Section 12-1

Angles and Degree Measures

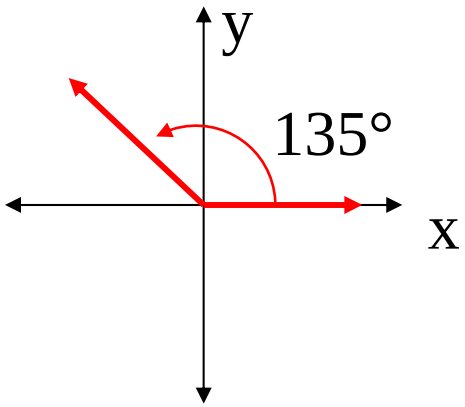
Objectives

- to sketch an angle in standard position
- to find the measure of an angle from a diagram
- to write a formula for coterminal angles
- to express degrees and minutes as a decimal to the nearest tenth
- to express degrees, minutes, seconds as an angle to the nearest hundredth
- to express tenths as degrees and minutes
- to express hundredths as degrees, minutes and seconds
- to estimate the measure of an angle given a point on the terminal side
- to find multiples of a given angle that would land in a specific quadrant

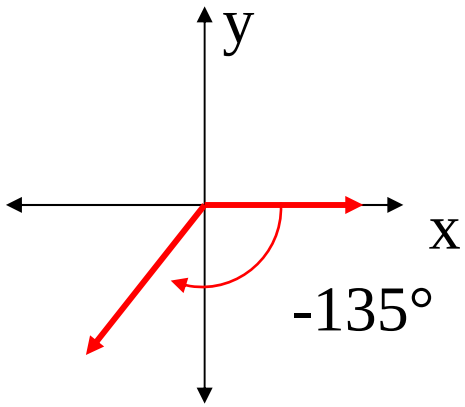
Angles and Degree Measure

- degree: one $\frac{1}{360}$ of a revolution
- initial side: one of the rays forming an angle
- terminal side: one of the rays forming an angle generated by rotating the initial side
- positive angles: an angle formed by a counterclockwise rotation
- negative angles: an angle formed by a clockwise rotation
- directed angle: an angle generated by rotation
- standard position: an angle whose initial side coincides with the positive x-axis
- quadrantal angle: an angle in standard position whose terminal side lies on a coordinate axis
- coterminal: two angles in standard position whose terminal sides coincide
- minute: $1'$ is $\frac{1}{60}$ of $1°$
- second: $1''$ is $\frac{1}{60}$ of $1'$

Examples for 1-12



II

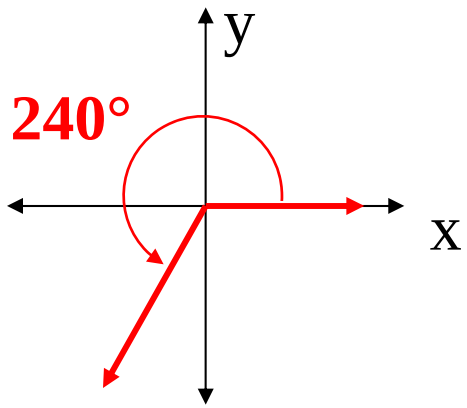


III

Examples for 13-18

$\frac{2}{3}$ of a counterclockwise revolution

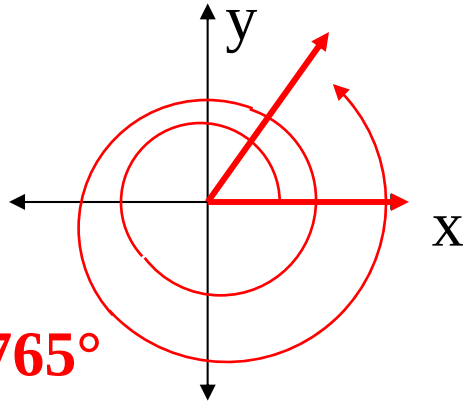
$$\left(\frac{2}{3}\right)(360°) = 240°$$

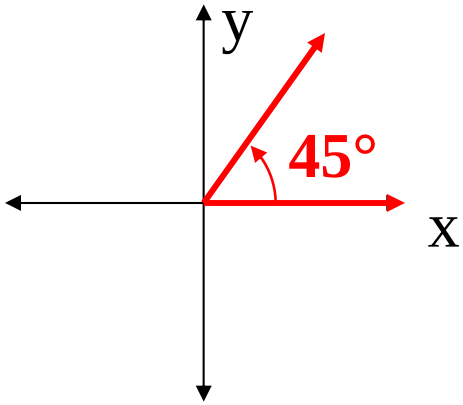


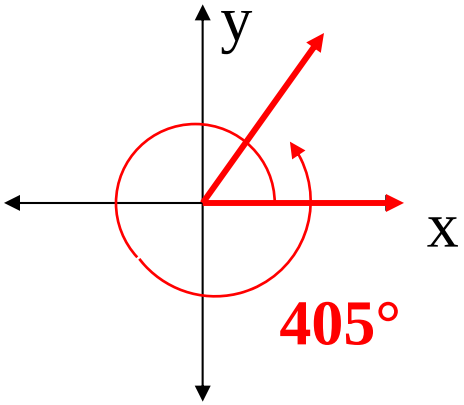
Examples for 19-22

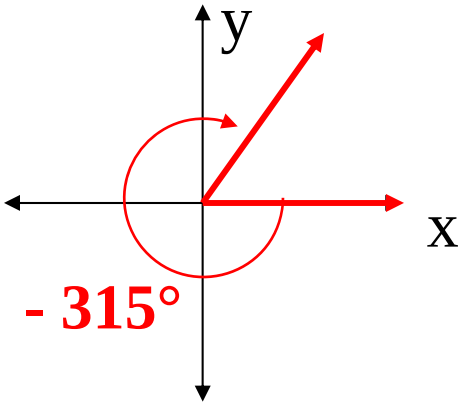
$45° + n(360°)$

n = 0 then 45 + 0 = 45°
n = 1 then 45 + 360 = 405°
n = 2 then 45 + 720 = 765°
n = - 1 then 45 – 360 = - 315 °









