Section 10-1

Rational Exponents

Objectives

- to simplify rational exponents
- to convert radicals to rational exponents
- to simply radicals by using rational exponents
- to solve radical equations by using rational exponents

Rational Exponents

- All radicals can be written as a rational exponent.
  - The numerator of the exponent is the exponent of the radicand (base).
  - The denominator of the exponent is the index of the radical.

$$\sqrt[q]{b^{p}} = b^{\frac{p}{q}}$$

Example for 1-20

$$\left(16^{-5}\right)^{\frac{1}{20}}$$

$$\left(2^{4}\right)^{-\frac{1}{4}}$$

$$2^{-1}$$

$$\frac{1}{2}$$

Example for 21-28

$$\sqrt[4]{\frac{16^{3}a^{-2}}{b^{6}}}$$

$$\frac{2^{3}a^{-\frac{1}{2}}}{b^{\frac{3}{2}}}$$

$$\sqrt[4]{\frac{2^{12}a^{-2}}{b^{6}}}$$

$$8a^{-\frac{1}{2}}b^{-\frac{3}{2}}$$

$$\frac{2^{\frac{12}{4}}a^{-\frac{2}{4}}}{b^{\frac{6}{4}}}$$

Example for 29-36

$$\sqrt[4]{27} \bullet \sqrt[8]{9}$$

$$\sqrt[4]{3^3} \bullet \sqrt[8]{3^2}$$

$$3^{\frac{3}{4}} \bullet 3^{\frac{1}{4}}$$

$$3^{\frac{3}{4}+\frac{1}{4}}$$

3

Example for 37-42

$$a^{\frac{1}{2}}\left(a^{\frac{3}{2}}-2a^{\frac{1}{2}}\right)$$

$$a^{\frac{1}{2}}\bullet a^{\frac{3}{2}}-2a^{\frac{1}{2}}\bullet a^{\frac{1}{2}}$$

$$a^{2}-2a$$

Example for 43-50

$$\left(x^2+4\right)^{\frac{2}{3}}=25$$

$$\left[\left(x^2+4\right)^{\frac{2}{3}}\right]^{\frac{3}{2}}=(25)^{\frac{3}{2}}$$

