Section 4-1

Polynomials

Objectives

- simplify polynomials
- determine the term with the largest degree within a polynomial
- determine the degree of a polynomial
- add and subtract polynomials
- simplify polynomial expressions by applying order of operations and all the rules of real numbers that you have learned to date
- find the values of coefficients that will make two polynomials equal.

Polynomials

- constant: a number
- monomial: a constant, a variable or a product of constants and variables
- coefficient: the numerical factor of a monomial
- degree of a variable in a monomial: the exponent of the variable
- degree of a monomial: the sum of all the exponents in the variable
- similar monomials: have the same variables to the same degrees
- polynomial: the sum of monomial and constant terms
- simplified polynomial: no two terms are similar
- degree of a polynomial: is the same as the term with the largest degree

Example #5, $x^2y^2 - x^2 + 8x^2y^2 + 5xy^2 - 2x^2$

I. Identify similar terms by looking for the same variables with the same exponents and you find that

$$x^2y^2 - x^2 + 8x^2y^2 + 5xy^2 - 2x^2$$

$$9x^2y^2 - 3x^2 + 5xy^2$$

II. Degrees of each term $2 + 2 = 4$; 2; 3

III. Therefore the polynomial is already written in decreasing order of degree. $9x^2y^2 - 3x^2 + 5xy^2$ and the degree of the polynomial is 4.

Example #9, $5m - 4$, $2m + 3$.

I. $(5m - 4) + (2m + 3) = 5m - 4 + 2m + 3$

II. $7m - 1$

III. $7m - 1$

IV. $(5m - 4) - (2m + 3) = 5m - 4 - 2m - 3 = 3m - 7$

Example #23, $3[2p^2 - q(3p + 4q)] - 2[4q^2 - 3p(p - 2q)]$

I. First distribute into the parentheses

$$3[2p^2 - 3pq - 4q^2] - 2[4q^2 - 3p^2 + 6pq]$$

II. Second distribute into the brackets

$$6p^2 - 9pq - 12q^2 - 8q^2 + 6p^2 - 12pq$$

III. Third identify and group similar terms

$$6p^2 - 9pq - 12q^2 - 8q^2 + 6p^2 - 12pq = 12p^2 - 21pq - 20q^2$$

IV. Fourth rewrite in decreasing order of degree

$$12p^2 - 21pq - 20q^2$$

Section 4-2

Laws of Exponents

Objectives

- to use the laws of exponents to multiply monomials and polynomials
- to solve for unknown exponents

Laws of Exponents

Let a and b be real numbers and m and n be positive integers.

1. When multiplying like bases add the exponents.

   $a^m a^n = a^{m + n}$

2. Power of a product equals the power of each factor in the product.

   $(ab)^m = a^m b^m$

3. Power raised to a power multiply the exponents.

   $(a^m)^n = a^{mn}$

First rule example #1: $3z^2 \bullet 2z^3$

Rule to use: When multiplying like bases add the exponents.

$$a^m a^n = a^{m+n}$$

I. multiply the coefficients: (3)(2) = 6

II. write down the variables in alphabetical order: 6z

III. add all the exponents of the variables 2 + 3 = 5 and set this as the exponent of the variable: $6z^5$

Second rule example #3: $(-t^4)^3$

Rule to use: Power of a product equals the power of each factor in the product. $(ab)^m = a^m b^m$

I. raise the coefficient to the power of the quantity: $(-1)^3 = -1$

II. write down the variable and multiply the exponent of the variable to the exponent of the quantity
$(-1)[t^{(4)(3)}] = (-1)[t^{12}] = -t^{12}$

Third Rule example #9: $(4a^3b^2)^2$

Rule to use: Power raised to a power multiply the exponents.

$$(a^m)^n = a^{mn}$$

I. raise the coefficient to the power of the quantity: $4^2 =$

II. write the variables in alphabetical order: $16(a^3b^2)^2$

III. multiply the exponents of the variable by the exponent of the quantity: $16a^{(3)(2)}b^{(2)(2)} = 16a^6b^4$

Problems 35-38: Solve for n.