Examples for 23-30

35°

35 + n(360)

n = 1 then 35 + 360 = 395°

n = - 1 then 35 – 360 = - 325°

Examples for 31-34

$15°30'$

$$15 + \frac{30}{60}$$

$15 + 0.5$

15.5°

Examples for 35-38

$25°45'$

$$25 + \frac{45}{60}$$

$25 + 0.75$

$25.75°$

Examples for 39-42

$25.4°$

$25°(0.4)(60)'$

$25°\ 24'$

Examples for 43-46

$34.41°$

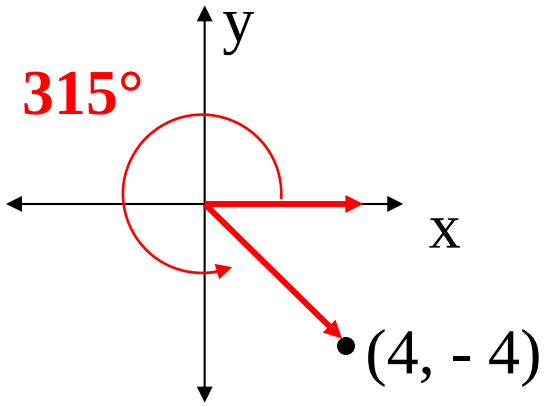
$34°(0.41)(60)′$

$34°\ 24.6′$

$34°\ 24′(0.6)(60)″$

$34°\ 24′\ 36″$

Examples for 47-54



Examples for 55-57

the first quadrant

$$0 < 4\theta < 90$$

$$\frac{0}{4} < \theta < \frac{90}{4}$$

$0 < \theta < 22.5$

Examples for 58-60

the third quadrant

$$180 < 6\theta < 270$$

$$\frac{180}{6} < \theta < \frac{270}{6}$$

$30 < \theta < 45$

Examples for 61-63

the first quadrant

$$0 < \frac{1}{2}\theta < 90$$

$$(2)(0) < \theta < (2)(90)$$

$$\mathbf{0 < \theta < 180}$$

Examples for 64-66

the second quadrant

$$90 < \frac{1}{5}\theta < 180$$

$$(5)(90) < \theta < (5)(180)$$

$450 < \theta < 900$

Section 12-2

Trigonometric Functions of Acute Angles

Objectives

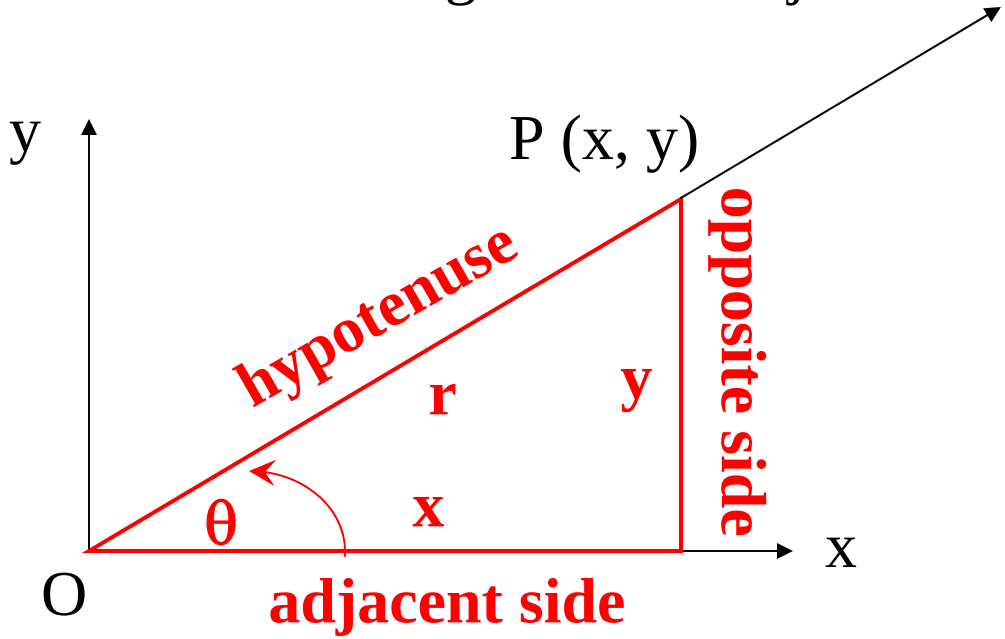
- to find the values of the six trigonometric functions of an angle of a triangle
- to find the values of the six trigonometric functions of an angle given a point on its terminal side
- to complete a table of trigonometric values
- to find the measure of an acute angle through the cofunction identities
- to complete the angles and sides of a given right triangle
- to find the lengths of segments in a right triangle in their exact radical form by using the 30-60-90 rule and the 45-45-90 rule
- to prove basic trigonometric identities

Trigonometric Functions of Acute Angles

$$\sin\theta = \frac{y}{r} = \frac{\text{length of the side opposite } \theta}{\text{length of the hypotenuse}}$$

$$\cos\theta = \frac{x}{r} = \frac{\text{length of side adjacent to } \theta}{\text{length of the hypotenuse}}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta}$$