$$x^2+4=125$$

$$x^2=121$$

$$x=\pm 11$$

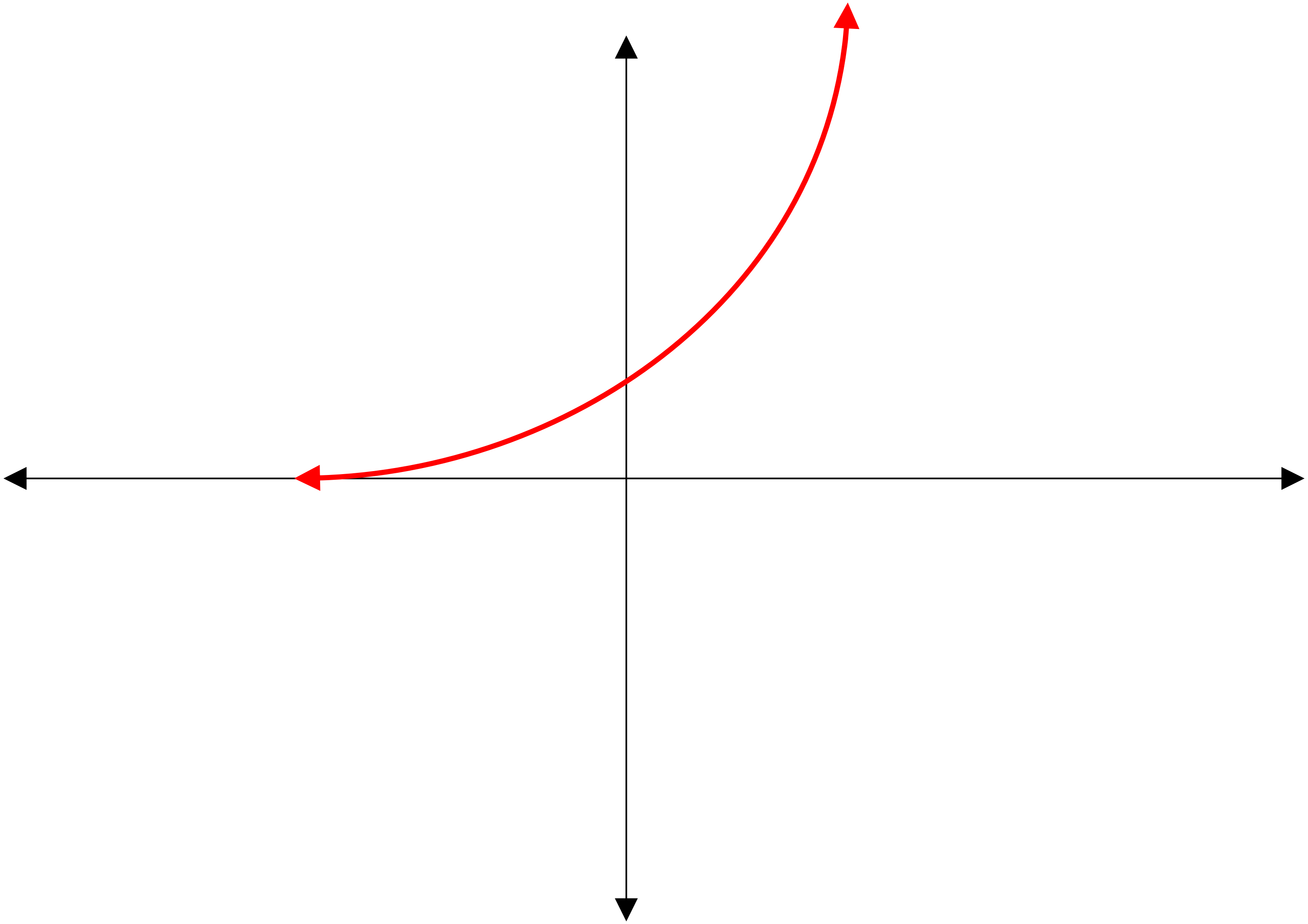
Section 10-2

Real Number Exponents

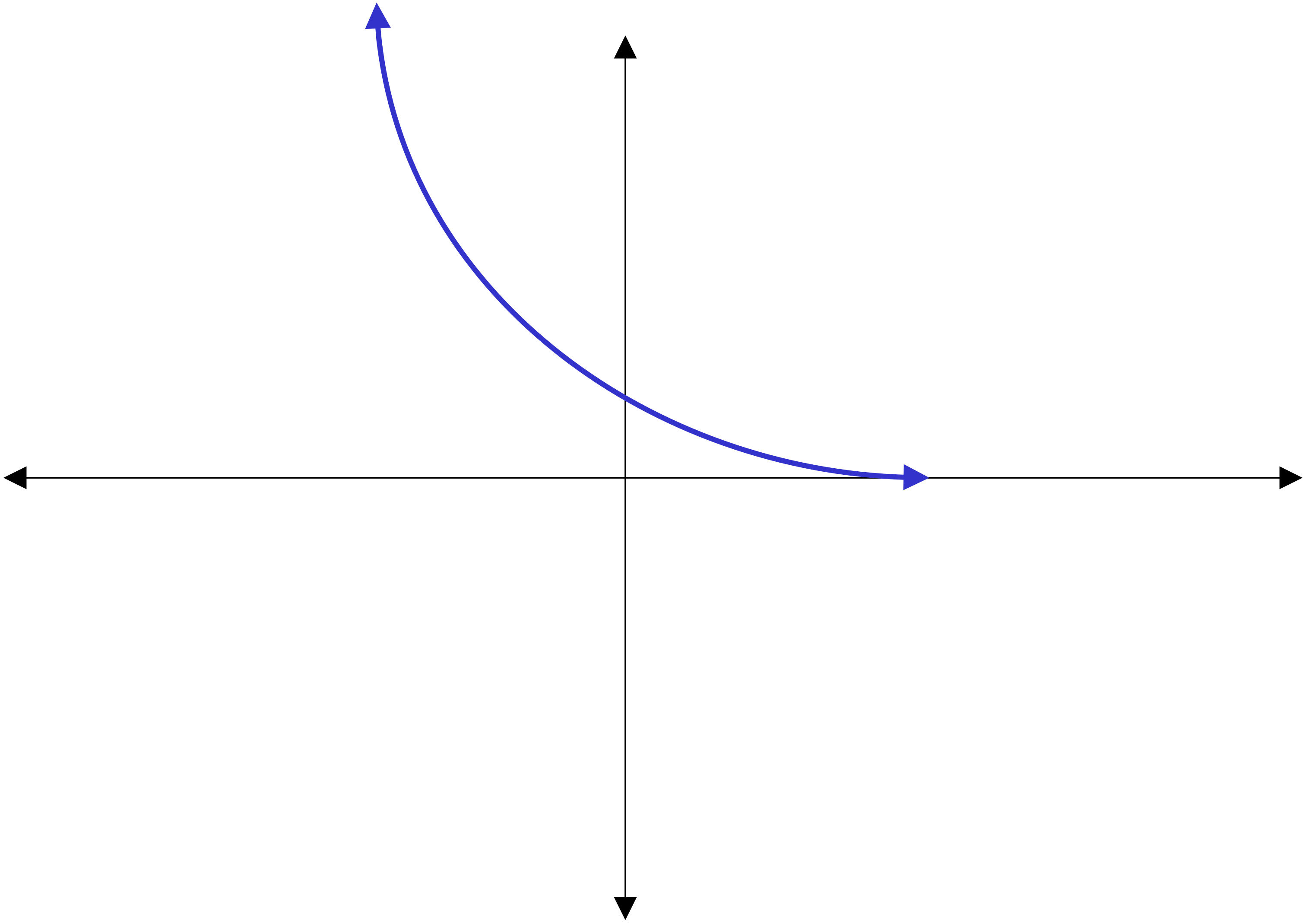
Objectives

- to simplify radical exponents
- to simplify real exponents
- to solve equations with real exponents
- to graph exponential equations

Graph of $y = b^x$ if $b > 1$



Graph of $y = b^x$ if $0 < b < 1$



Example for 1 & 2

$$3^{\sqrt{2}} \bullet 3^{\sqrt{2}} = 3^{2\sqrt{2}} = 9^{\sqrt{2}}$$

$$\left(3^{\sqrt{2}}\right)^{2} = 3^{2\sqrt{2}} = 9^{\sqrt{2}}$$

$$\left(3^{\sqrt{2}}\right)^{\sqrt{2}} = 3^{2} = 9$$

$$\frac{3^{\sqrt{2}+2}}{3^{\sqrt{2}-2}} = 3^{\sqrt{2}+2-\sqrt{2}+2} = 3^{4} = 81$$

Example for 3-10

$$\sqrt{6^{2\pi}}$$

$$\left(6^{2\pi}\right)^{\frac{1}{2}}$$

$$6^{\pi}$$

Example for 11-18

$$\frac{\left(1+\sqrt{3}\right)^{\pi-1}}{\left(1+\sqrt{3}\right)^{\pi+1}}$$

$$\left(1+\sqrt{3}\right)^{\pi-1-\pi-1}$$

$$\left(1+\sqrt{3}\right)^{-2}$$

$$\frac{1}{\left(1+\sqrt{3}\right)^{2}}$$

$$\frac{1}{4+2\sqrt{3}}$$

$$\frac{1}{4+2\sqrt{3}}\left(\frac{4-2\sqrt{3}}{4-2\sqrt{3}}\right)$$

$$\frac{4-2\sqrt{3}}{16-12}$$

$$\frac{4-2\sqrt{3}}{4}$$

$$\frac{2-\sqrt{3}}{2}$$

Example for 19-29

$$25^{2x} = 5^{x+6}$$

$$\left(5^{2}\right)^{2x} = 5^{x+6}$$

$$5^{4x} = 5^{x+6}$$

$$4x = x + 6$$

$$3x = 6$$

$x = 2$

Example for 30-32

$$4^{2x} - 63 \bullet 4^{x} - 64 = 0$$

$$4^{x} = y$$

$$y^{2} - 63y - 64 = 0$$

$$(y - 64)(y + 1) = 0$$

$$y = 64 \text{ or } y = -1$$

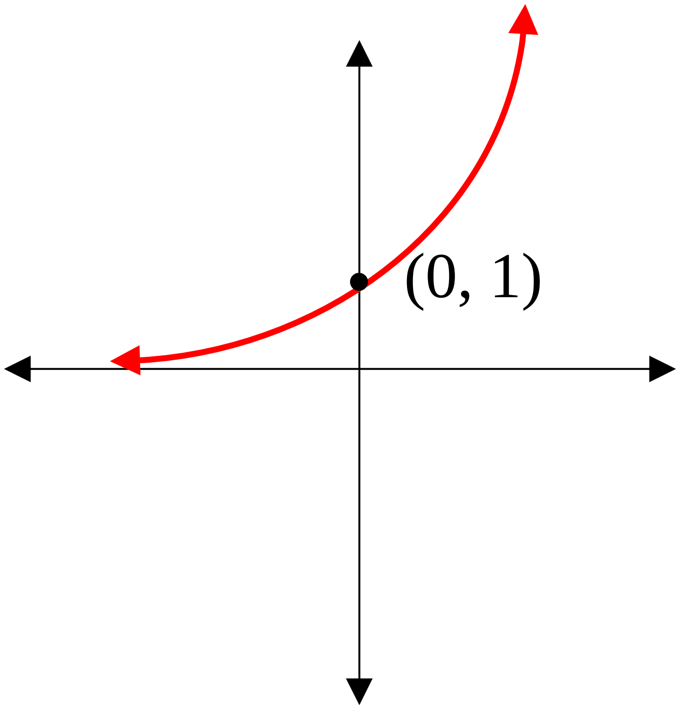
$$4^{x} = 64 \text{ or } 4^{x} = -1$$

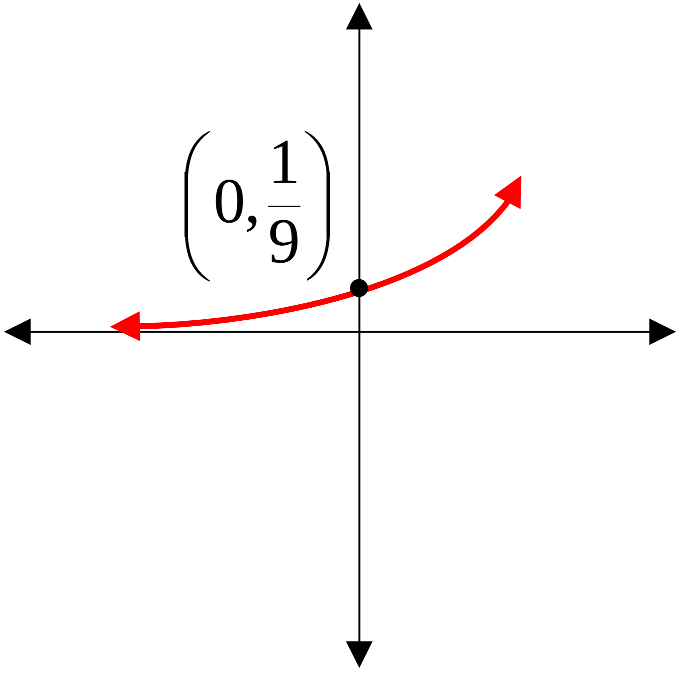
$$4^{x} = 4^{3} \text{ or } \phi$$

$x = 3$

Example for 33-36

$$y = 3^{x} \quad \text{and } y = 3^{x-2}$$





Section 10-3

Composition and Inverses of Functions

Objectives

- to find the composites of exponential equations
- to test if an exponential relation is a function
- to reflect into the line $y = x$ to graph the inverse of a relation
- to determine if an inverse of a relation is a function
- to write and graph the equation of an inverse

