentence in two variables: any equation or inequality that uses two variables
- solution of an open sentence in two variables: is an ordered pair of numbers representing points on the graph of the equation
- solution set: the set of all ordered pairs that satisfy the open sentence

Example for 1-12

$$2x + 3y = 7 \quad x \in \{-1, 0, 2\}$$

$$2(-1) + 3y = 7$$

$$-2 + 3y = 7$$

$$2 + (-2) + 3y = 2 + 7$$

$$0 + 3y = 9$$

$$3y = 9$$

$y = 3$ yields the point $(-1, 3)$

$x = 0$ then $y = \frac{7}{3}$

$x = 2$ then $y = 1$

$$\left\{(-1,\ 3)\left(0,\ \frac{7}{3}\right)(2,\ 1)\right\}$$

Example for 13-20

$3x + 2y = 12$ (0, ___) (___, 0) (2, ___)

I have the x value in the first point and third point and the y value in the second point.

For the first point substitute the 0 in for x to get

$3(0) + 2y = 12$

Solve this equation for y: $0 + 2y = 12$, $2y = 12$, $y = 6$ so the first point is (0, 6)

The second point would be (4, 0)

The third point would be (2, 3).

Example for 21-26

$$2x + y = k; \quad (2, 1)$$

$$2(2) + 1 = k$$

$$4 + 1 = k$$

$$5 = k$$

Example for 27-32

$$x + y = 4$$

$x \in$ {whole numbers: 0, 1, 2, 3, ...}

$0 + y = 4$ yields (0, 4)

$1 + y = 4$ yields (1, 3)

$2 + y = 4$ yields (2, 2)

$3 + y = 4$ yields (3, 1)

$4 + y = 4$ yields (4, 0)

Example for 33-38

$$x + y < 5$$

$$x \in \{1, 2, 3, 4, ...\}$$

$1 + y < 5$ yields $y < 4$ $\therefore$ (1, 1) (1, 2) (1, 3)

$2 + y < 5$ yields $y < 3$ $\therefore$ (2, 1) (2, 2)

$3 + y < 5$ yields $y < 2$ $\therefore$ (3, 1)

$4 + y < 5$ yields $y < 1$ and since there are no positive integers less than one we get no more points in the solution set.

{(1, 1) (1, 2) (1, 3) (2, 1) (2, 2) (3, 1)}

Sections 3-2 & 3-3

Graphs of Linear Equations in Two Variables

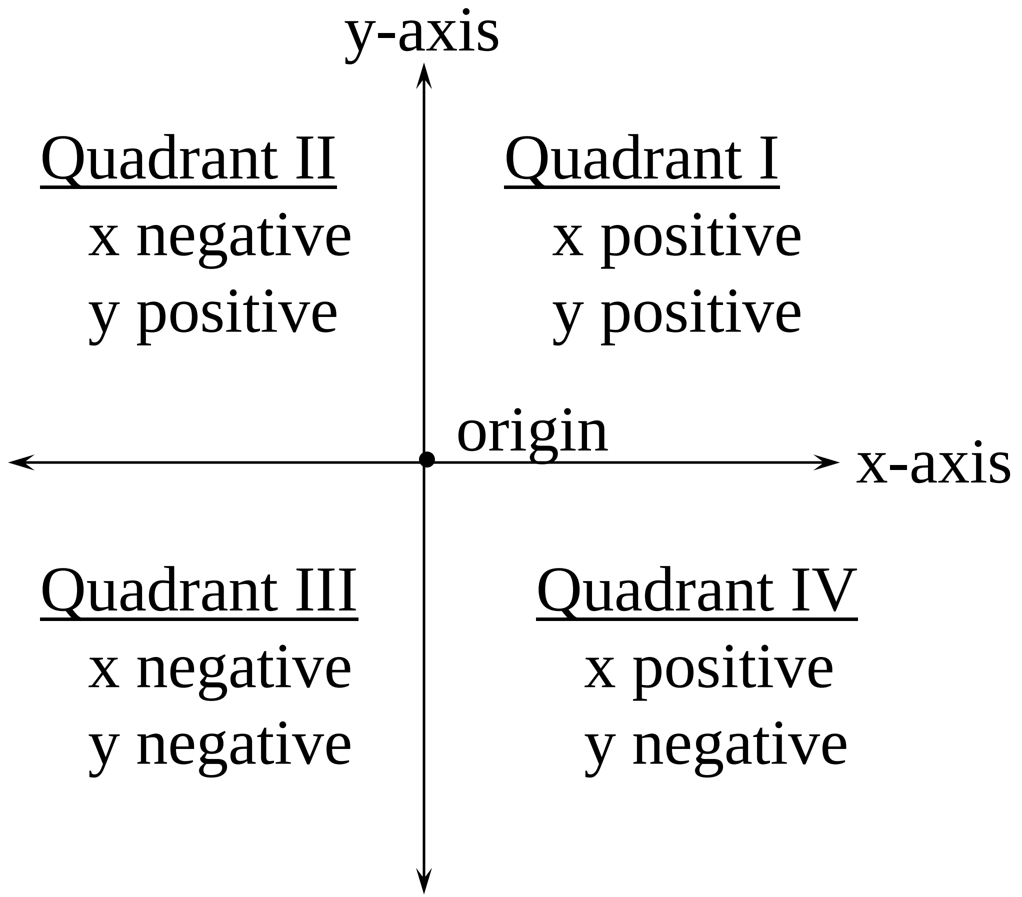
The Slope of a Line

Objectives

- graph ordered pairs
- graph linear equations
- find the value of coefficients or constant of an equation so that a point lies on the graph of the equation
- solve a system of linear equations by graphing & check your solution
- graph equations involving absolute value
- find the slope of a line given two points on the line
- find the slope of the line given the equation of the line
- graph a line given its slope and a point on the line
- find the value of either a coefficient or constant of an equation so that is has a given slope

Graphs of Linear Equations in Two Variables

- Cartesian coordinate plane: (xy-coordinate plane or plane rectangular coordinate system) created by a pair of perpendicular number lines that intersect at their origins



Graphs of Linear Equations in Two Variables

- x-coordinate: abscissa or independent variable
- y-coordinate: ordinate or dependent variable
- graphing: when plotting ordered pairs on a coordinate plane the x-coordinate will always be the first number in the ordered pair.
- graph of an open sentence in two variables is the set of all points that satisfy the open sentence. It is a picture of the solution set.
- linear equation in two variables: The graph of any equation in the form $Ax + By = C$ (where A & B are not both zero and are integers) is a line. This is called the Standard Form of a line.