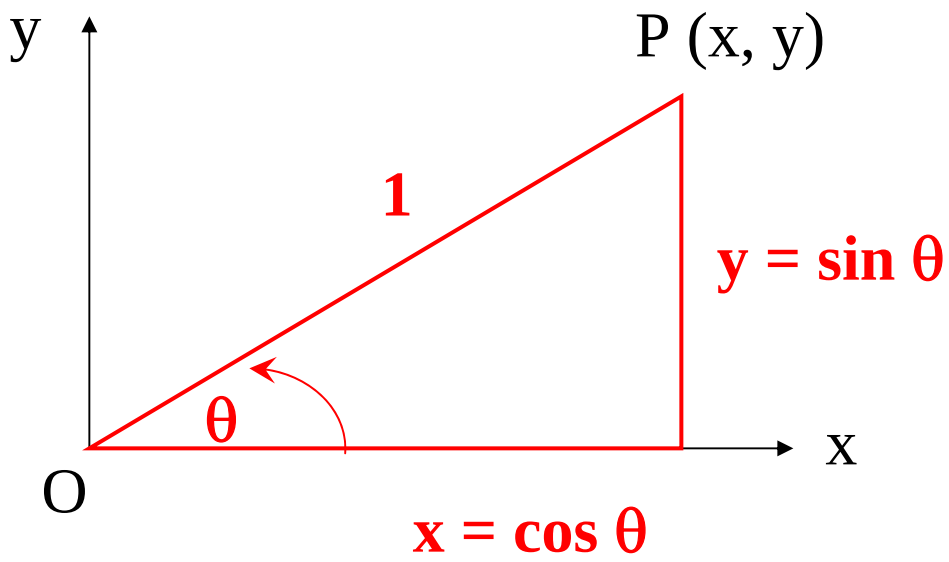
Reciprocal Functions

$$\text{cosecant of } \theta = \csc\theta = \frac{1}{\sin\theta}$$

$$\text{secant of } \theta = \sec\theta = \frac{1}{\cos\theta}$$

$$\text{cotangent of } \theta = \cot\theta = \frac{1}{\tan\theta}$$

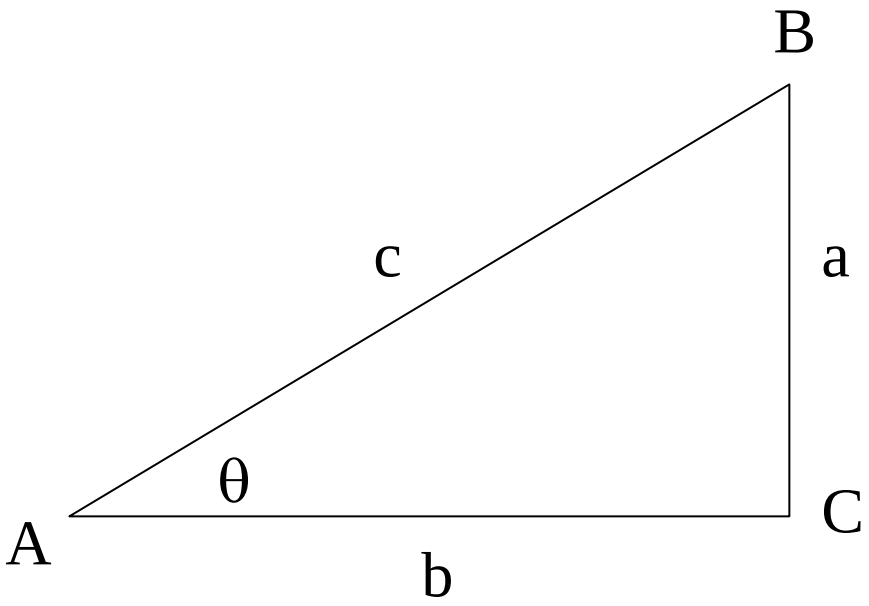
Pythagorean Identity



$$x^2 + y^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

Cofunctions



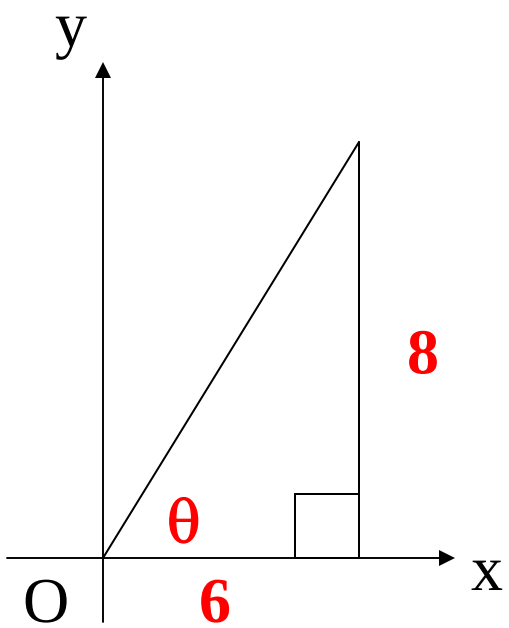
$$\sin A = \frac{a}{c} = \cos B \qquad \sec A = \frac{c}{b} = \csc B \qquad \cot A = \frac{b}{a} = \tan B$$

$$\sin\theta = \cos(90-\theta) \qquad \sec\theta = \csc(90-\theta) \qquad \cot\theta = \tan(90-\theta)$$

$$\tan A = \frac{a}{b} = \cot B \qquad \cos A = \frac{b}{c} = \sin B \qquad \csc A = \frac{c}{a} = \sec B$$

$$\tan\theta = \cot(90-\theta) \qquad \cos\theta = \sin(90-\theta) \qquad \csc\theta = \sec(90-\theta)$$

Examples for 1-4



This is a pattern right triangle so the hypotenuse is 10.

$$\sin\theta = \frac{8}{10} = \frac{4}{5}$$

$$\cos\theta = \frac{6}{10} = \frac{3}{5}$$

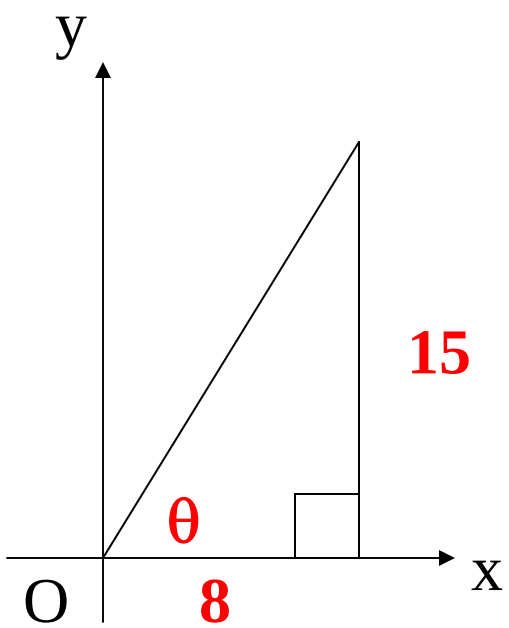
$$\tan\theta = \frac{8}{6} = \frac{4}{3}$$

$$\csc\theta = \frac{5}{4}$$

$$\sec\theta = \frac{5}{3}$$

$$\cot\theta = \frac{3}{4}$$

Examples for 5-8



This is a pattern right triangle so the hypotenuse is 17.

$$\sin\theta = \frac{15}{17}$$

$$\cos\theta = \frac{8}{17}$$

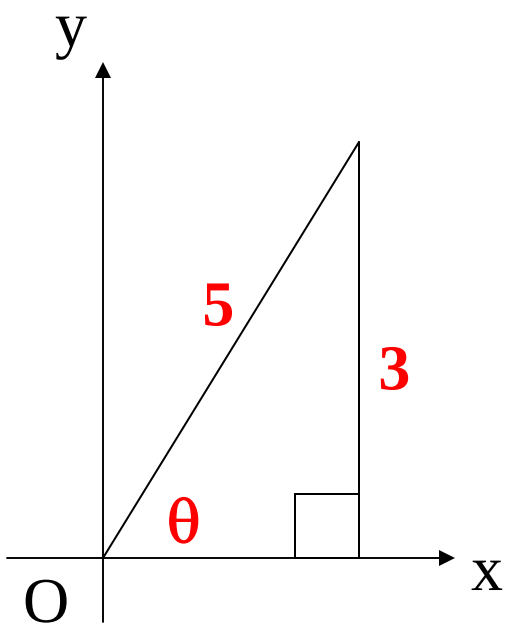
$$\tan\theta = \frac{15}{8}$$

$$\csc\theta = \frac{17}{15}$$

$$\sec\theta = \frac{17}{8}$$

$$\cot\theta = \frac{8}{15}$$

Examples for 9-15



$$\cos\theta = \frac{4}{5}$$

$$\tan\theta = \frac{3}{4}$$

This is a pattern right triangle
so the x value is 4.