I. rewrite all factors with the same base.

II. combine the exponents using the three laws of exponents

III. solve the exponent equation for n.

Example, #37: $(3)(9^{2n}) = (3^{n + 1})^3$

I. $(3)(9^{2n}) = (3^{n + 1})^3$

$(3)(3^2)^{2n} = (3^{n + 1})^3$

II. $(3)(3^{4n}) = (3^{3n + 3})$

$3^{4n + 1} = 3^{3n + 3}$

III. $4n + 1 = 3n + 3$

$n = 2$

35. $3^{5n} = 3^5(3^{2n})^2$

Section 4-3

Multiplying Polynomials

Objectives

- Multiply two binomials by using the FOIL method.
- Distribute monomials and polynomials into other polynomials
- Multiply 3 or more polynomial factors.

Multiplying Polynomials

- When asked to multiply polynomials you must multiply each term of the first factor to each term of the second factor.
- If there are more than two factors, then you may only work with two at a time.
- When both factors are binomials you can simplify the distribution process by using the FOIL (firsts-outsides-insides-lasts) method
- Common FOIL patterns are:
  - $(a + b)^2 = a^2 + 2ab + b^2$
  - $(a - b)^2 = a^2 - 2ab + b^2$
  - $(a + b)(a - b) = a^2 - b^2$

Problems 1-20: Multiply.

Foil to find the product of the binomials.

For example #1 $(3v + 1)(2v - 5)$

$(3v)(2v) + (3v)(-5) + (1)(2v) + (1)(-5)$

$6v^2 - 15v + 2v - 5$

$6v^2 - 13v - 5$

Problems 21-24: Multiply. Assume that variable exponents represent positive integers.

I. Multiply two factors at a time.

II. Foil the binomials and distribute the monomial over the product.

For example, #21 $t(t - 2)(t + 1)$

I. $t(t^2 - t - 2)$

II. $t(t^2) + t(- t) + t(- 2)$

$t^3 - t^2 - 2t$

Problems 25-48: Multiply. Assume that variable exponents represent positive integers.

Distribute every term in the first factor into the second factor.
For example # 31, $(x^2 - x + 2)(x^2 + x - 1)$

$$x^2(x^2) + x^2(x) + x^2(-1) + (-x)(x^2) + (-x)(x) + (-x)(-1) + 2(x^2) + 2(x) + 2(-1)$$

$$x^4 + x^3 - x^2 - x^3 - x^2 + x + 2x^2 + 2x - 2$$

$$x^4 + 3x - 2$$

Section 4-4

Using Prime Factorization

Objectives

- prime factor an integer
- prime factor a monomial
- find the GCF and LCM of integers
- find the GCF and LCM of monomials

Using Prime Factorization

- factor set: any pair of numbers that multiply to make the given product
- prime number: a number whose only factors are 1 and itself
- prime factorization: to break a number down into a product of prime numbers
- greatest common factor (GCF): to find the largest number that divides evenly into any number.
  - first, perform prime factorization
  - second, multiply any factors that appear in the prime factorization of all of the given values.
- least common multiple (LCM): the smallest number that two given values will divide into.
  - first, perform prime factorization
  - second, multiply every factor that appears in each number to its highest degree

Problems 1-8: Find the prime factorization of each integer.

I. Break each number down into its prime factors.

II. Remember even numbers are divisible by 2,

III. Numbers ending in 0 or 5 are divisible by 5

IV. Numbers where the sum of the digits is divisible by 3 are divisible by 3.

For example #1, 140.

$$140 = (2)(70) = (2)(7)(10) = (2)(7)(2)(5) = (2^2)(5)(7)$$

For example, #17 $9p^3q$ & $15p^2$

I. (3)(3)(p)(p)(p)(q) & (3)(5)(p)(p)

II. (3)(p)(p) = $3p^2$

III. (3)(3)(p)(p)(p)(q)(5) = $45p^3q$

Example #27, The GCF of 84 and another integer is 42, and their LCM is 252. Find the other integer.