Slope of a Line

- slope: the steepness of a line
  - vertical lines have no slope
  - horizontal lines have slope = 0

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

  - slope of a line written in Ax + By = C form is $-\frac{A}{B}$
- slope intercept form of a line: $y = mx + b$, where m is the slope of the line and b is the y-coordinate of the y-intercept
- point-slope form of a line: $y - y_1 = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) is any known point on the line

Slope of a Line

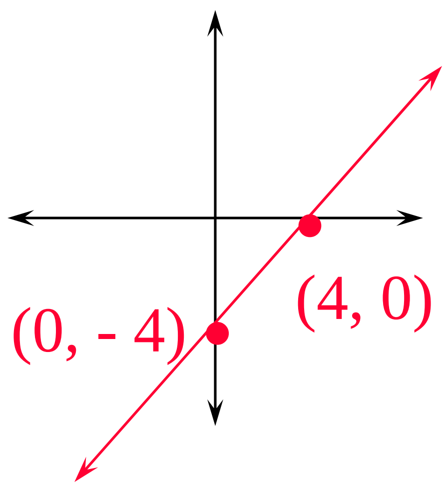
- graphing a line:
  - plot two points (solutions of the open sentence) and connect
  - plot one point and use the slope as a set of rise and run instructions to move from that point to another point on the graph of the line, then connect the two

Examples for 5-22

x - y = 4

0 - y = 4 then y = - 4

x - 0 = 4 then x = 4



Example for 23-26

P(2, 1), L: $3x + ky = 8$

$3(2) + 1(k) = 8$

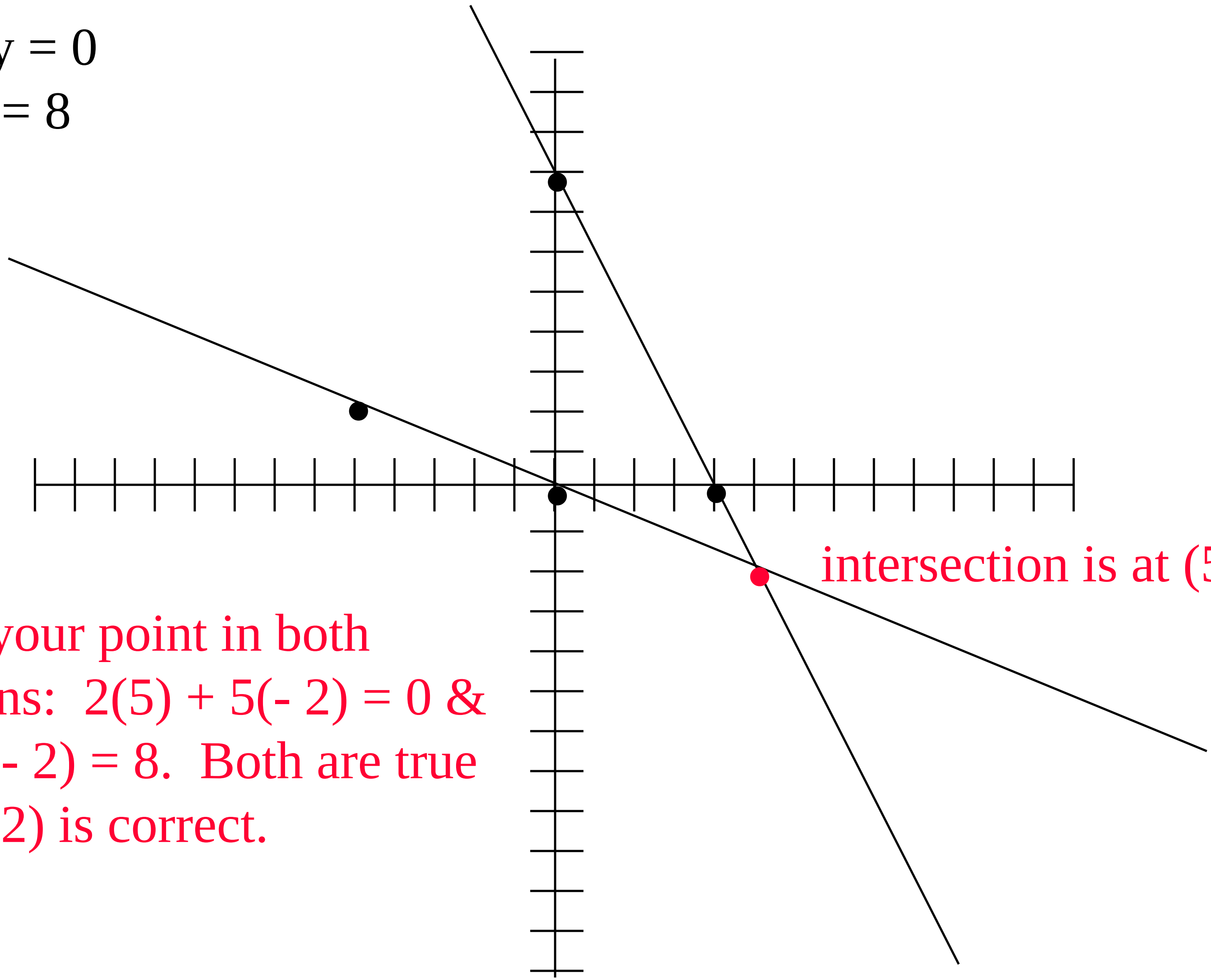
$6 + k = 8$

$k = 2$

Example for 27-30

2x + 5y = 0

2x + y = 8



intersection is at (5, - 2)

Check your point in both equations: 2(5) + 5(- 2) = 0 & 2(5) + (- 2) = 8. Both are true so (5, - 2) is correct.

Example for 31-33

$y = |x|$

$y = x$ and $y = -x$

The first line crosses at the origin and has a slope of 1.