Examples for 16-19

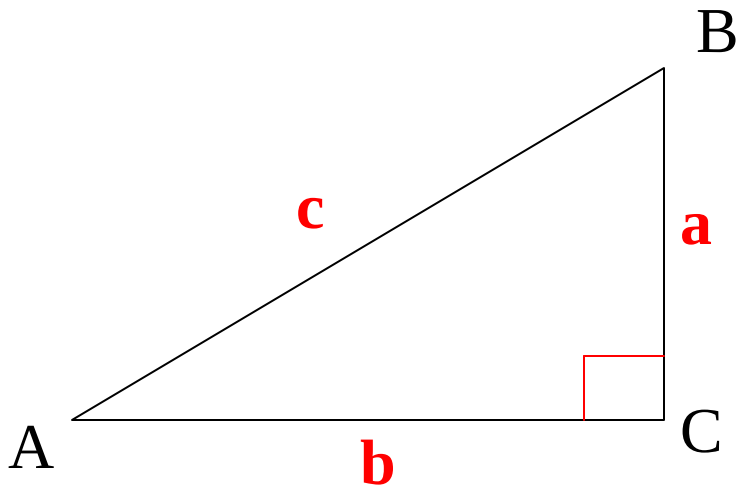
$\sin \phi = \cos 50°$

ϕ will always be the complementary angle

$\phi = 40°$

Examples for 20-25

$b = 2, \angle A = 45°$



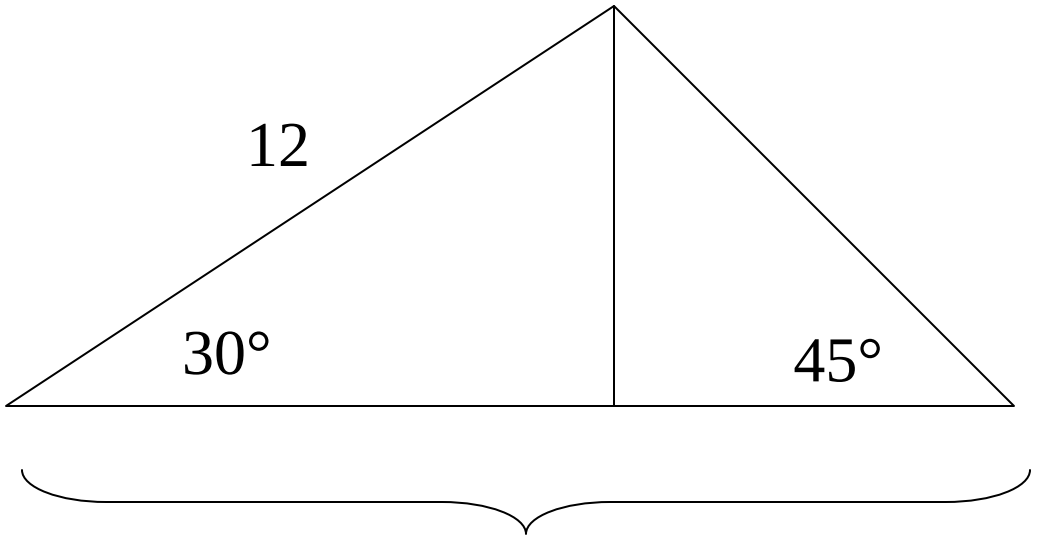
You will be using the 45-45-90 and the 30-60-90 patterns you learned in geometry last year.

$a = 2$

$c = 2\sqrt{2}$

$m\angle B = 45°$

Examples for 26-29



You will be using the 45-45-90 and the 30-60-90 patterns you learned in geometry last year.

$x = 6 + 6\sqrt{3}$

Section 12-3

Trigonometric Functions of General Angles

Objectives

- to find the six trigonometric values given a point on the terminal side
- to complete a table of trigonometric functions
- to find the reference angle
- to write an angle as a function of an acute angle
- to find the six exact trigonometric values of an angle
- to identify the quadrant of an angle and its six trigonometric values
- to solve a simple trigonometric equation

Trigonometric Functions of General Angles

$$\sin\theta = \frac{y}{r}$$

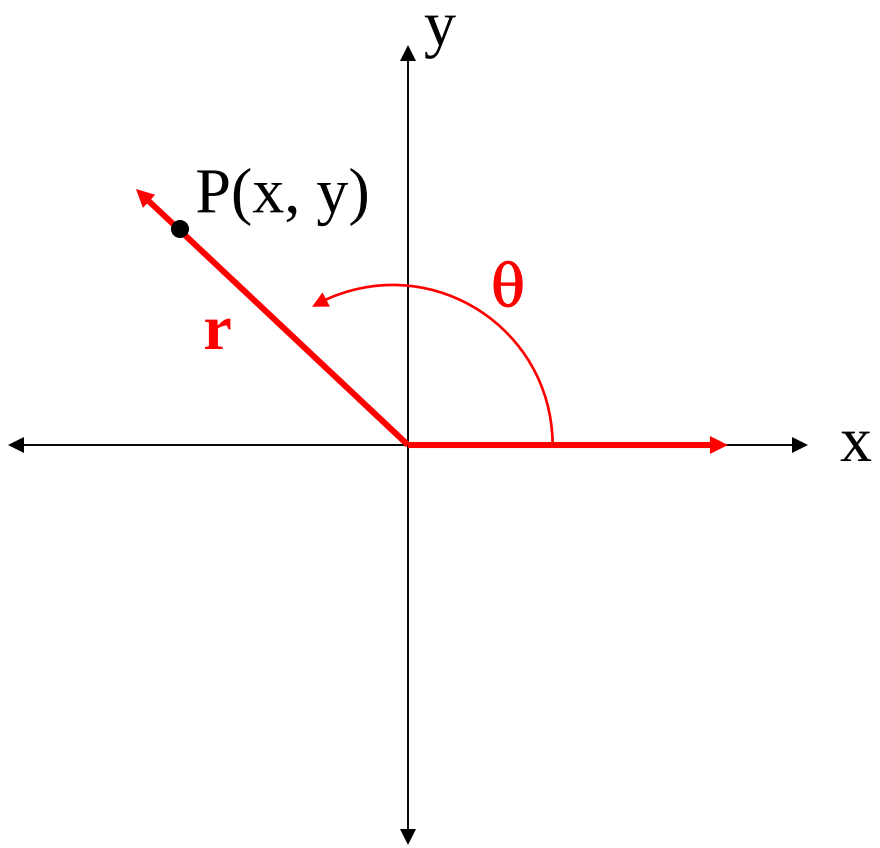
$$\csc\theta = \frac{r}{y}, \text{ if } y \neq 0$$

$$\cos\theta = \frac{x}{r}$$

$$\sec\theta = \frac{r}{x}, \text{ if } x \neq 0$$

$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}, \text{ if } y \neq 0$$

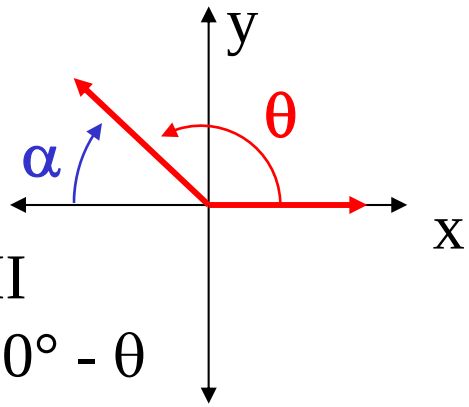


Sine and Cosine Functions

Function value	Quadrant of θ			
	I	II	III	IV
sin θ csc θ	+	+	-	-
cos θ sec θ	+	-	-	+
tan θ cot θ	+	-	+	-