I. given term = 84, GCF = 42, LCM = 252
$84 = (2)(42) = (2)(2)(21) = (2^2)(3)(7)$
$42 = (2)(21) = (2)(3)(7)$
$252 = (2)(126) = (2)(2)(63) = (2^2)(3)(21) = (2^2)(3)(3)(7)$
$= (2^2)(3^2)(7)$

II. Since the given term only has a single factor of 3 and the LCM has a factor of 3^2 then the missing term must have a factor of 3^2 also.

III. Since the GCF indicates that 2 & 7 appear in both terms and so far we have not put a 2 or a 7 into the list of factors for our missing term we must include them.

IV. The factors of the missing term are: $(2)(3^2)(7)$ = 126

Section 4-5

Factoring Polynomials

Objectives

- to factor polynomials that represent small numbers (2 prime factors)
- to learn and begin to recognize the pattern polynomials
- to learn the process for factoring non-prime polynomials
- to factor polynomials that represent large numbers (3 or more prime factors)

Factoring Polynomials

I. Factor out the GCF if any exists.

II. Look for one of the 5 patterns of factoring.

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

III. Arrange the terms in decreasing order by degree to see if they fit the standard form of a quadratic equation: $ax^2 + bx + c$

IV. Standard form quadratic equations are created by FOILing binomials; therefore, ax^2 comes from the product of Firsts, c comes from the product of Lasts & bx comes from the sum of the Outside and Inside products.

Factoring Polynomials

V. Set up two sets of parentheses.

VI. In the Firsts position write the factors that will have the product ax^2.

VII. Multiply (a)(c) and find a factor pair of that product with a sum of b. The numbers in the factor pair represent either your Outside or Inside products.

VIII. Insert the necessary values into the Lasts positions so that the product of your Outsides and Insides are the numbers in your factor pair.

IX. Check your answer by reFOILing. You should get the original polynoial back.

Problems 1-20: Factor each polynomial. Some will have a GCF, but all will follow one of the 5 patterns.

Examples of patterns:

#3: $t^2 + 18t + 81 = t^2 + 2(9)(t) + 9^2 = (t + 9)^2$

$a^2 + 2(a)(b) + b^2 = (a + b)^2$

#11: $121s^2 - 66st + 9t^2 = (11s)^2 - 2(11s)(3t) + (3t)^2 = (11s - 3t)^2$

$a^2 - 2(a)(b) + b^2 = (a - b)^2$

#5: $16k^2 - 1 = (4k)^2 - 1^2 = (4k + 1)(4k - 1)$

$a^2 - b^2 = (a + b)(a - b)$

#17: $t^3 - 27 = t^3 - 3^3 = (t - 3)(t^2 + 3t + 9)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

#19: First factor out GCF: $2rs(8r^3 + s^3)$

Then $2rs[(2r)^3 + s^3] = 2rs(2r + s)(4r^2 - 2rs + s^2)$

$2rs[a^3 + b^3] = 2rs(a + b)(a^2 - ab + b^2)$

Problems 21-24: Factor each polynomial.

Remember a GCF can be a quantity so long as that quantity is a factor of the term. Follow the steps for factoring polynomials.

For example #21, $x(y - 3) + 2(y - 3)$.

GCF: $(y - 3)$

Factoring out GCF leaves $(y - 3)(x + 2)$

Sample Problem #23: $x(y - 3) + 2(3 - y)$

Problems 25-32: Factor each polynomial.

- If there is a term with both variables in it, then group them into binomials that have GCF's.
- For example #25, $pq - 2q + 2p - 4$ has a term with both variables.

  First: Regroup so that $(pq - 2q) + (2p - 4)$

  Second: Factor out GCF from binomials to get

  $q(p - 2) + 2(p - 2)$

  Third: Factor out GCF quantity to get $(p - 2)(q + 2)$
- Sample Problem: #27, $ab - 2 - 2b + a$

Problems 25-32: Factor each polynomial.