Inverse Equations

- Two functions f and g are inverse functions if:
  - $f(g(x)) = x$ for all x in the domain of g
  - $g(f(x)) = x$ for all x in the domain of f.
- $f^{-1}(x)$ denotes the inverse function of f(x)
- A function f(x) will have an inverse function $f^{-1}(x)$ if and only if the graph of f(x) can pass a horizontal line test.
- To find the inverse equation switch the positions of x and y in your equation and then isolate y.

Example for 1-6

$$f(x)=\frac{x}{2},\ \ g(x)=x-3,\ \ h(x)=\sqrt{x}$$

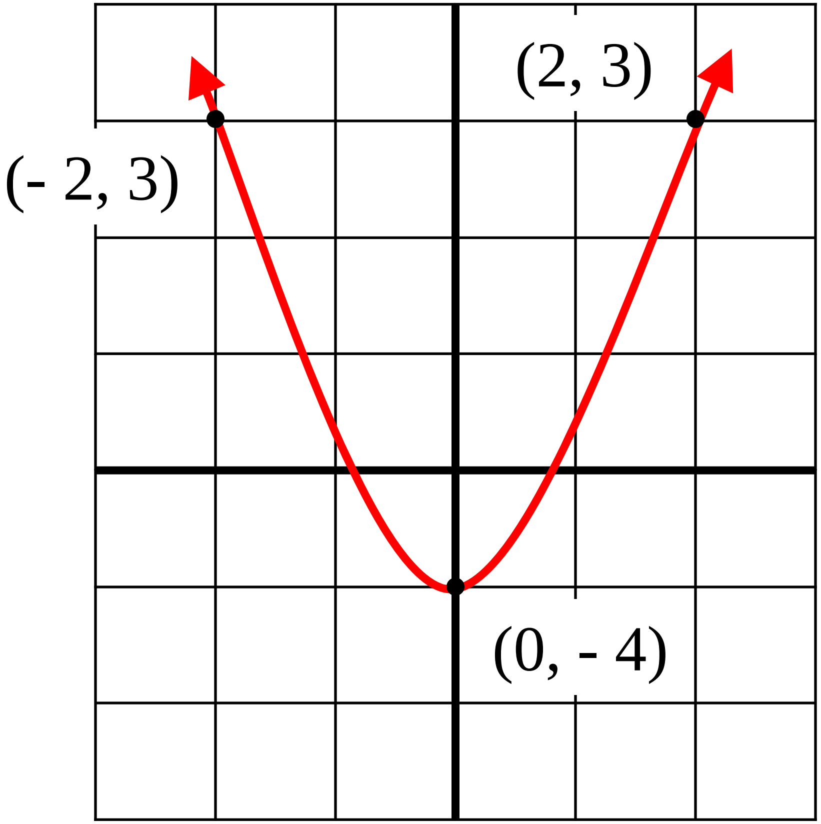
$$f(g(8))=\frac{8-3}{2}=\frac{5}{2}$$

$$f(g(-5))=\frac{-5-3}{2}=-4$$

$$f(g(0))=\frac{0-3}{2}=-\frac{3}{2}$$

$$f(g(x))=\frac{x-3}{2}$$

Example for 7-10



f(x) is a function

$f^{-1}(x)$ is not a function

Example for 11-14

$$f(x) = 2x - 3$$

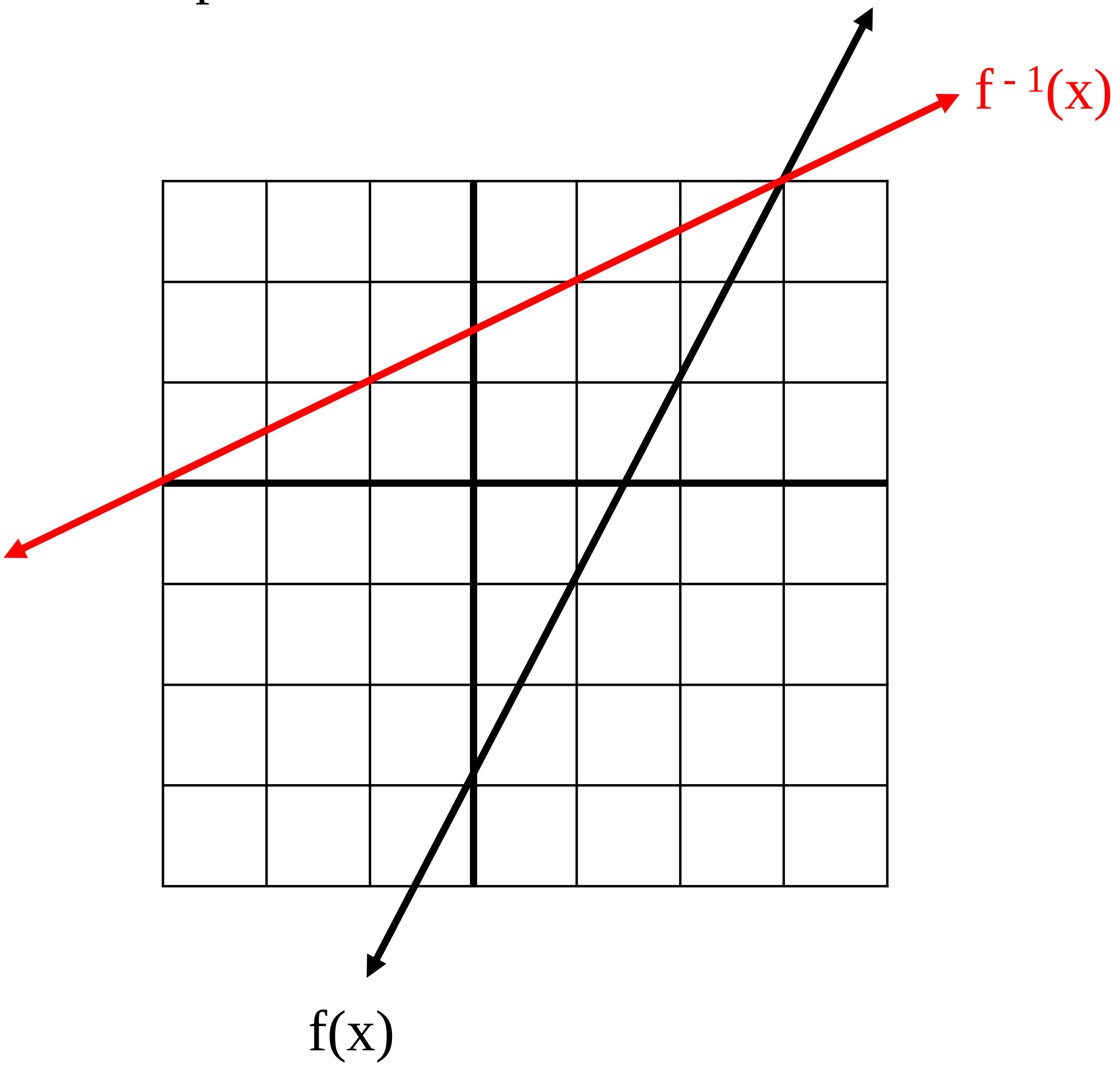
$$y = 2x - 3$$

$$x = 2y - 3$$

$$x + 3 = 2y$$

$$\frac{x+3}{2} = y$$

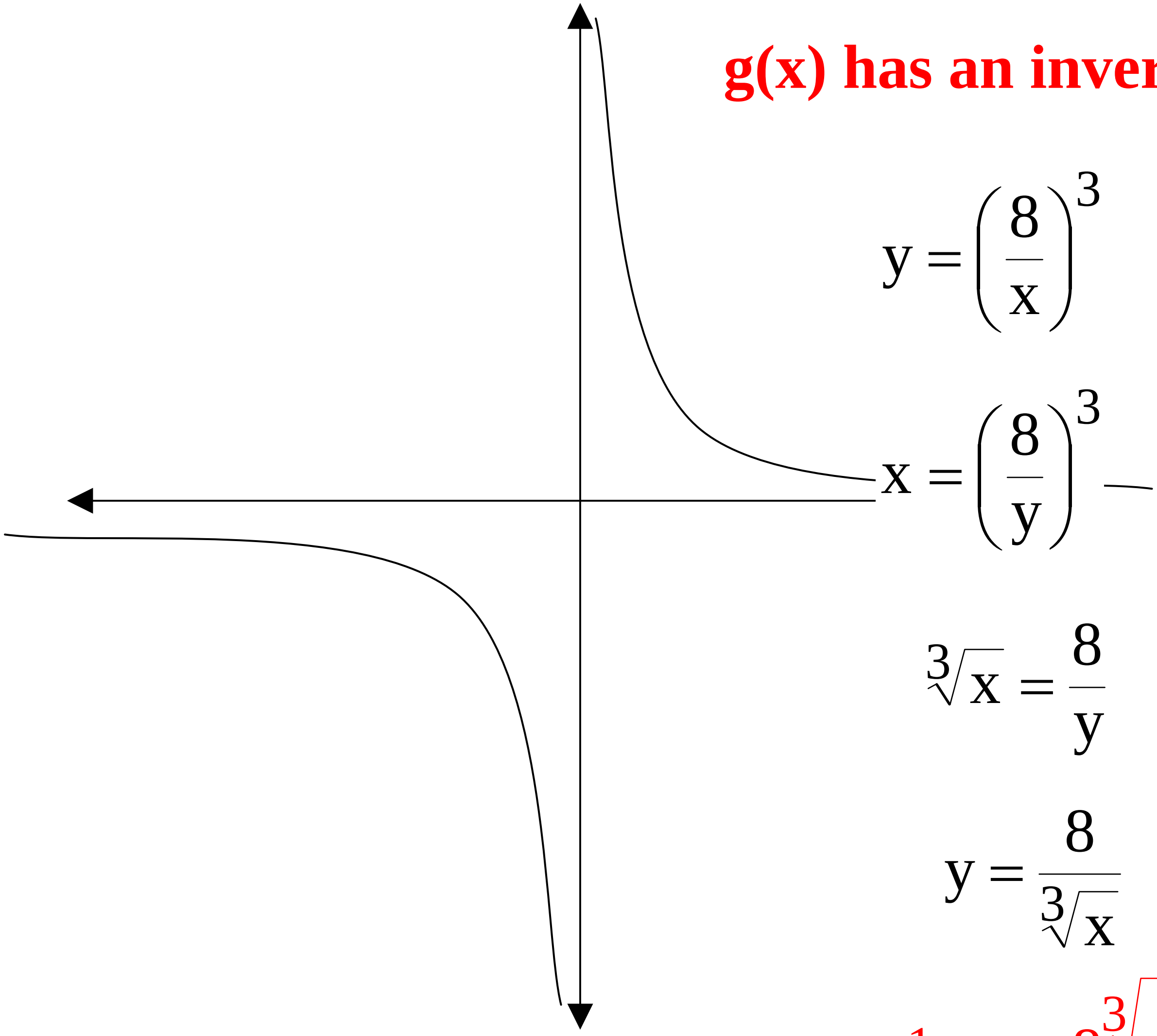
$$f^{-1}(x) = \frac{x+3}{2}$$



Example for 15-22

$$g(x) = \left(\frac{8}{x}\right)^3$$

x	g(x)
- 8	- 1
- 4	- 8
- 2	- 64
- 1	- 512
0	ud
1	512
2	64
4	8
8	1



g(x) has an inverse

$$y = \left(\frac{8}{x}\right)^3$$

$$x = \left(\frac{8}{y}\right)^3$$

$$\sqrt[3]{x} = \frac{8}{y}$$

$$y = \frac{8}{\sqrt[3]{x}}$$

$$g^{-1}(x) = \frac{8\sqrt[3]{x^2}}{x}$$

Section 10-4

Definition of Logarithms

Objectives

- to simplify a logarithm