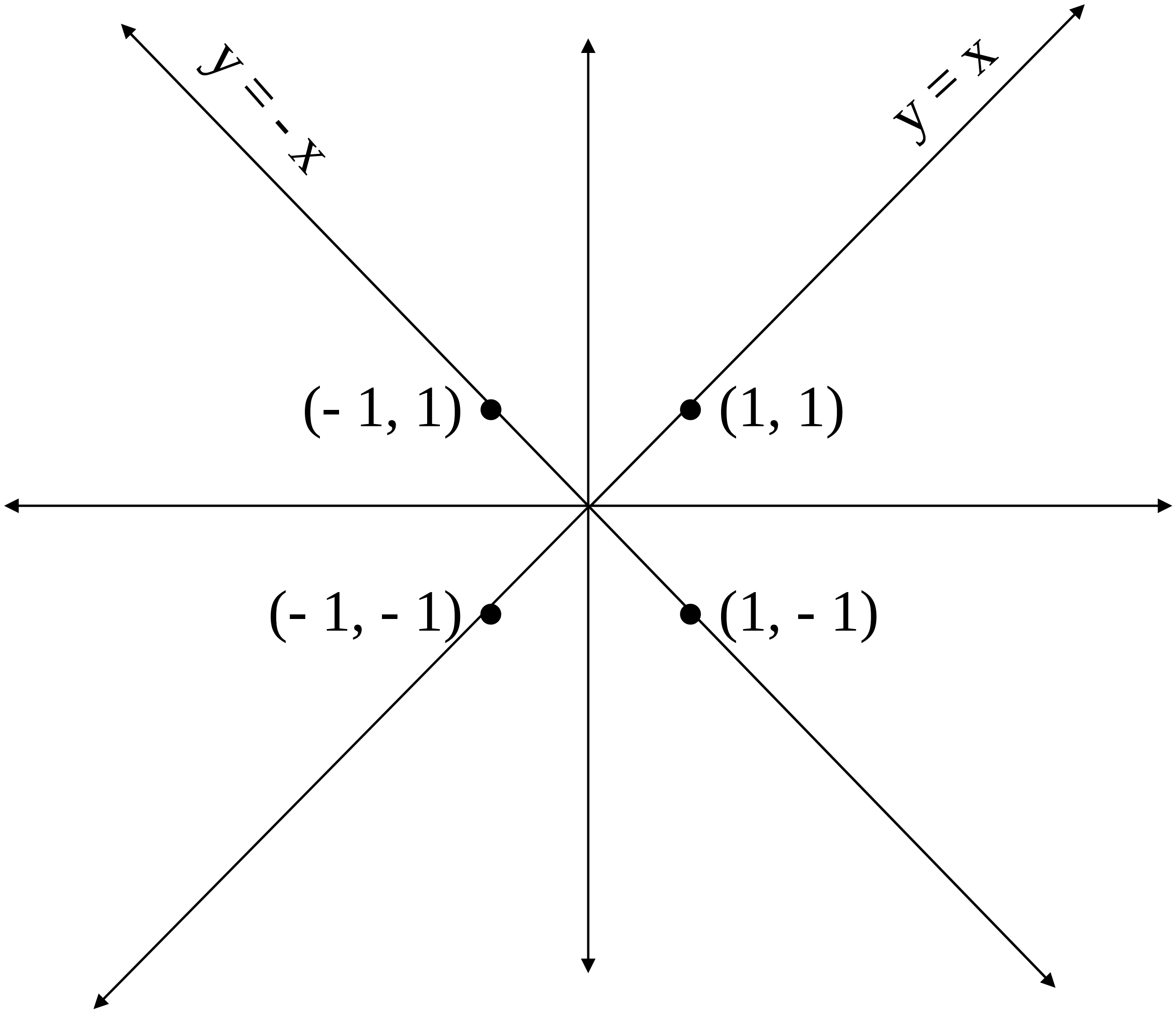
The second line also crosses at the origin but has a slope of

- 1.

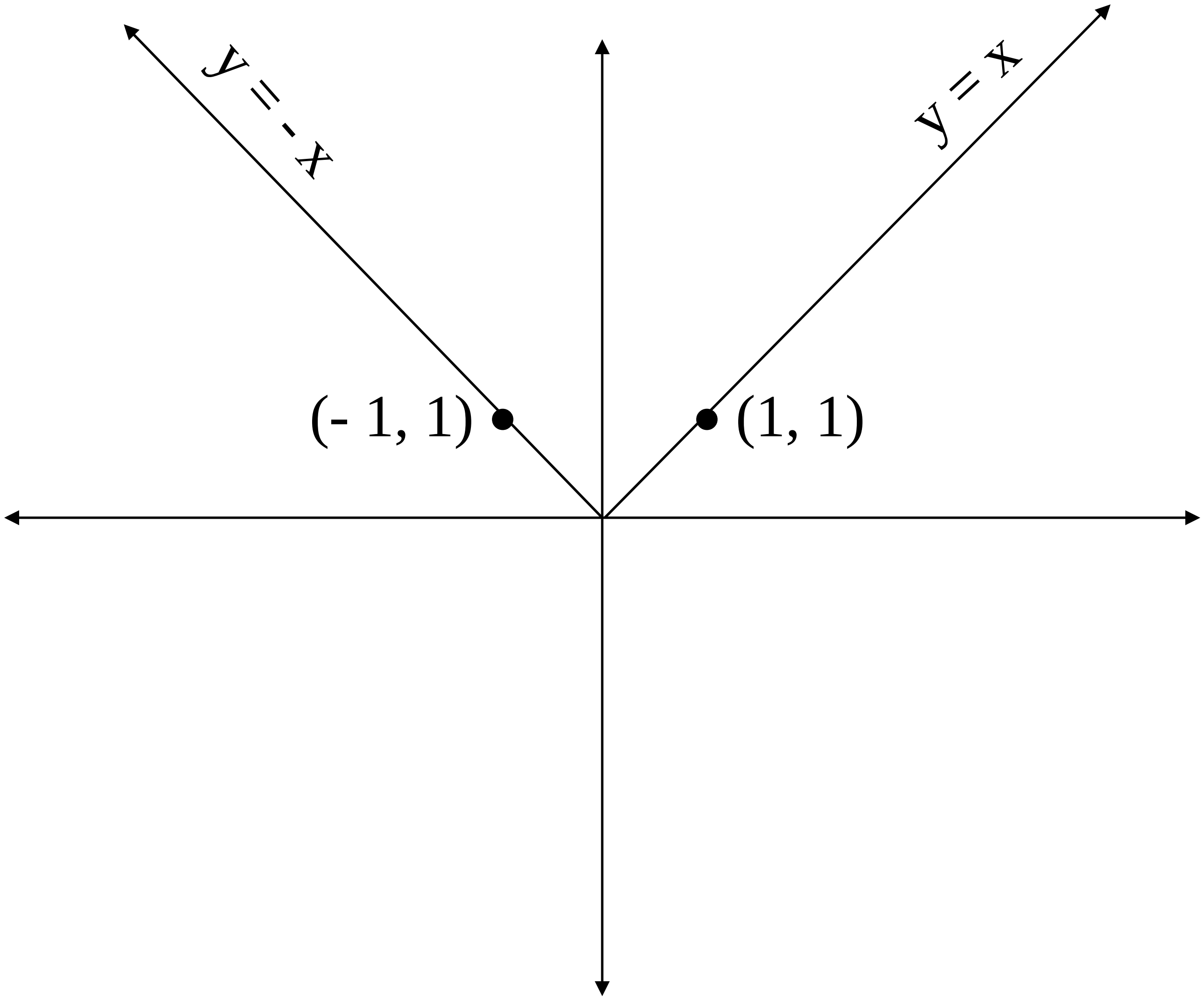
Test the points (1, 1) (1, - 1) (- 1, 1) & (- 1, - 1) in the original problem

Only (1, 1) & (- 1, 1) work so you would erase the part of the graph below the x-axis.

Graph of y = | x |



Example: Problem # 33. $y = |x|$



Example for 1-12

(3, 1) (5, 5)

Let (3, 1) be the second point and (5, 5) be the first point.

$$\text{slope} = \frac{1-5}{3-5}$$

$$\text{slope} = \frac{1-5}{3-5} = \frac{-4}{-2} = 2$$

Example for 13-24

x = 3y + 2

move the 3y to the left side by subtraction: x - 3y = 2

The coefficient of x is (1) and the coefficient of y is (- 3).

Take the opposite of (1) and divide it by (- 3) to get the slope.