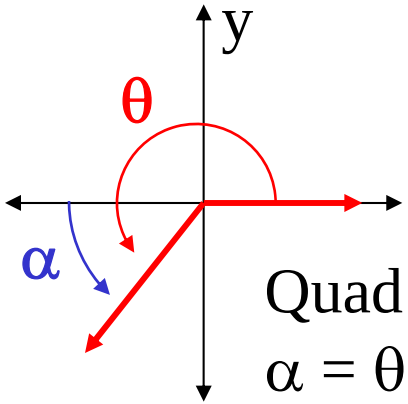
Reference Angles

reference angle: for every non-quadrantal angle θ, there is a unique acute angle α, corresponding to θ, formed by the terminal side of θ and the positive or negative x-axis



Quad II

$\alpha = 180° - \theta$

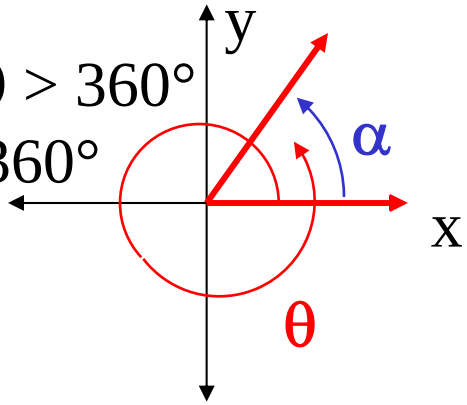


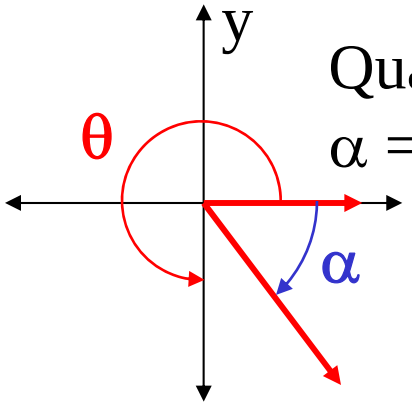
Quad III

$\alpha = \theta - 180°$

$720° > \theta > 360°$

$\alpha = \theta - 360°$



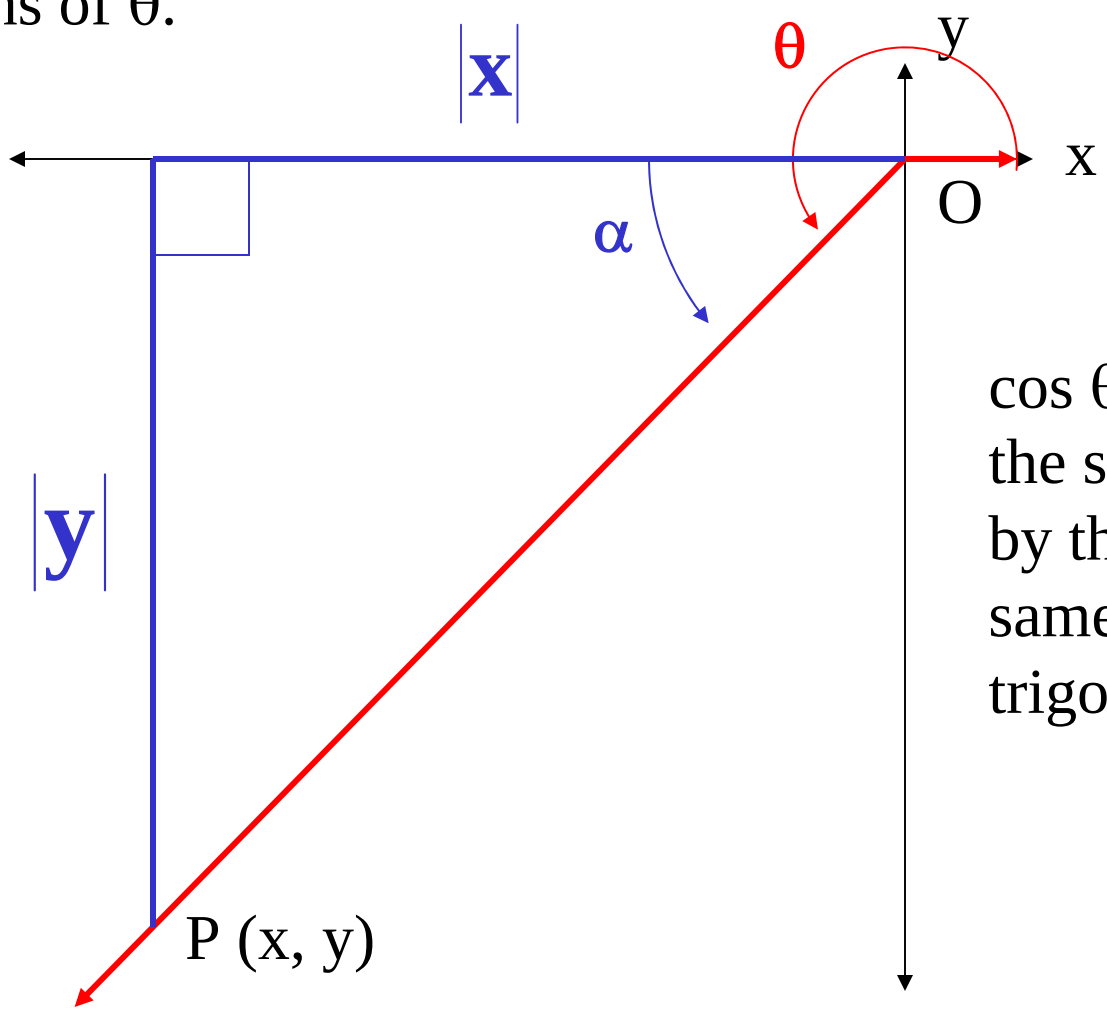


Quad IV

$\alpha = 360° - \theta$

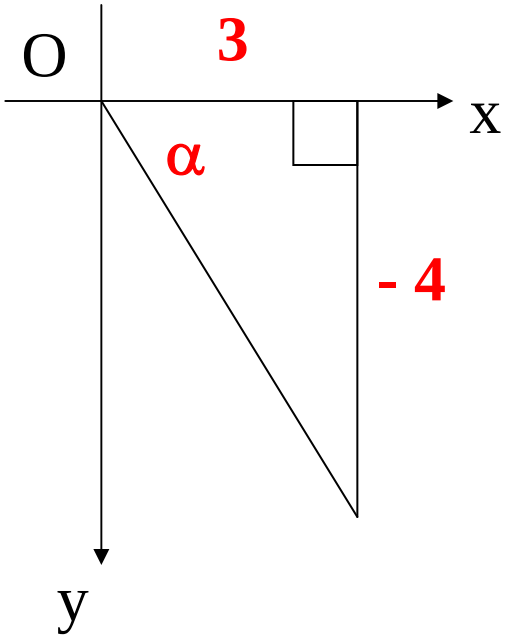
Reference Triangles

reference triangle: using the reference angle α for any angle θ you can draw a reference triangle to find the values of the trigonometric functions of θ.