- to solve for the value of a logarithm
- to solve for the base of the logarithm
- to write inverse equations of logarithms
- to identify the domain and range of logarithm equations
- to graph logarithmic equations

Logarithms

- Logarithms are another way of writing exponents.
- Logarithms follow all of the rules that exponents follow.
- If b and N are positive numbers ($b \neq 1$), then $\log_b N = k$ if and only if $b^k = N$.
- Properties of logarithms:

$$\log_b N = \log_b M \text{ if and only if } N = M$$

$$\log_b b^k = k$$

$$b^{\log_b N} = N$$

$$\log_b b = 1$$

$$\log_b 1 = 0$$

Example for 1-18

$$\log_6 6\sqrt{6}$$

$$\log_6 6\sqrt{6} = x$$

$$6^x = 6\sqrt{6}$$

$$6^x = \left(6^1\right)\left(6^{\frac{1}{2}}\right)$$

$$6^x = 6^{\frac{3}{2}}$$

$$x = \frac{3}{2}$$

Example for 19-24

$$\log_9 x = -\frac{1}{2}$$

$$9^{-\frac{1}{2}} = x$$

$$\frac{1}{9^{\frac{1}{2}}} = x$$

$$\frac{1}{\sqrt{9}} = x$$

$$\frac{1}{3} = x$$

Example for 25-30

$$\log_x 7 = -\frac{1}{2}$$

$$x^{-\frac{1}{2}} = 7$$

$$\left(x^{-\frac{1}{2}}\right)^{-2} = (7)^{-2}$$

$$x = \frac{1}{7^2}$$

$$x = \frac{1}{49}$$

Example for 33 & 34

$$f(x) = 6^x$$

$$y = 6^x$$

$$x = 6^y$$

$$\log_6 x = y$$

$$f^{-1}(x) = \log_6 x$$

$$f^{-1}(36) = \log_6 36$$

$$y = \log_6 36$$

$$6^y = 36$$

$$6^y = 6^2$$

$$y = 2$$

$$f^{-1}\left(\frac{1}{\sqrt{6}}\right) = \log_6\left(\frac{1}{\sqrt{6}}\right)$$