Reduce if necessary.

$$\text{slope} = \frac{-1}{-3} = \frac{1}{3}$$

Example for 25-36

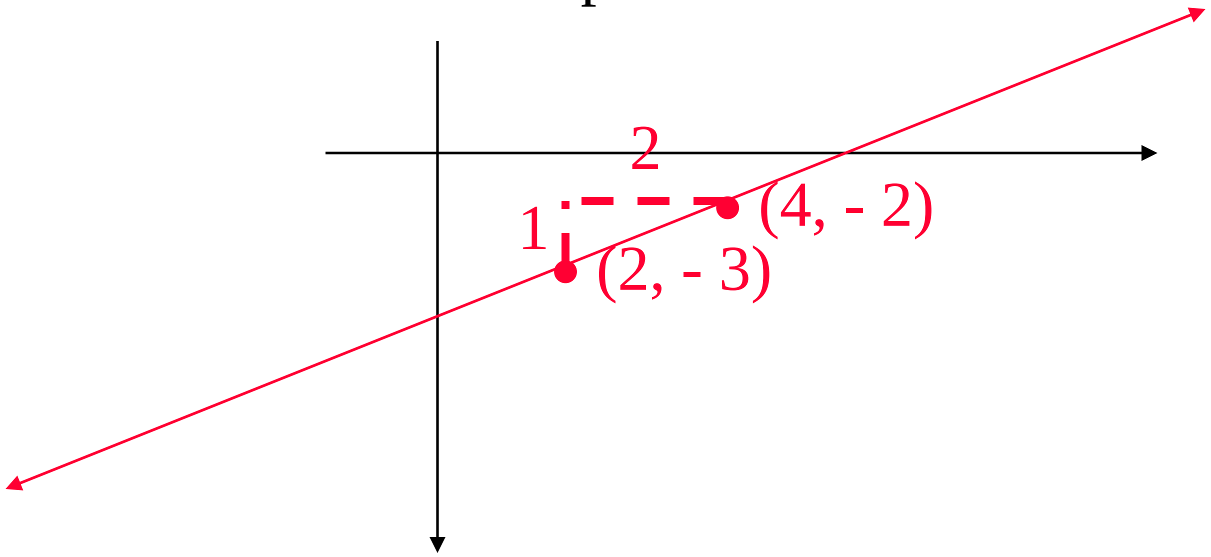
P(2, -3), m = ½.

Plot P on a graph.

The slope says to change the 2 by adding 2 and to change the - 3 by adding 1. Which creates the new point (4, - 2).

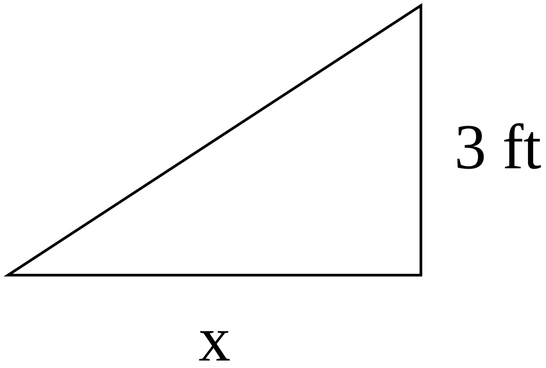
Plot the point (4, - 2).

Connect these two points with a line.



Example for 37 & 38

A ramp to provide handicapped people with access to a building is to be constructed with a 5% slope. If the entrance to the building is 3 ft above ground level, how long should the base of the ramp be?



$$5\% = \frac{3}{x} \Rightarrow \frac{5}{100} = \frac{3}{x} \Rightarrow 5x = 300 \Rightarrow x = 60\text{ft}$$

Example for 39-42

(k + 3)x - 3y = 1; m = k

This equation is already in standard form.

The standard form shortcut says that

$$\frac{-(k+3)}{-3} = k$$

$$\frac{-(k+3)}{-3} = k \Rightarrow \frac{(k+3)}{3} = \frac{k}{1} \Rightarrow k+3 = 3k \Rightarrow 3 = 2k \Rightarrow k = \frac{3}{2}$$

Example for 43-46

(2k, 3) & (1, k); m = 2

$$\frac{3-k}{2k-1}$$

$$\frac{3-k}{2k-1} = 2 \Rightarrow \frac{3-k}{2k-1} = \frac{2}{1}$$

3 - k = 2(2k - 1); 3 - k = 4k - 2; 5 = 5k;

k = 1

Section 3-4

Finding Equations of a Line

Objectives

- know the three forms of a line
- be able to write the equation of a line in any form given a point on the line and the slope of the line
- be able to write the equation of a line in any form given the slope of the line and the y-intercept of the line
- be able to write the equation of a line in any form given two points on the line
- be able to write the equation of a line in any form given a point on the line and the equation of a line parallel to the line
- be able to write the equation of a line in any form given a point on the line and the equation of a line perpendicular to the line

Writing Equations of the Line

- Review of Forms of Equations of a Line:
  - standard form: $Ax + By = C$ where A, B & C are all integers
    - slope: $-\frac{A}{B}$
    - x-intercept: $\frac{C}{A}$
    - y-intercept: $\frac{C}{B}$
  - slope-intercept form: $y = mx + b$ where m is the slope and b is the y-intercept
  - point-slope form: $y - y_1 = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) is any known point on the line

Writing Equations of the Line

- Writing an equation of a line:
  - identify either one or two points on the line
  - identify or solve for the slope of the line using either two points, a line parallel or a line perpendicular to the line whose equation you are trying to write
  - substitute the slope value for m in point-slope form & substitute the coordinates of one of the points for x_1 & y_1 in point-slope form
  - rearrange into standard form

Writing Equations of the Line

- Given two lines L_1 and L_2 with slopes m_1 and m_2 respectively:
  - If L_1 and L_2 are parallel then $m_1 = m_2$.
  - If L_1 and L_2 are perpendicular then $(m_1)(m_2) = -1$ or m_1 and m_2 are the negative reciprocals of one another.

Example for 1-12

$$P(4, -3) \text{ \& } m = \frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = m(x - 4)$$

$$y - (-3) = \frac{1}{5}(x - 4)$$

$$y - (-3) = \frac{1}{5}x - \frac{4}{5}$$

$$5\left[y - (-3) = \frac{1}{5}x - \frac{4}{5}\right] \Rightarrow 5y - 5(-3) = 5\left(\frac{1}{5}\right)x - 5\left(\frac{4}{5}\right)$$

$$5y + 15 = x - 4 \Rightarrow -x + 5y = -19$$

Example for 13-18

m = -1 & b = 2

y = - 1x + 2 or y = - x + 2

there are no fractions to eliminate

add x to both sides to get: x + y = 2

Example for 19-30

(0, 0) (5, - 2)

$$\text{slope} = \frac{0-(-2)}{0-5} = \frac{2}{-5}$$

$$y-0 = \frac{2}{-5}(x-0)$$

$$y = \frac{2}{-5}x$$

$$-5y = 2x$$

$$2x + 5y = 0$$

Example for 31-38

P(2, 0); x + 2y = 3

slope for (a) is $\frac{-1}{2}$ slope for (b) is 2

(a) $y - 0 = \frac{-1}{2}(x - 2)$ (b) y - 0 = 2(x - 2)