$\cos \theta = \pm \cos \alpha$ where the sign is determined by the quadrant. The same is true for all six trigonometric functions.

Examples for 1-4



This is a pattern right triangle so the hypotenuse is 5.

$$\sin\theta = \frac{-4}{5}$$

$$\cos\theta = \frac{3}{5}$$

$$\tan\theta = \frac{-4}{3}$$

$$\csc\theta = \frac{5}{-4}$$

$$\sec\theta = \frac{5}{3}$$

$$\cot\theta = \frac{3}{-4}$$

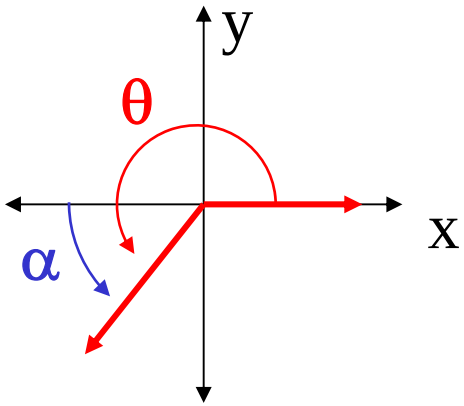
Examples for 5-8

	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°	**0**	**1**	**0**	**undefined**	**1**	**undefined**

Remember that when the hypotenuse is equal to one $x = \cos\theta$ and $y = \sin\theta$.

Examples for 9-24

$\theta = 233°$



Quad III: $\alpha = \theta - 180°$

Quad III: $\alpha = 233° - 180°$

$\alpha = 53°$

Examples for 25-36

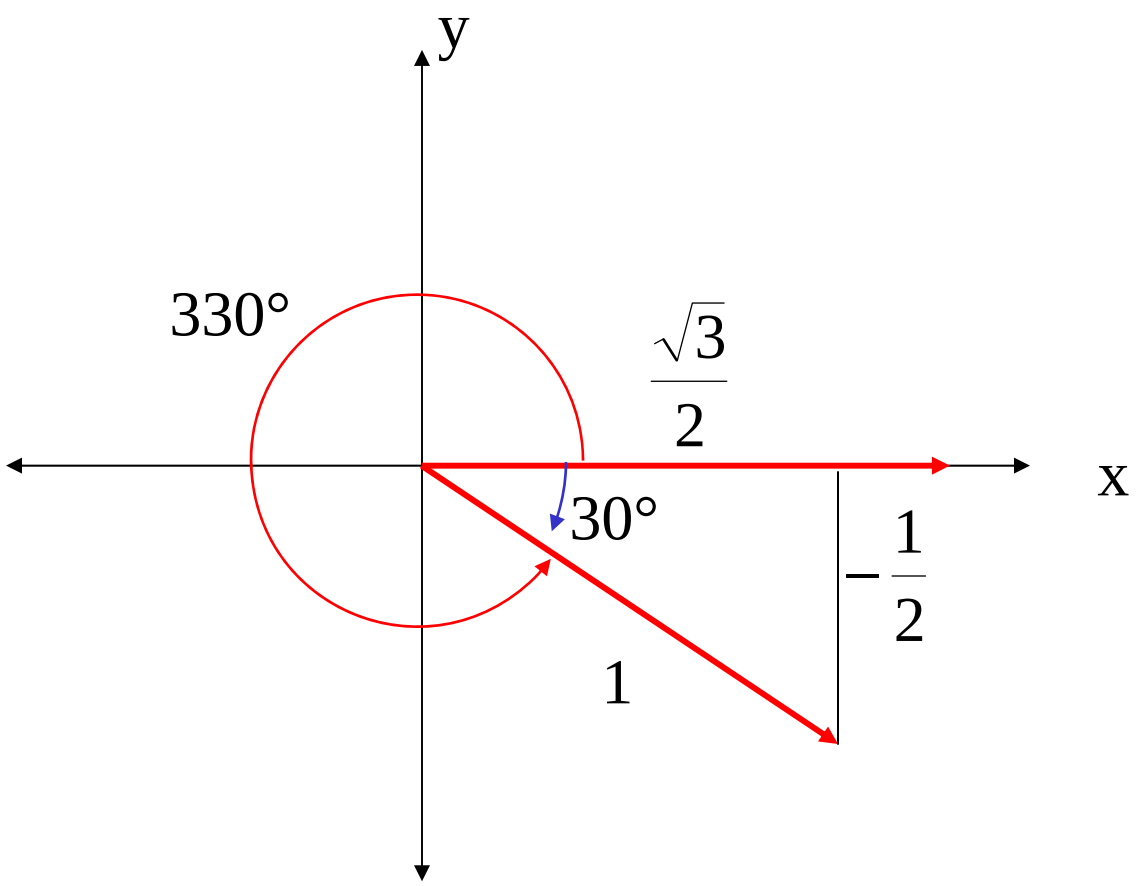
cos 216°

The reference angle is 216° -180° = 36°

216° is in Quadrant III which makes the cosine negative.

- cos 36°

Examples for 37-44



$$\sin 330° = -\frac{1}{2}$$

$$\cos 330° = \frac{\sqrt{3}}{2}$$

$$\tan 330° = -\frac{\sqrt{3}}{3}$$

$$\csc 330° = -2$$

$$\sec 330° = \frac{2\sqrt{3}}{3}$$

$$\cot 330° = -\sqrt{3}$$

Examples for 45-52

$$\cos\theta = -\frac{8}{17}, 0° < \theta < 180°$$

$0° < \theta < 180°$ implies that the angle is in either Quadrant I or II.

Cosine is only negative in Quadrant II.

According to the table in Quadrant II sine is (+), tan is (-), csc is (+), sec is (-) and cot is (-).

This is an 8, 15, 17 pattern right triangle.

$$\sin\theta = \frac{15}{17}$$

$$\tan\theta = -\frac{15}{8}$$

$$\csc\theta = \frac{17}{15}$$

$$\sec\theta = -\frac{17}{8}$$

$$\cot\theta = -\frac{8}{15}$$

Examples for 53-67

$\tan \theta = 1$

Tangent is positive only in Quadrant I and III.

Tangent is equal to one only when x = y, which makes a 45-45-90 triangle.