$$y = \log_6\left(\frac{1}{\sqrt{6}}\right)$$

$$6^y = \frac{1}{\sqrt{6}}$$

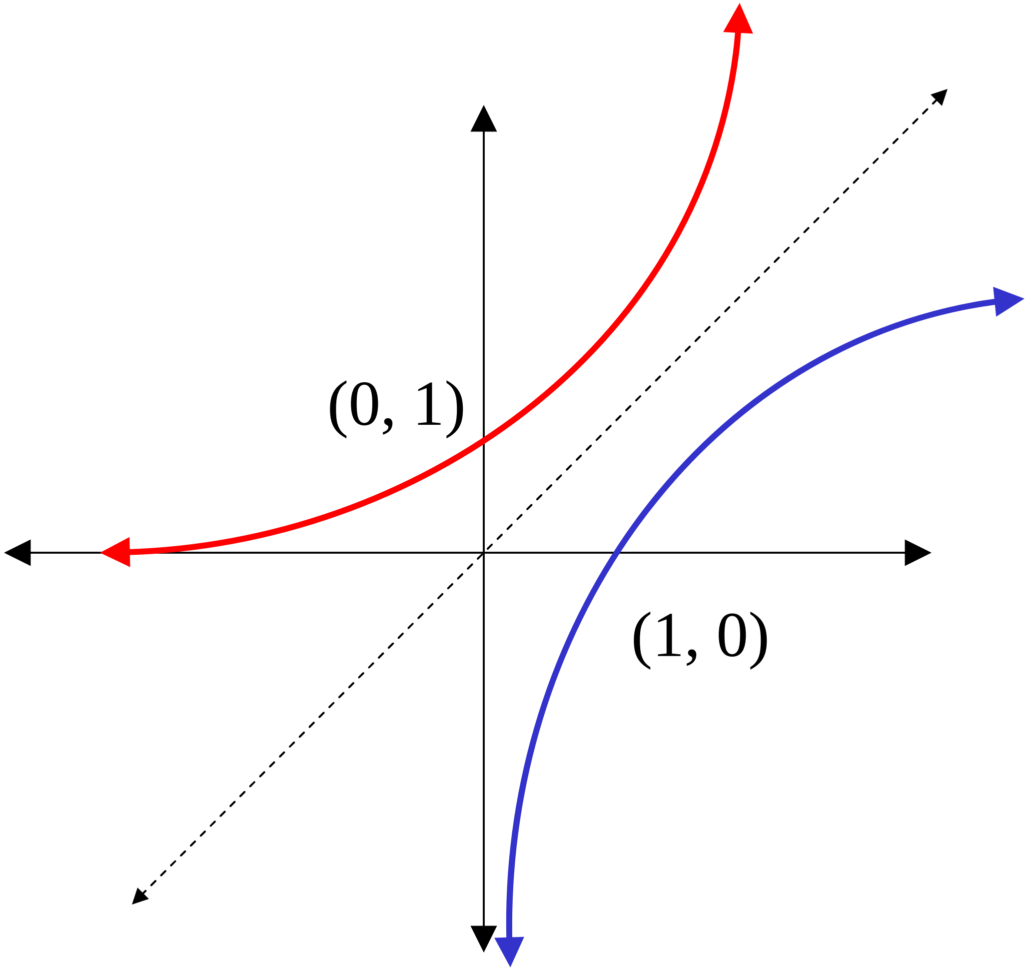
$$6^y = 6^{-\frac{1}{2}}$$

$$y = -\frac{1}{2}$$

Example for 35-38

$f(x) = 6^x$

$f^{-1}(x) = \log_6 x$



Section 10-5

Laws of Logarithms

Objectives

- to rewrite powers, products and quotients of logarithms
- to use powers, products and quotients to evaluate logarithms
- to express a logarithm as a single value
- to simplify logarithmic statements
- to solve logarithmic equations

Laws of Logarithms

- $\log_b MN = \log_b M + \log_b N$
- $\log_b \frac{M}{N} = \log_b M - \log_b N$
- $\log_b M^k = k \log_b M$

Example for 1-8

$$\log_2 \sqrt{\frac{M}{N^3}} \qquad \log_2 M^{\frac{1}{2}} - \log_2 N^{\frac{3}{2}}$$

$$\log_2 \left(\frac{M}{N^3}\right)^{\frac{1}{2}} \qquad \frac{1}{2}\log_2 M - \frac{3}{2}\log_2 N$$

$$\log_2 \left(\frac{M^{\frac{1}{2}}}{N^{\frac{3}{2}}}\right)$$

Example for 9-20

$$\log_{10} \frac{20}{9}$$

$$\log_{10} 20 - \log_{10} 9$$

$$\log_{10} (2)(10) - \log_{10} 9$$

$$\log_{10} 2 + \log_{10} 10 - \log_{10} 9$$

$$0.3 + 1 - 0.95$$

0.35

Example for 21-28

$$1 - 3\log_5 x$$

$$\log_5 5 - 3\log_5 x$$

$$\log_5 5 - \log_5 x^3$$

$$\log_5 \frac{5}{x^3}$$

Example for 29-32

$$2\log_{10} 5 + \log_{10} 4$$

$$\log_{10} 5^2 + \log_{10} 4$$

$$\log_{10} 25 + \log_{10} 4$$

$$\log_{10} (25)(4)$$

$$\log_{10} 100$$

2

Example for 33-40

$$\log_a x - \log_a (x-5) = \log_a 6$$

$$\log_a \frac{x}{x-5} = \log_a 6$$

$$\frac{x}{x-5} = 6$$

$$6x - 30 = x$$

$$5x = 30$$

$$\mathbf{x = 6}$$

Example for 41 & 42

If $f(x) = \log_2 x$ and $g(x) = 4^x$ find $f(g(3))$, $g\left(f\left(\frac{1}{2}\right)\right)$, $f(g^{-1}(16))$

$\log_2 4^3$

$3\log_2 4$

$(3)(2)$

6

$4^{\log_2 \frac{1}{2}}$

4^{-1}

$\frac{1}{4}$

$\log_2(\log_4 16)$

$\log_2 2$

1

Section 10-6

Applications of Logarithms

Objectives

- to evaluate powers to three significant digits
- to evaluate logarithms to three significant digits
- to solve exponential equations with logarithms and to evaluate answers to three significant digits
- to solve exponential equations to exact values
- to evaluate to three significant digits non-base ten logarithms

Applications of Logarithms

- A common logarithm is base 10.
- The values of common logs can be accessed through a table like the one in the back of your textbook (p 812) or by accessing the same information on your calculator through the “log” button.
- To use the table to find common logs:
  - Write the number in scientific notation.
  - The decimal portion of the number should be between 0 and 10. Look up the first two digits in the column marked N; find the column marked with the third digit.
  - Write the 4 digit decimal from the table. This is your mantissa.
  - Add this decimal to the power of 10 (the characteristic) in your scientific number.
  - The result is the value of your logarithm.

Change of Base Formula

- To convert all other logs to a common log so that you can solve for their value using either a table or a calculator use the change of base formula.

$$\log_a x = \frac{\log_b x}{\log_b a}$$

- Most often you will have b = 10 because common logs can be solved with a calculator, but you can use this formula to convert to any base value.

Example for 1-8

$(0.38)^5$

$\left(3.8\text{x}10^{-1}\right)^5$

$3.8^5 \text{ x } 10^{-5}$

$7.92 \text{ x } 10^2 \text{ x } 10^{-5}$

0.00792

$(0.38)^5$

$3.8^5 \text{ x } 10^{-5}$

Let $x = 3.8^5$

$\log x = 5 \log 3.8$

$\log x = 5(0.5798)$

$\log x = 2.899$

$x = \text{antilog } .8990 \text{ x } 10^2$

$x = 7.92 \text{ x } 10^2 \text{ x } 10^{-5}$

0.00792

Example for 9-14

$\log x = 0.8531$

$x = \text{antilog } 0.8531$

Find the closest number to 8531 on the table and then identify its mantissa 713.

$x = 7.13$

$\log x = -1.8$

Rewrite the problem as a sum of its characteristic and its mantissa;
$\log x = -2 + 0.2$

Find the closest number to 2000 on the table and then identify its mantissa 158.