(a) $2\left[y = \frac{-1}{2}(x - 2)\right]$ (b) y = 2x - 4 or - 2x + y = - 4

2y = - x + 2

x + 2y = 2

Sections 3-5 & 3-6

Systems of Linear Equations in Two Variables

Problem Solving Using Systems

Objectives

- solve systems of linear equation in two variables
- graph a system of linear equations and estimate the answers
- determine whether a system is infinite (and provide several solutions) or has no solution
- determine whether a system is consistent or inconsistent
- identify two variables in a word problem
- set up a system of linear equations using these two variables from the information contained in a word problem
- solve the system of linear equations in two variables and provide a solution(s) to the word problem

Systems of Linear Equations in Two Variables

- system of linear equations: a set of equations in the same two variables
- solution: all ordered pairs that satisfy both equations, or the intersection of the two graphs
  - a linear system may have a single point as a solution when the lines intersect
  - a linear system may have no solution if the lines are parallel
  - a linear system may have an infinite set of points in its solution when the lines coincide i.e. the two equations may look different but are really describing the same line

Systems of Linear Equations in Two Variables

- consistent equations: a system with at least one solution
- dependent equations: a system with infinite solutions
- inconsistent equations: a system with no solutions
- equivalent systems: systems with the same solution set

Systems of Linear Equations in Two Variables

- the goal of the solution process is to eliminate one of the variables.
  - graph the equations and identify the intersection: not a very reliable method since your answer is only as accurate as your graph
  - add the two equations together: works only if the coefficients of the same variable are opposites
  - subtract the two equations: works only if the coefficients of the same variable are equal
  - multiply then add/subtract: works with reliably with any system. You may pick any constant to multiply through either equation to create opposite /equal coefficients
  - substitution: works reliably with any system but is easiest when one of the coefficients is either 1 or - 1. You must rearrange one of the equations into “x =“ or “y =“

Systems of Linear Equations in Two Variables

- solve for the remaining unknown variable
- substitute the value back into one of the original equations and solve for the second variable
- check to make sure your point works in both of the original equations.
- Remember that if both x and y cancel and you are left with a true statement then the answer is infinite; if both variables cancel and you are left with nonsense the answer is null set.

Problem Solving Using Systems

- First, you should follow the same steps to solve these word problems as you should have been using in the past.
- Second, for these problems you will have two variables to work with.
- Third, when you begin translating you will have to have two different equations to work with i.e. you must set up a system of equations.
- Fourth, you may use whichever method is appropriate to solve your system of equations.
- Fifth, remember to check your answer not only in your system but also to make sure that it is a plausible answer for the word problem.

Section 3-7

Linear Inequalities in Two Variables

Objectives

- graph the solution of a linear inequality
- graph the solutions of linear inequalities

Linear Inequalities in Two Variables

- When you replace the equals sign in a linear equation with two variables with an inequality symbol you create a linear inequality.
  - the solution of a linear inequality in two variables is still any ordered pair of numbers that makes a true statement of order.
  - all linear inequalities have an infinite solution set since the graph of a linear inequality is either an open or closed half-plane.

Linear Inequalities in Two Variables

- To graph a linear inequality you must:
  - first graph its boundary which is the linear equation associated to the inequality (the equation you get when you replace the inequality symbol with an equals sign).
  - second this line will be solid if the inequality was either ≥ or ≤ . It will be a dashed line if the inequality was either < or >.
  - third choose a point from one side of the boundary line and test it in the inequality if it makes a true statement of order then shade that side of the boundary line; if it makes a false statement of order then shade the other side of the boundary line.

Linear Inequalities in Two Variables

- Systems of linear inequalities: you must solve a system of linear inequalities by graphing
  - first, graph each inequality in the system on the same coordinate plane.
  - second, the solution to the system occurs everywhere that the shaded regions overlap.

Example for 1-18

$x + y > 1$

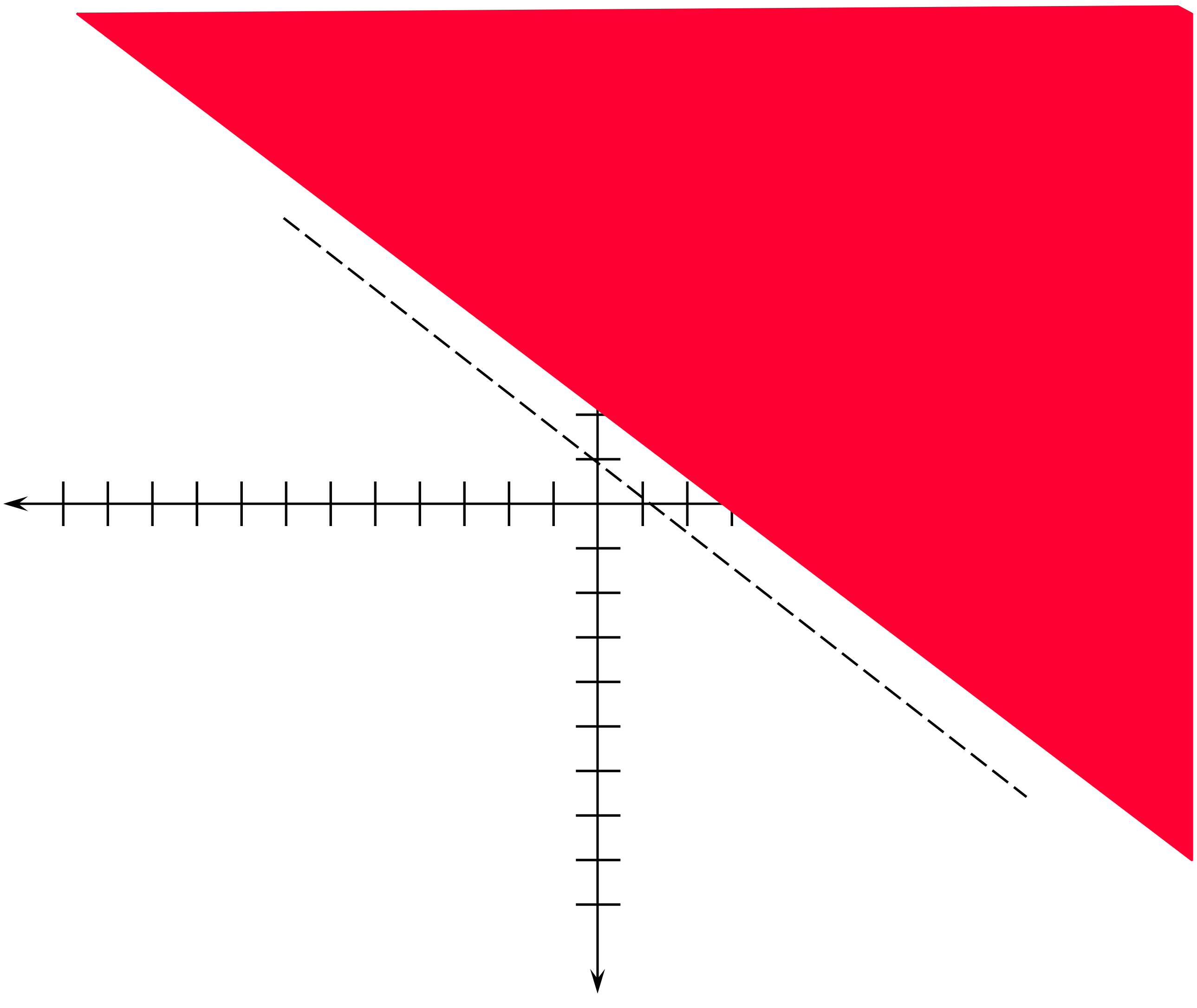
boundary is the dotted line $x + y = 1$

which has x-int (1, 0) & y-int (0, 1)

plot these two points on a graph and connect them with a dotted line

choose a point from the graph that is on one side of the boundary in this problem the origin will work and substitute it into the problem $0 + 0 > 1$ is false so shade the other side of the boundary.