$\theta = 45°$ and $225°$

Section 12-4

Values of Trigonometric Functions

Objectives

- to estimate the value of a trigonometric function to four significant digits
- to estimate the value of an angle to the nearest tenth given one of its trigonometric values
- to estimate the value of an angle to the nearest minute given one of its trigonometric values
- to find the measure of two angles in a unit circle with a given trigonometric value
- to find the measure of an angle with a given trigonometric value in a specific quadrant

Using a Calculator to Find Trigonometric Values

- Remember to convert all minutes into decimal degrees before entering the numbers into your calculator.
- Remember that a calculator only returns acute angles or reference angles; you must use the **sign** of the trigonometric function to determine what quadrant your angle is in.
- It is your responsibility to know how to properly use the calculator you purchased for this class.

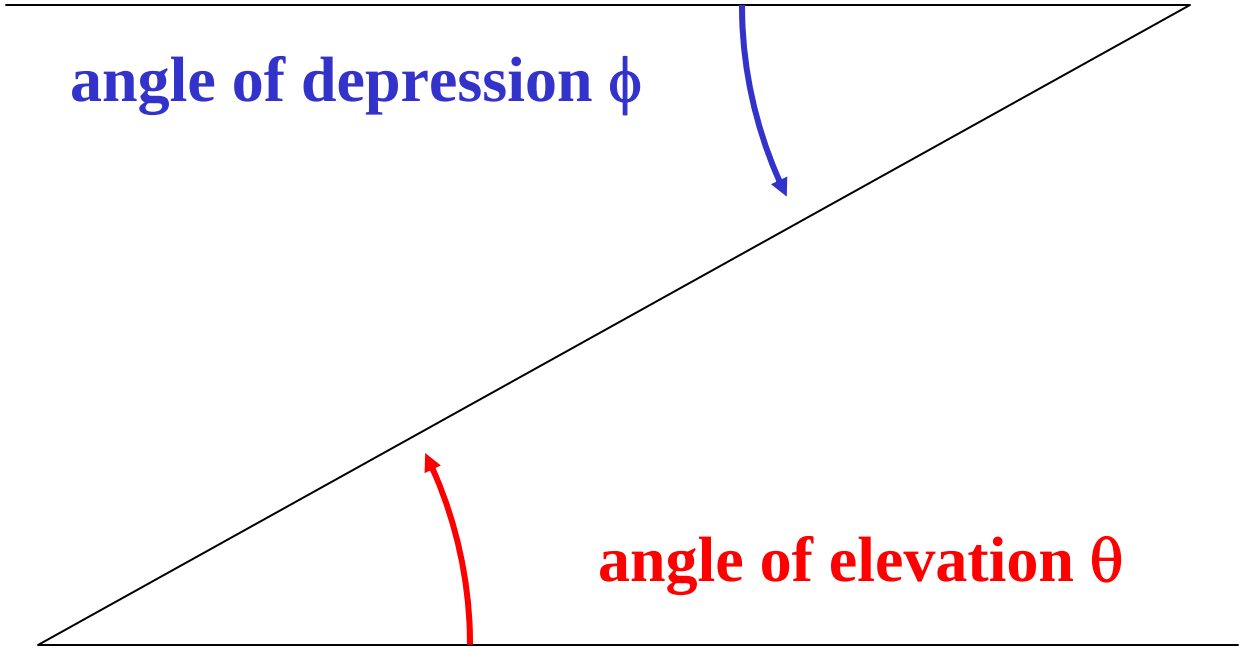
Section 12-5

Solving Right Triangles

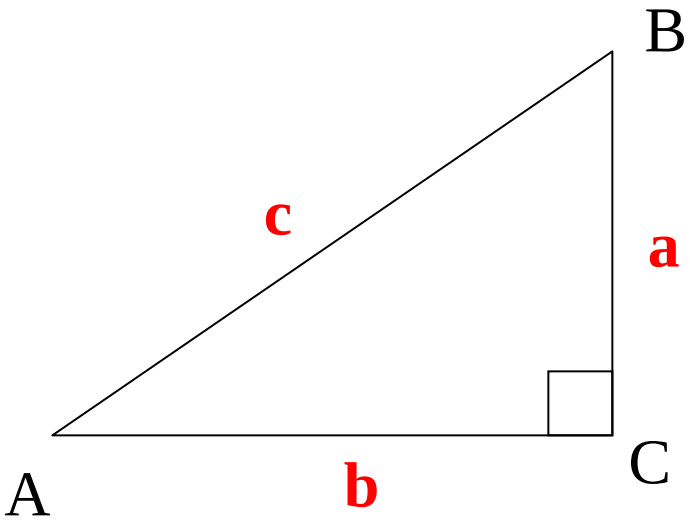
Objectives

- to solve a right triangle using trigonometry
- to find the measures of the sides and angles of different polygons using trigonometry