Rewrite the characteristic as a power of 10: 10^{-2}

$\log x = \log 10^{-2} + \log 1.58$

$\log x = \log (1.58 \times 10^{-2})$

$x = 1.58 \times 10^{-2} = 0.0158$

Example for 15-22

$$3^x = 30$$

$$\log 3^x = \log 30$$

$$x \log 3 = \log 30$$

$$x = \frac{\log 30}{\log 3}$$

$$\mathbf{x = 3.10}$$

Example for 23-26

$$4^{x} = 8\sqrt{2}$$

$$\left(2^{2}\right)^{x} = \left(2^{3}\right)\left(2^{\frac{1}{2}}\right)$$

$$2^{2x} = 2^{\frac{7}{2}}$$

$$2x = \frac{7}{2}$$

$$x = \frac{7}{4}$$

Example for 27-34

$$\sqrt[3]{x^4} = 60$$

$$x^{\frac{4}{3}} = 60$$

$$\left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} = 60^{\frac{3}{4}}$$

$$x = 60^{\frac{3}{4}}$$

$$\log x = \log 60^{\frac{3}{4}}$$

$$\log x = \frac{3}{4} \log 60$$

$$\log x = \frac{3}{4}(1.7782)$$

$$\log x = 1.3337$$

$$x = \text{antilog } 1.3337$$

$$\mathbf{x = 21.6}$$

Example for 35-38

$$\log_2 9$$

$$\frac{\log 9}{\log 2}$$

$$\frac{0.9542}{0.3010}$$

3.17

Example for 39 & 40

$$3^{2x} - 7 \bullet 3^{x} + 10 = 0$$

$$y = 3^{x}$$

$$y^{2} - 7y + 10 = 0$$

$$(y - 2)(y - 5) = 0$$

$$y = 2 \text{ or } 5$$

$$3^{x} = 2 \text{ or } 3^{x} = 5$$

$$\log 3^{x} = \log 2 \text{ or } \log 3^{x} = \log 5$$

$$x \log 3 = \log 2 \text{ or } x\log 3 = \log 5$$

$$x = \frac{\log 2}{\log 3} \text{ or } x = \frac{\log 5}{\log 3}$$

$$x = \frac{0.3010}{0.4771} \text{ or } x = \frac{0.6990}{0.4771}$$

x = 0.631 or 1.46

Section 10-7

Exponential Growth and Decay

Objectives

- to calculate compound interest
- to calculate compound depreciation
- to calculate exponential growth
- to calculate exponential decay

Exponential Formulas

- Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

  - (A)mount \$ coming out of the account at the maturity date of the investment
  - (P)rincipal \$ put into account at the beginning of the investment period
  - (r)ate of annual interest
  - (n)umber of times in a year the interest is calculated
  - (t)ime the investment takes to mature

Exponential Formulas

- Compound Depreciation

$$A = P\left(1 - \frac{r}{n}\right)^{nt}$$

  - (A)mount $ coming out of the account at the maturity date of the investment
  - (P)rincipal $ put into account at the beginning of the investment period
  - (r)ate of annual interest
  - (n)umber of times in a year the interest is calculated
  - (t)ime the investment takes to mature

Exponential Formulas

- Monthly Loan Payments & Total Cost of a Loan

$$\text{Payment} = P \div \left[\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \right]$$

  - (P)rincipal $ put into account at the beginning of the investment period
  - (r)ate of annual interest
  - (n)umber of times in a year the interest is calculated
  - (t)ime the investment takes to mature
- The cost of a loan is the product of the monthly payment, the number of times in a year the interest is calculated and the total time of the loan

Exponential Formulas

- Exponential Growth (population doubling)

$$N = N_0 \bullet 2^{\frac{t}{d}}$$

  - (N)umber of individual in the population at the end of the period of growth
  - (N_0)umber of individuals in the population at the beginning of the period of growth
  - (t)ime the population is allowed to grow
  - (d)oubling time it takes to complete one generational cycle of growth
- 2 is the constant because our populations will be doubling. The constant can be any value > 1.

Exponential Formulas

- Exponential Decay (half-life)

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

  - (N)umber of individual in the population at the end of the period of growth
  - (N_0)umber of individuals in the population at the beginning of the period of growth
  - (t)ime the population is allowed to grow
  - (h)alf-life it takes to complete one cycle of decay
- ½ is the constant because we are calculating half-life. The constant can be any value between 0 and 1, exclusive.

Example for Compound Interest

You invest $50 into an account bearing 6% interest compounded quarterly and leave it there for six years (from age 16 to 21)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = what we are asked to find

P = 50

r = . 06

n = 4

t = 6

$$A = 50\left(1 + \frac{0.06}{4}\right)^{(4)(6)}$$

$$A = 50(1.015)^{24}$$

A = $71.48

If you added $50 to this account each week at the end of the six years you would now have$21,444. Earning yourself $5,844 in interest.

If you deposited $100 into this account initially and each week at the end of the six years you would have$42,888. Earning yourself $11,688 in interest.

If you deposited $50 a week but the interest rate was 20% then after 6 years you would have $28,770; earning yourself $13,170 in interest.

If you deposited $100 a week but the interest rate was 20% then after 6 years you would have $57,540; earning yourself $26,340 in interest.

If you contributed $50 at 6% from age 16 to 75 (60 years) then you would have $1,703,090.50; earning $1,547,090.50 in interest.

If you contributed $100 at 6% from age 16 to 75 (60 years) then you would have $3,406,181; earning $3,094,181 in interest.

If you contributed $50 at 20% from age 16 to 75 (60 years) then you would have $1,521,700,000.

If you contributed $100 at 20% from age 16 to 75 (60 years) then you would have $3,043,400,000.

Average annual return as of 11/06/2003

1 Year	5 Year	10 Year	Since Acct Open
2.92%	N/A	N/A	2.84%

Average annual return as of 11/06/2003

1 Year	5 Year	10 Year	Since Acct Open
38.88%	N/A	N/A	19.77%

Average annual return as of 11/06/2003

1 Year	5 Year	10 Year	Since Acct Open
18.85%	N/A	N/A	9.85%

Example for Compound Depreciation

The value of a new \$12,500 automobile decreases 20% per year. Find its value after 3 years.

$$A = P\left(1 - \frac{r}{n}\right)^{nt}$$

A = what we are asked to find

P = 12,500

r = .20

n = 1

t = 3

$$A = 12500\left(1 - \frac{.20}{1}\right)^{(1)(3)}$$

$$A = 12500(.8)^3$$

A = \$6400

Example for Monthly Payments & Total Cost of a Loan

You get a 10 year student loan at 8% to pay for $20,000 of your college tuition that is not covered by any grants,scholarships or your parents. How much is your monthly payment going to be and what will be the ultimate cost of your $20,000 of tuition?

$$\text{Payment} = 20000 \div \left[\frac{1 - \left(1 + \frac{.08}{12}\right)^{-(12)(10)}}{\frac{.08}{12}} \right]$$

payment = $242.66   cost = $29,118.62

The majority of your first 37.58 (3+ years of) payments will go towards paying the $9,118.66 in interest off and not the $20,000 principal that you borrowed.