- If there is not a term with both variables but there is a term with one of the variables squared, then group them into a monomial and a quadratic trinomial that can be factored.
- For example #29, $x^2 - 6x + 9 - 4y^2$ does not have a term with two variables.

  First: Regroup so that $(x^2 - 6x + 9) - 4y^2$

  Second: Factor the trinomial$(x - 3)^2 - 4y^2$

  Third: Factor a second time $(x - 3)^2 - (2y)^2 =$

  $[(x - 3) + 2y][(x - 3) - 2y]$
- Sample Problem: #31, $u^2 - v^2 + 2v - 1$

Problems 33-46: Factor each polynomial.

- Look for patterns within patterns or ways to simplify the initial problem through substitution.
- For example #33, $x^4 - 2x^2y + y^2$.

I. Let $w = x^2$ so that the problem then becomes $w^2 - 2wy + y^2$.

II. You can simplify the problem further by temporarily removing the y's so that the problem becomes $w^2 - 2w + 1$ which factors to $(w - 1)^2$

- Now that it is factored we need to begin to rebuild the original problem.

I. First replace the y's so that the answer becomes $(w - y)^2$.

II. Second, replace the w's with x^2 so the answer is $(x^2 - y)^2$.

Section 4-6

Non-Pattern Factoring

Objectives

- to factor non-pattern polynomials
- to find GCF of two polynomials
- to find LCM of two polynomials

Factoring Quadratic Polynomials

- to factor quadratic polynomials use the same process that we described in the previous section.
- standard form of a quadratic equation: $y = ax^2 + bx + c$ or
  $f(x) = ax^2 + bx + c$
  - ax^2 is the quadratic term
  - bx is the linear term
  - c is the constant
- A polynomial is prime (unfactorable) if no factor pair of the product ac can be found that has a sum of b.

Example: # 25, $3p^2 - 7p - 6$

I. $3p^2 - 7p - 6$

II. (ac) = (3)(- 6) = - 18

factor pairs: {1, - 18} {- 1, 18} {2, - 9} {- 2, 9} {3, - 6} {- 3, 6}

{2, - 9} is the factor pair we want to use because 2 + (- 9) = - 7

III. (3p   )(p   )

IV. (3p + 2)(p - 3)

V. (3p)(p) + (3p)(- 3) + (2)(p) + (2)(- 3)

$3p^2 - 9p + 2p - 6$

$3p^2 - 7p - 6$

Problems 45-50: Find (a) the GCF and (b) the LCM of the given polynomial.

I. Factor each polynomial.

II. Find the GCF and the LCM for the polynomials the same way that you did for the numbers on page 181.

Example: #45, $x^2 - 3x + 2$; $x^2 - 4x + 4$

I. $(x - 1)(x - 2)$; $(x - 2)(x - 2)$

II. The only factor that appears in both is $(x - 2)$

III. Write down all the factors of the first polynomial and then any in the second polynomial that you don't already have $(x - 1)(x - 2)(x - 2)$

Section 4-7

Solving Polynomial Equations

Objectives

- to solve polynomial equations
- to solve polynomial equations with fractional coefficients
- to solve polynomial equations written in function form
- to solve polynomial equations with variable coefficients.

Solving Polynomial Equations

- a polynomial equation describes a curved line
- the roots of a polynomial equation are the numbers where the graph of the curve crosses the x-axis; therefore, they are the values of the variable that make the value of the polynomial equal to 0.
- Steps for solving:
  - move all terms to one side--the side where the quadratic term will be positive. If you are moving variables you may have to expand and then re-factor.
  - factor the polynomial side of the equation
  - set each factor equal to 0 and solve for the variable

Example #21, $(y - 4)^2 = 2y$

I. $y^2 - 8y + 16 = 2y$

II. $y^2 - 8y - 2y + 16 = 0$

III. $y^2 - 10y + 16 = 0$

IV. $(y - 2)(y - 8) = 0$

V. Either $y - 2 = 0$ or $y - 8 = 0$; therefore, $y = 2$ or 8

Section 4-8

Problem Solving Using Polynomial Equations

Objectives

- to solve simplified applications of quadratic polynomials.