Graph Example for 1-18



Example for 19-42

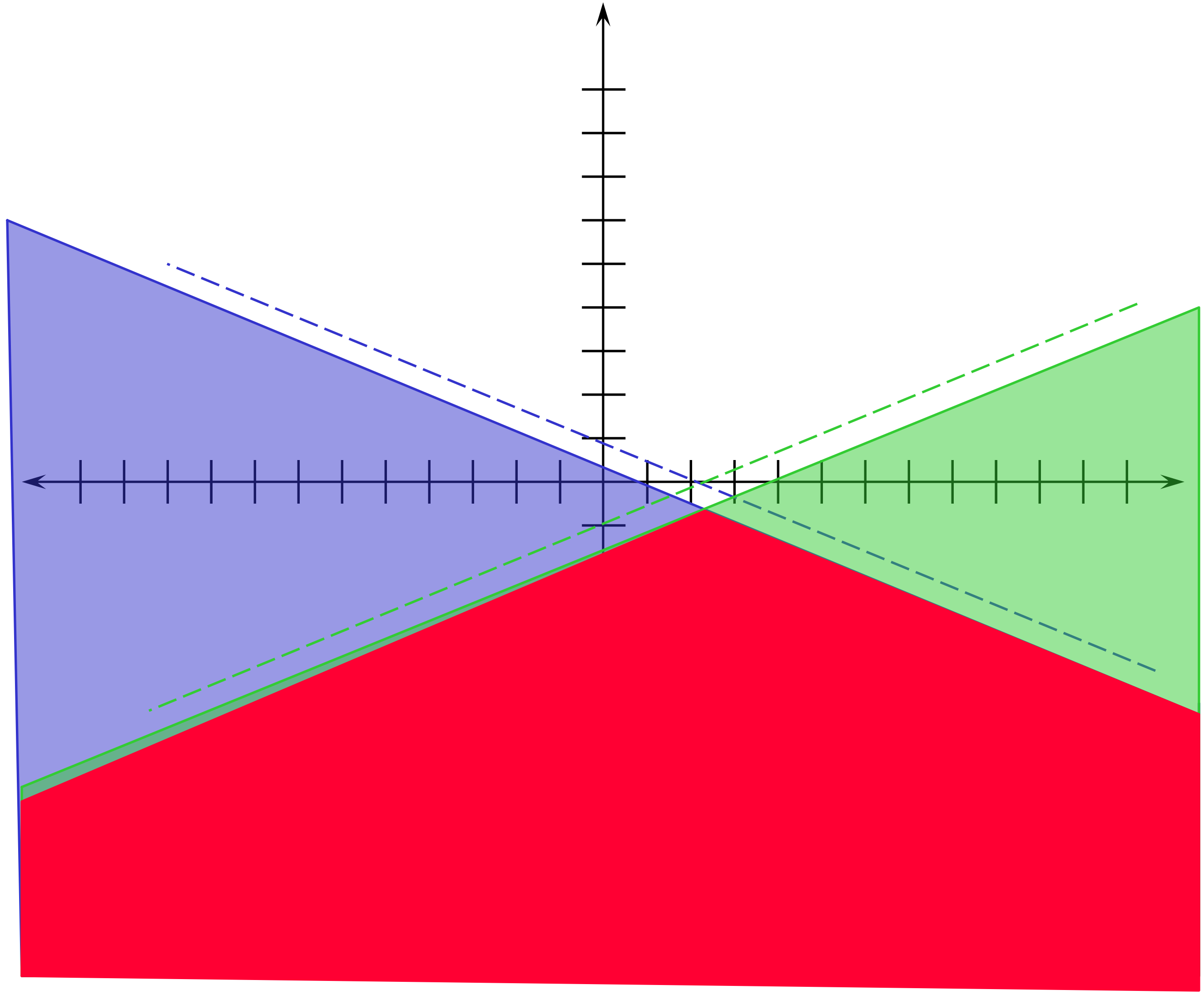
$x + 2y < 2$ & $x - 2y > 2$

boundary for $x + 2y < 2$ is $x + 2y = 2$ x-int (2, 0) y-int (0, 1)

boundary for $x - 2y > 2$ is $x - 2y = 2$ x-int (2, 0) y-int (0, - 1)

see graph on next slide

Graph Example for 19-42



Section 3-8

Functions

Objectives

- read and understand the different ways of writing functions
- solve for the range of a function given its rule and its domain
- write the equation of a function given a diagram of its domain and range
- identify the domain of a function given its equation
- find the values of composites of functions

Functions

- mapping diagram: pictures a correspondence between two sets, the domain D and the range R
- function: is a correspondence between two sets D & R, that assigns to each member of D exactly one member of R. In other words and x-value can be used only once.
- functional notation: not all equations in two variables are written as open sentences. They may also be written in f(x) or f: x→ form.
- values of a function: the members of the range
- graph of a function: all points (x, y) such that x is in the domain and y is in the range.