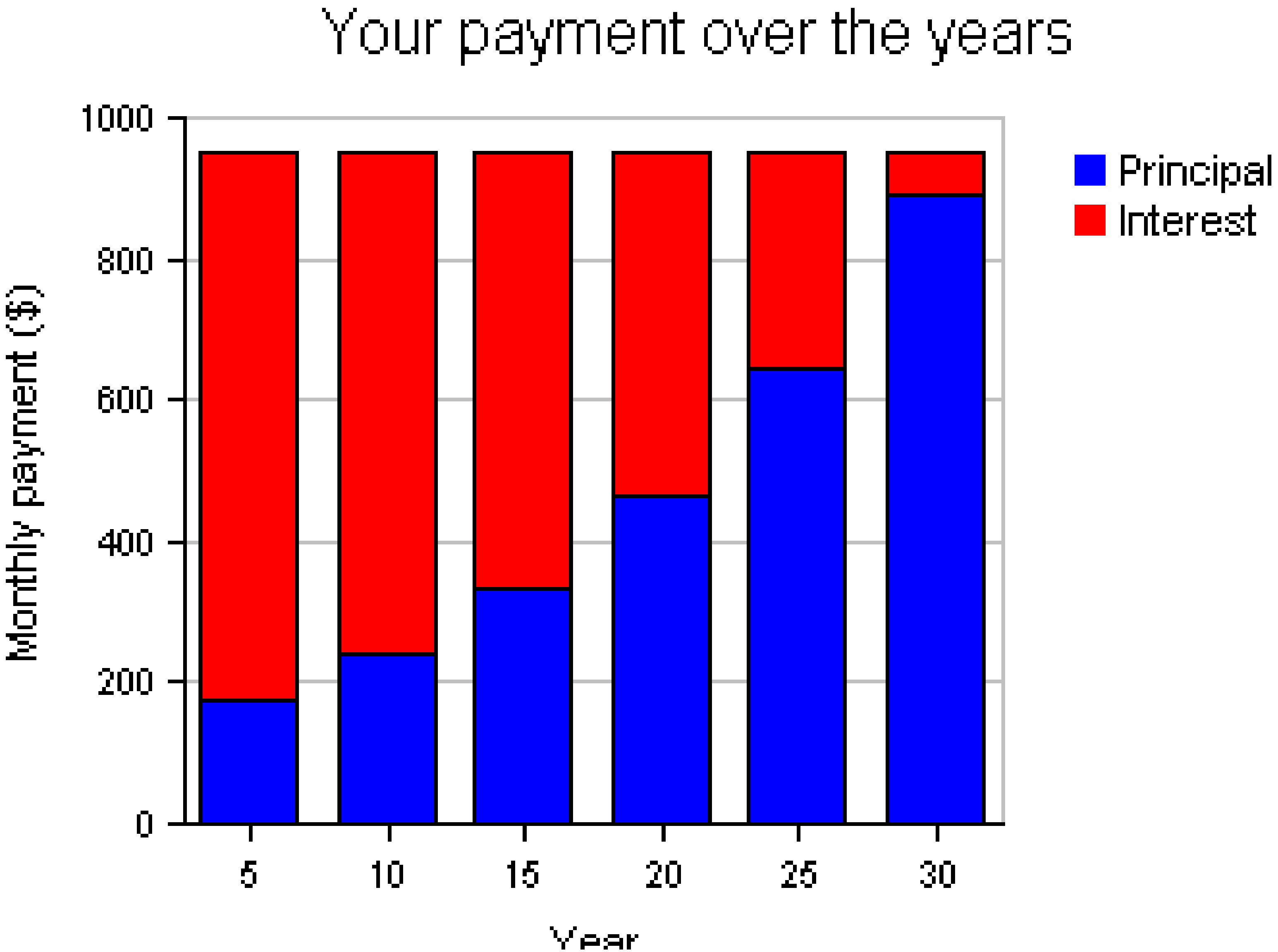
Example for Monthly Payments & Total Cost of a Loan

You get a 30 year fixed mortgage at 6.5% to pay for a $150,000 home. How much is your monthly payment going to be and what will be the ultimate cost of your $150,000 home?

$$\text{Payment} = 150000 \div \left[\frac{1 - \left(1 + \frac{.065}{12}\right)^{-(12)(30)}}{\frac{.065}{12}} \right]$$

$948 (principal & interest)
\+ $183 (taxes & insurance)
\+ $70 (mortgage insurance)
= $1201 monthly payment

total cost to buy the house = $432,360



Your payment over the years
Monthly payment ($)
1000
800
600
400
200
0
5
10
15
20
25
30
Year
Principal
Interest

Your annual schedule of payments

Month	Monthly payment	Remaining amount owed	Principal paid	Interest paid	Cumulative interest paid
1	$948	$149,864	$135	$812	$812
2	$948	$149,728	$136	$811	$1,624
3	$948	$149,590	$137	$811	$2,435
4	$948	$149,453	$137	$810	$3,245
5	$948	$149,314	$138	$809	$4,055
6	$948	$149,175	$139	$808	$4,863
7	$948	$149,035	$140	$808	$5,671
8	$948	$148,894	$140	$807	$6,479
9	$948	$148,752	$141	$806	$7,285
10	$948	$148,610	$142	$805	$8,091
11	$948	$148,467	$143	$804	$8,896
12	$948	$148,323	$143	$804	$9,700

Your annual schedule of payments

Month	Monthly payment	Remaining amount owed	Principal paid	Interest paid	Cumulative interest paid
349	$948	$10,097	$888	$59	$190,985
350	$948	$9,204	$893	$54	$191,040
351	$948	$8,306	$898	$49	$191,090
352	$948	$7,403	$903	$44	$191,135
353	$948	$6,495	$908	$40	$191,175
354	$948	$5,582	$912	$35	$191,210
355	$948	$4,664	$917	$30	$191,240
356	$948	$3,741	$922	$25	$191,265
357	$948	$2,813	$927	$20	$191,286
358	$948	$1,880	$932	$15	$191,301
359	$948	$942	$937	$10	$191,311
360	$948	$0	$942	$5	$191,316