Problem Solving with Polynomial Equations

- all of the problems in this section require that you set up a quadratic equation that is factorable.
- most of the equations will have you establish the quadratic by substituting into a product of variables.
- make sure that the answers you provide make sense with what the word problem asks.   Not all algebraic solutions apply in the real world.
- make sure you read through the section because it has formulas which you need for some of the problems

Problem Solving with Polynomial Equations

- Make sure that you use the 5 step process to solve these problems.
- Make sure that you are translating correctly by looking back at your chapter 1 notes.
- Make sure you are using the correct formulas by reading the section or recalling them from Geometry.

Section 4-9

Polynomial Inequalities

Objectives

- find the solution to a polynomial inequality
- graph the solution of a polynomial inequality

Solving Polynomial Inequalities

- move every thing to one side of the inequality and factor as you would a polynomial equation.
- if the polynomial is greater than 0, then find the combinations of (+) and (-) factors that will make a positive product.
- if the polynomial is less than 0, then find the combinations of (+) and (-) factors that will make a negative product.
- to say that a factor is negative is to say that it is less than 0
- to say that a factor is positive is to say that it is greater than 0

Example #11, $t^2 > 9(t - 2)$

I. $t^2 - 9t + 18 > 0$ which factors to $(t - 6)(t - 3) > 0$

II. the product is positive

III. (+)(+) or (-)(-)

IV. $t - 6 > 0$ and $t - 3 > 0$ OR $t - 6 < 0$ and $t - 3 < 0$

$t > 6$ and $t > 3$ OR $t < 6$ and $t < 3$

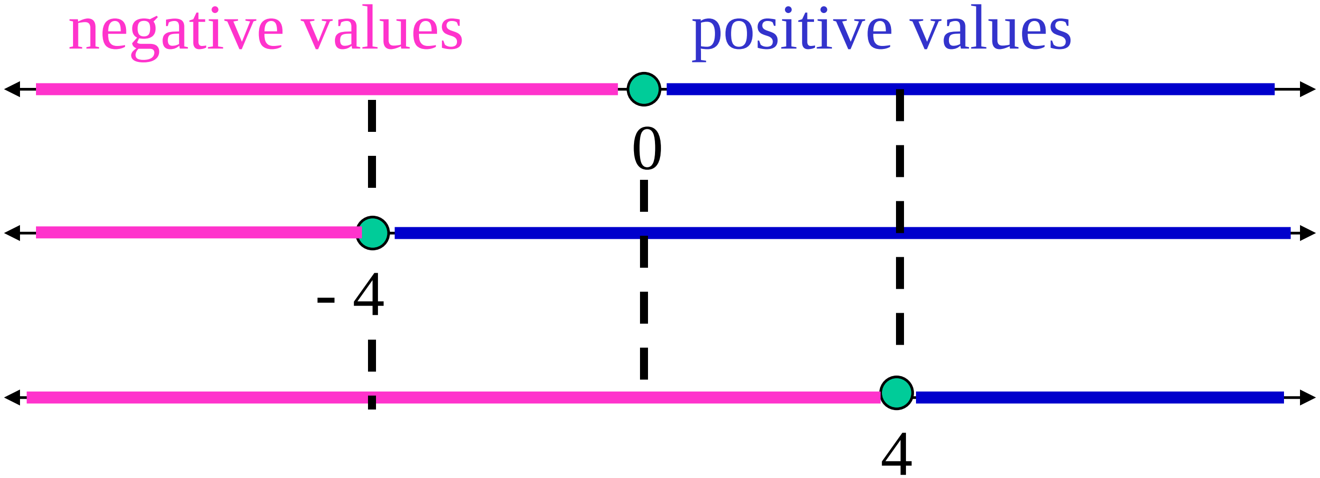
V.

Example # 19, $x^3 - 16x > 0$

I. $x^3 - 16x > 0$

II. $x(x + 4)(x - 4)$

III. x



IV. The product is positive so that answers will only be in regions where all three number lines indicate positive values or two negative values.