Example for 1-24

F: $x \rightarrow 3 - 2x$; $D = \{0, 1, 2, 3\}$

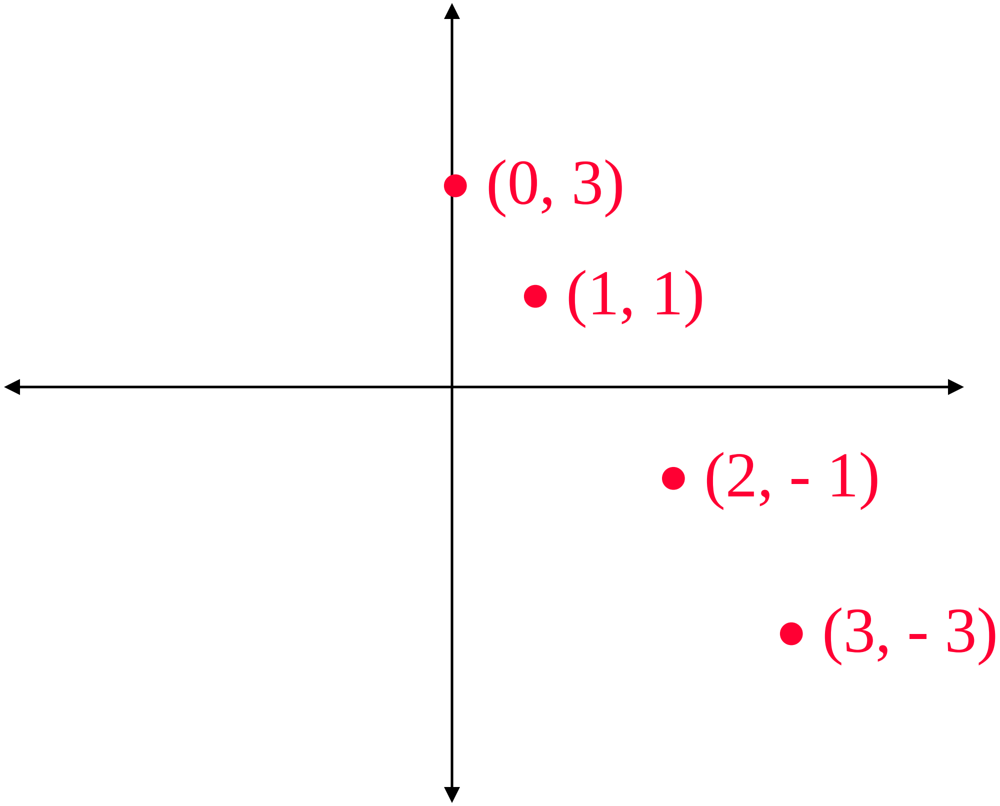
F: $0 \rightarrow 3 - 2(0) = 3$

F: $1 \rightarrow 3 - 2(1) = 1$

F: $2 \rightarrow 3 - 2(2) = -1$

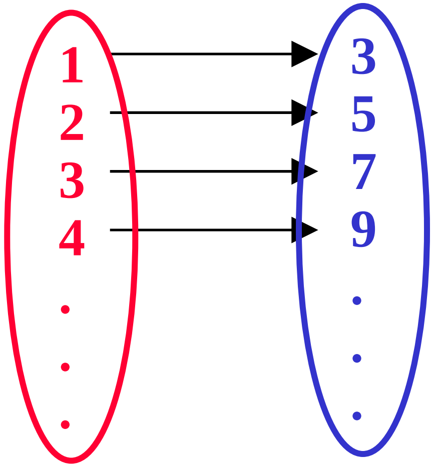
F: $3 \rightarrow 3 - 2(3) = -3$

$R = \{3, 1, -1, -3\}$



Example for 25-28

$f(x) = \_\_\_\_ x + 1$



Find the value that makes all of the statements true by first subtracting 1 from both sides and then dividing by the number in the parentheses.

$3 = \_\_\_(1) + 1$

$5 = \_\_\_(2) + 1$

$7 = \_\_\_(3) + 1$

$9 = \_\_\_(4) + 1$

2 = ___

Example for 29-34

$$h(x) = \frac{2}{x^2 + 3}$$

$x^2 + 3$ is the denominator so we would exclude from the domain any values of x where $x^2 + 3 = 0$.

$$x^2 = -3$$

$$\sqrt{x^2} = \sqrt{-3}$$

$$x = \sqrt{-3}$$

since this is not a real number no real numbers get excluded and $D = \{x: x \in \Re\}$

Example for 35-40

$f(x) = x^2 - 1$ and $g(x) = 1 - 2x$

find $f(g(1))$

Find $g(1) = 1 - 2(1) = -1$

Substitute -1 into $f(-1) = (-1)^2 - 1 = 0$

Section 3-9

Linear Functions

Objectives

- find the equation of a linear function given information about the function
- complete a table of values that describe a linear function
- find the values of a function for elements of the domain given several ordered pairs of the function
- solve word problems involving linear functions

Linear Functions

- linear function: is defined by the equation f(x) = mx + b
- constant function: is defined by the equation f(x) = b

$$\text{rate of change of m} = \frac{\Delta f(x)}{\Delta x}$$

Example for 1-22

$f(1) = 2$ and $f(2) = 5$

$$m = \frac{2-5}{1-2} = \frac{-3}{-1} = 3$$

$5 = 3(2) + b$ $5 = 6 + b$ $-1 = b$

$f(x) = 3x - 1$

Example for 23-26

x	g(x)
3	4
1	- 2
0	-5
	- 8

Differences: x from 3 to 1: - 2; x from 1 to 0: - 1. g(x) from 4 to - 2: - 6; g(x) from - 2 to -5: w.

$$\frac{-6}{-2} = \frac{w}{-1}$$

$$-2w = 6$$

$$w = -3$$

x	g(x)
3	4
1	- 2
0	-5
-1	- 8

Differences: x from 1 to 0: - 1; x from 0 to -1: z. g(x) from - 2 to -5: - 3; g(x) from -5 to - 8: - 3.

$$\frac{-3}{-1} = \frac{-3}{z}$$

$$-3z = 3$$

$$z = -1$$

Example for 27-30

If f(6) = 7 & f(3) = 2 find f(- 3) & f(10).

$$m = \frac{7-2}{6-3} = \frac{5}{3}$$

Plugging in the slope and a point to solve for b

$$2 = \frac{5}{3}(3) + b$$

$$2 = 5 + b$$

$$-3 = b$$

The equation is: $f(x) = \frac{5}{3}x - 3$

Substitute - 3 into this function to get f(- 3) = - 8 and

substitute 10 into this function to get f(10) = $\frac{41}{3}$

Example for 1-10

A photocopying machine purchased new for \$4500 loses \$900 in value each year.

a. Find the book value of the machine after 18 months.

b. When will the book value be \$1200?

x is the age of the photocopier machine in years

f(x) is the value of the machine

$f(x) = 4500 - 900x$ is the linear function for value.