Example for Monthly Payments & Total Cost of a Loan

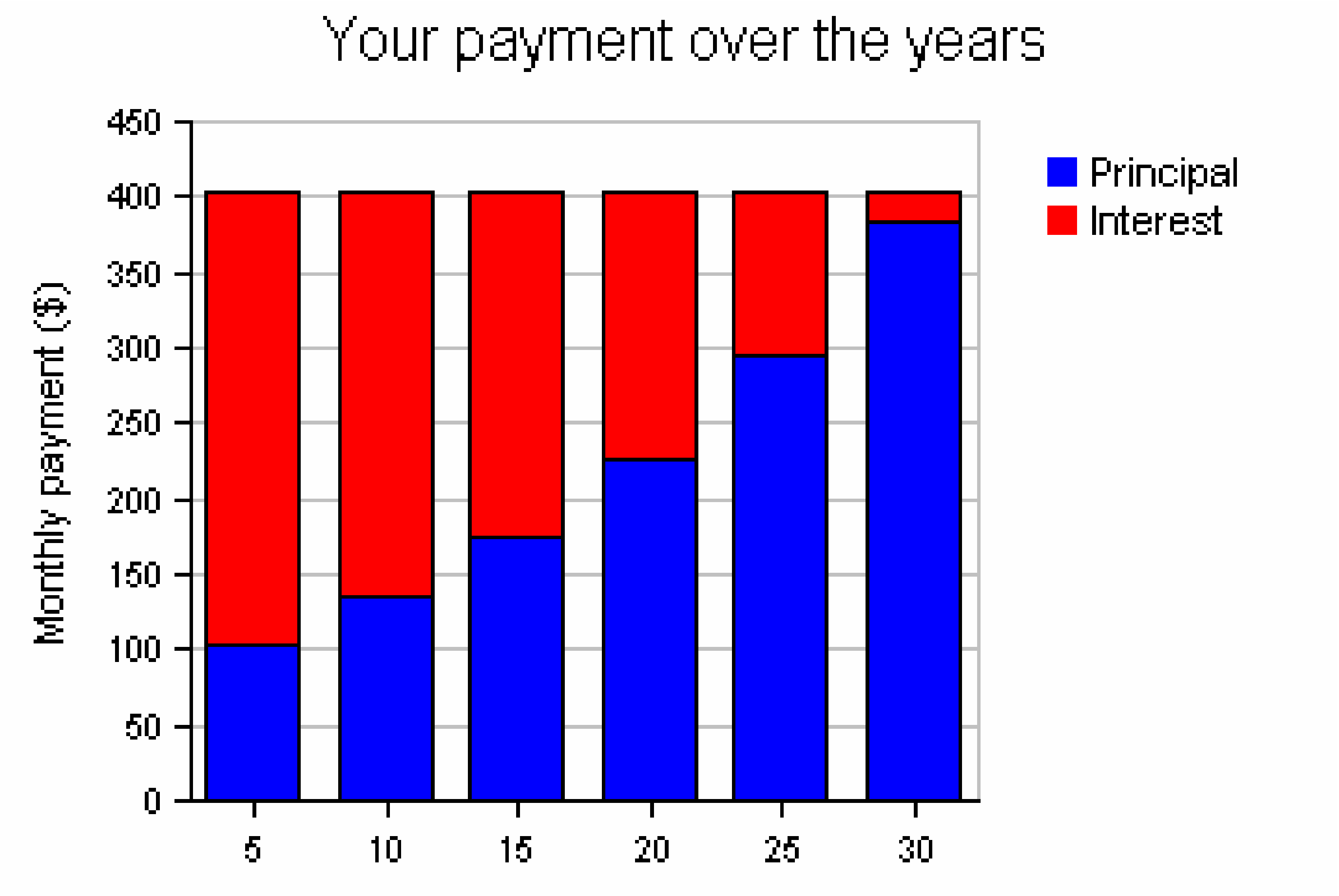
I got a 30 year fixed mortgage at 5.25% to pay for a $158,500 home but I only borrowed $73,000 I put the rest down in cash as a down payment and to but points which lowered my interest rate to 5.25%. Compare this to the last example where a person put no money down and bought a less expensive house

$$\text{Payment} = 73000 \div \left[\frac{1-\left(1+\frac{.0525}{12}\right)^{-(12)(30)}}{\frac{.0525}{12}} \right]$$

$403 (principal & interest)
+ $183 (taxes & insurance)
+ $0 (mortgage insurance)
= $586 monthly payment

total cost to buy my house = $210,960 + $85,500 = $296,460

Because of the large down payment and lower interest rate my more expensive house will cost me $135,900 less than the other person who didn't put any money down and bought a less expensive house.



Your payment over the years
Monthly payment ($)
0
50
100
150
200
250
300
350
400
450
5
10
15
20
25
30
Principal
Interest

Your annual schedule of payments

Month	Monthly payment	Remaining amount owed	Principal paid	Interest paid	Cumulative interest paid
1	$403	$72,916	$83	$319	$319
2	$403	$72,832	$84	$319	$638
3	$403	$72,747	$84	$318	$957
4	$403	$72,662	$84	$318	$1,275
5	$403	$72,577	$85	$317	$1,593
6	$403	$72,492	$85	$317	$1,910
7	$403	$72,406	$85	$317	$2,227
8	$403	$72,319	$86	$316	$2,544
9	$403	$72,233	$86	$316	$2,861
10	$403	$72,145	$87	$316	$3,177
11	$403	$72,058	$87	$315	$3,492
12	$403	$71,970	$87	$315	$3,807

Your annual schedule of payments

Month	Monthly payment	Remaining amount owed	Principal paid	Interest paid	Cumulative interest paid
349	$403	$4,319	$382	$20	$72,004
350	$403	$3,935	$384	$18	$72,023
351	$403	$3,549	$385	$17	$72,041
352	$403	$3,162	$387	$15	$72,056
353	$403	$2,773	$389	$13	$72,070
354	$403	$2,382	$390	$12	$72,082
355	$403	$1,989	$392	$10	$72,092
356	$403	$1,594	$394	$8	$72,101
357	$403	$1,198	$396	$6	$72,108
358	$403	$800	$397	$5	$72,113
359	$403	$401	$399	$3	$72,117
360	$403	$0	$401	$1	$72,119

Example for Exponential Growth

A certain population of bacteria doubles every 3 weeks. The number of bacteria in the population is now N_0. Find its size in 15 weeks.

$$N = N_0 \bullet 2^{\frac{t}{d}}$$

N = what we are asked to find

$N_0 = N_0$

$t = 15$

$d = 3$

$$N = N_0 (2)^{\frac{15}{3}}$$

$$N = N_0 (2)^5$$

$$N = 32N_0$$

Example for Exponential Decay

The half-life of carbon-14 is approximately 6000 years. Determine how much of 100kg of this substance will remain after 24,000 years.

$$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

N = what we are asked to find

$N_0 = 100$

$t = 24{,}000$

$h = 6000$

$$N = 100\left(\frac{1}{2}\right)^{\frac{24000}{6000}}$$

$$N = 100(0.5)^4$$

$$N = 6.25\text{kg}$$

Section 10-8

Natural Logarithms

Objectives

- to write natural logarithms in exponential form
- to write exponents with base e as natural logarithms
- to simplify natural logarithms
- to write natural logarithm statements as a single natural logarithm expression
- to solve natural logarithmic equations
- to find the domain and range of natural logarithmic equations
- to graph natural logarithmic equations

Natural Logarithms

- Natural logarithms are base e (Euler's Number ≈ 2.71827...) which is an irrational number like π.
- $\log_e x = \ln x$
- Natural logarithms follow all of the rules for other logarithms.

Example for 1-4

$$\ln 8 = 2.08$$

$$\log_e 8 = 2.08$$

$$e^{2.08} = 8$$

Example for 5-8

$$e^3 = 20.1$$

$$\log_e 20.1 = 3$$

$$\ln 20.1 = 3$$

Example for 9-16

$$e^{\ln 5}$$

$$e^{\ln 5} = x$$

$$\log_e x = \ln 5$$

$$\ln x = \ln 5$$

$$x = 5$$

Example for 17-22

$$\frac{1}{3}\ln 8 + \ln 5 + 3$$

$$\ln 8^{\frac{1}{3}} + \ln 5 + \ln e^{3}$$

$$\ln 2 + \ln 5 + \ln e^{3}$$

$$\ln (2)(5)\left(e^{3}\right)$$

$$\ln 10e^{3}$$

Example for 23-28

$$\ln(x-4)=-1$$

$$\log_e(x-4)=-1$$

$$e^{-1}=x-4$$

$$4+e^{-1}=x$$

Example for 29-34

$$e^{x-2} = 2$$

$$\log_e 2 = x - 2$$

$$\ln 2 = x - 2$$

$$2 + \ln 2 = x$$

Example for 35-43

$$e^{2x} - 7e^{x} + 12 = 0$$

$$y = e^{x}$$

$$y^{2} - 7y + 12 = 0$$

$$(y-3)(y-4) = 0$$

$$y = 3, y = 4$$

$$e^{x} = 3, e^{x} = 4$$

$$\log_e 3 = x, \log_e 4 = x$$

$$\ln 3 = x, \ln 4 = x$$

Example for 44-47

$$f(x) = \ln(x - 5)$$

$$y = \ln(x - 5)$$

$$e^{y} = x - 5$$

Since e is a positive number, no exponent can make it have a negative value or be equal to zero.

$x - 5 > 0 \therefore$ **D{x: x > 5}**

Since y is an exponent, we have learned this year that any real number can be an exponent.

R{y: y $\in \Re$ }