a. $f(1.5) = 4500 - 900(1.5) = 4500 - 1350 =$ \$3150

b. $1200 = 4500 - 900x$

$-3300 = -900x$

3 years 8 months = x

Section 3-10

Relations

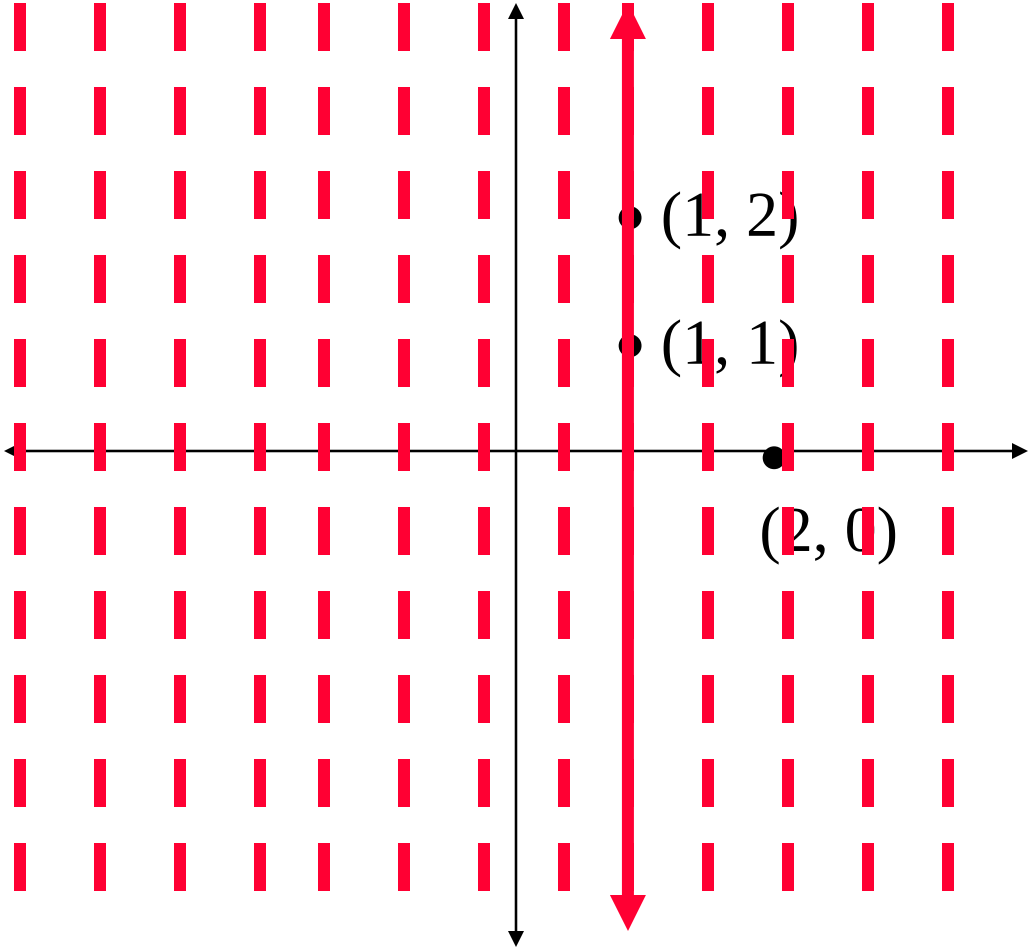
Objectives

- graph relations and determine from the graph whether or not it is a function
- identify the domain and the range of a relation and determine whether or not it is a function
- determine if a pair of functions are equal

Relations

- a relation is any set of ordered pairs.
  - as in a function or any other equation the first set of numbers in the ordered pair is the domain and the second set is the range
- a function is a relation where all of the ordered pairs have different first coordinates
- vertical line test: if the graph of a relation is crossed only once by each of the vertical lines on a graph then the relation is a function.

Example for 1-6



{(1, 2) (2, 0) (1, 1)}

This graph fails the vertical line test; therefore, it is not a function.

Example for 7-20

$\{(x, y): |x| = |y| \text{ and } |x| \leq 1\}$

Solving $|x| \leq 1$ by the methods we learned in Ch 2 we get that $-1 \leq x \leq 1$ and since x is an integer the domain is $\{-1, 0, 1\}$

When we substitute 1 in for x we get $1 = |y|$ and solving this means that y = 1 or - 1. When we substitute 0 in for x we get $0 = |y|$ and solving this means that y = 0. When we substitute - 1 in for x we get $1 = |y|$ and solving this means that y = 1 or - 1. Our range is {- 1, 0 1}

The points are: {(1, - 1) (1, 1) (0, 0) (- 1, 1) (- 1, - 1)}

This relation fails the vertical line test and therefore is not a function.

Example for 21-24

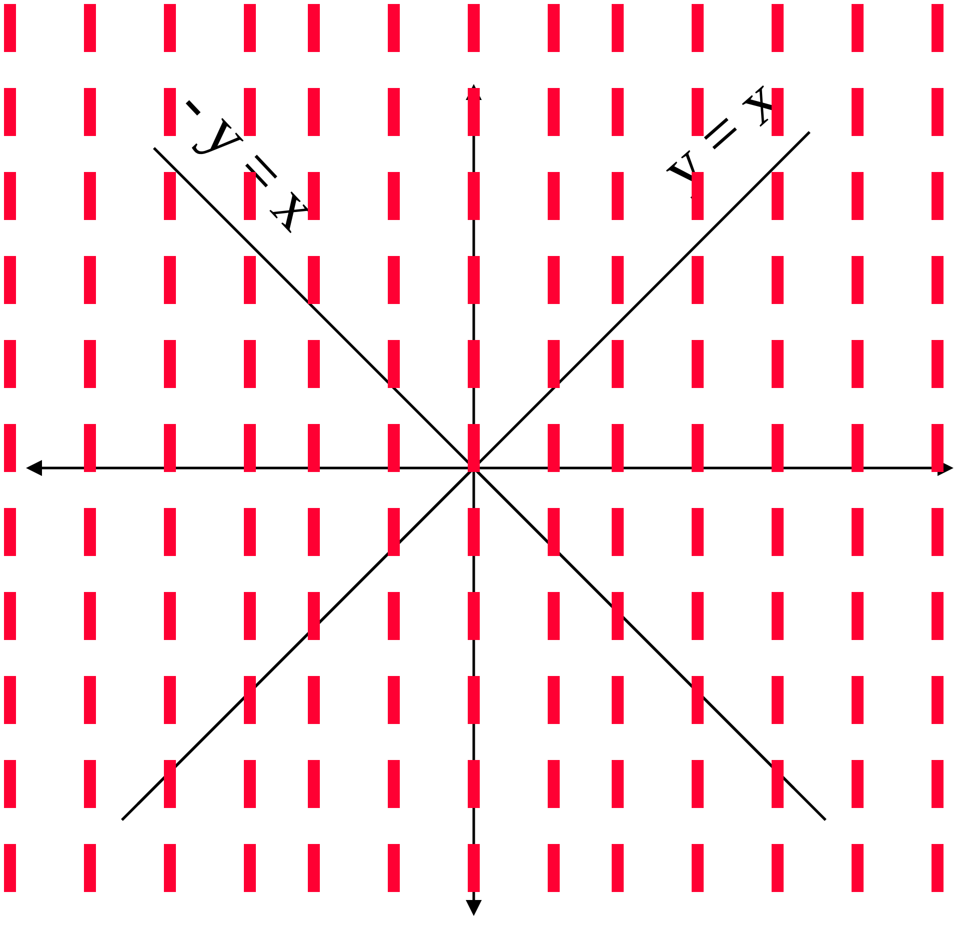
D = {- 1, 0, 1, 2, 3} f: x→2x + 1 g: x→5 - 2x

f: - 1→ - 1	g: - 1→ 7	
f: 0 → 1	g: 0 → 5	Since the values are not equal
f: 1 → 3	g: 1 → 3	the functions are not equal.
f: 2 → 5	g: 2 → 1	
f: 3 → 7	g: 3 → - 1	

Example for 25-30

{(x, y): y = - | x | } The two equations to be graphed are

- y = x and - y = - x or y = x



This passes the vertical line test; therefore, this relation is a function.