Angles of Elevation and Depression



Examples for 1-16



$\angle A = 36.2°$, $c = 68$

$\angle B = 90° - 36.2° = \mathbf{53.8°}$

$$\sin 36.2 = \frac{a}{68}$$

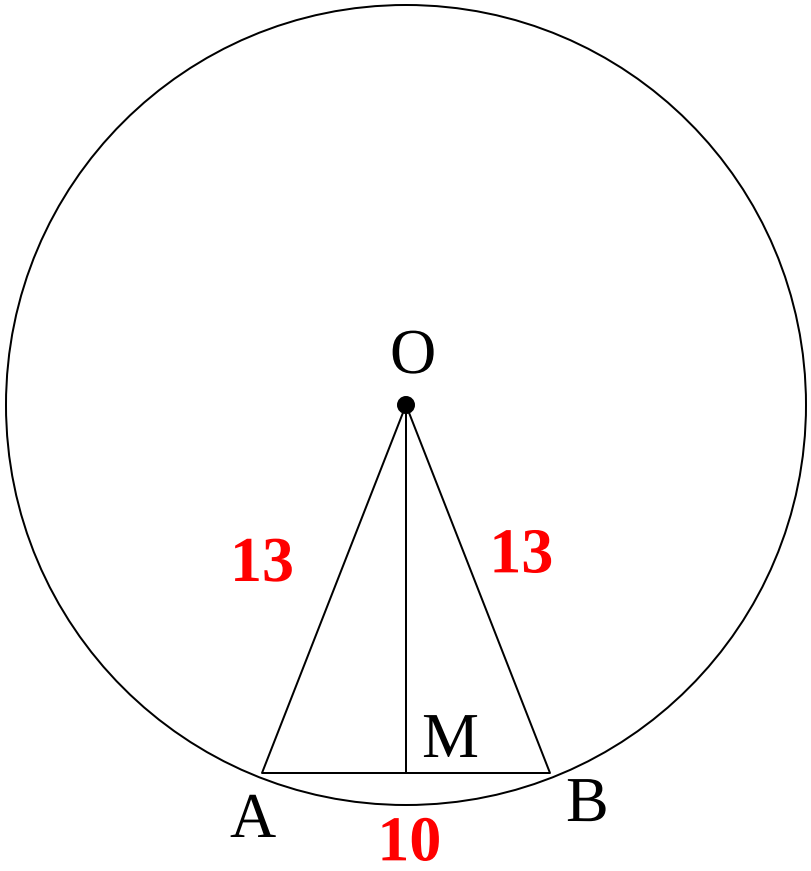
$a = 68(\sin 36.2) = \mathbf{40.2}$

$$\cos 36.2 = \frac{b}{68}$$

$b = 68(\cos 36.2) = \mathbf{54.9}$

Examples for 17-23

Find $\angle AOB$



$$\sin \angle AOM = \frac{5}{13}$$

$$\angle AOM = \sin^{-1}\left(\frac{5}{13}\right)$$

$$\angle AOM = 22.6$$

$\angle AOB = 2\angle AOM = (2)(22.6) = 45.2°$

Section 12-6

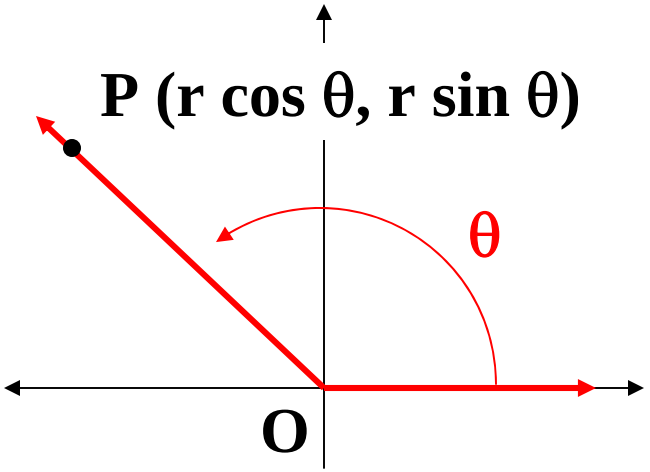
The Law of Cosines

Objectives

- to use the law of cosines to solve triangle problems

Law of Cosines

If θ is an angle in standard position and P is a point on its terminal side, then the coordinates of P are (r cos θ, r sin θ) where r = OP.

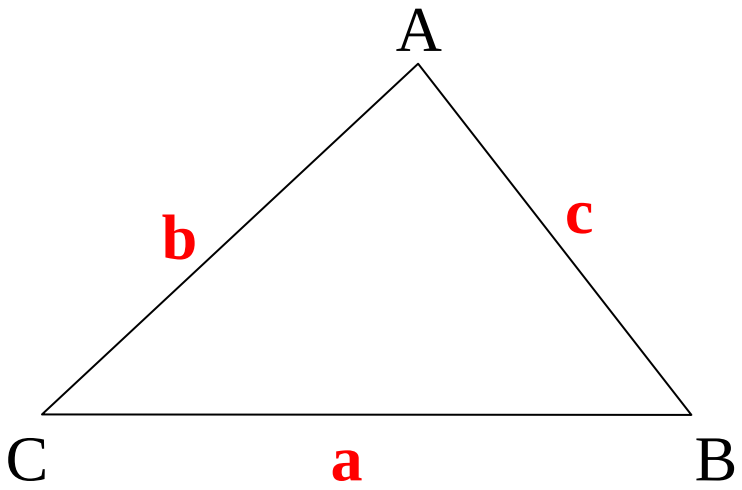


In any triangle ABC,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$



Examples for 1-12

$a = 6,\ b = 7,\ \angle C = 20°,\ c = ?$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 6^2 + 7^2 - 2(6)(7)(\cos 20)$$

$$c^2 = 36 + 49 - (84)(0.9397)$$

$$c^2 = 6.0652$$

$$c = \sqrt{6.0652} = 2.46$$

Section 12-7

The Law of Sines

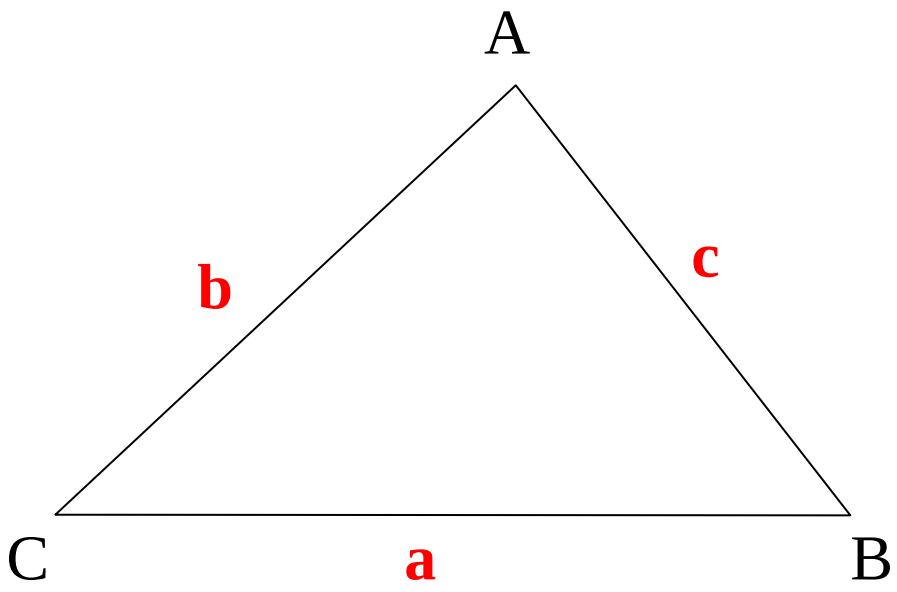
Objectives

- to use the law of sines to solve a triangle
- to find the value of the sine function when the cosine and tangent functions are known
- to prove basic trigonometric identities
- to solve triangle application problems

Law of Sines

In any triangle ABC,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Examples for 1-12

$a = 14, \angle A = 25°, \angle B = 75°, b = ?$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 25}{14} = \frac{\sin 75}{b}$$

$$\frac{0.4226}{14} = \frac{0.9659}{b}$$

$$b = \frac{(0.9659)(14)}{0.4226} = 32.0$$

Examples for 13-18

$$\sin A = \frac{2}{3}, \cos B = \frac{4}{5}$$

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

$$\frac{a}{b} = \frac{\frac{2}{3}}{\frac{3}{5}} = \left(\frac{2}{3}\right)\left(\frac{5}{3}\right) = \frac{10}{9}$$

Examples for 19-22

$$\frac{\sin A + \sin B}{\sin B} = \frac{a + b}{b}$$

$$\frac{\sin A}{\sin B} + \frac{\sin B}{\sin B} = \frac{a}{b} + \frac{b}{b}$$

$$\frac{\sin A}{\sin B} + 1 = \frac{a}{b} + 1$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

$$\frac{a}{b} + 1 = \frac{a}{b} + 1$$

Section 12-8

Solving General Triangles

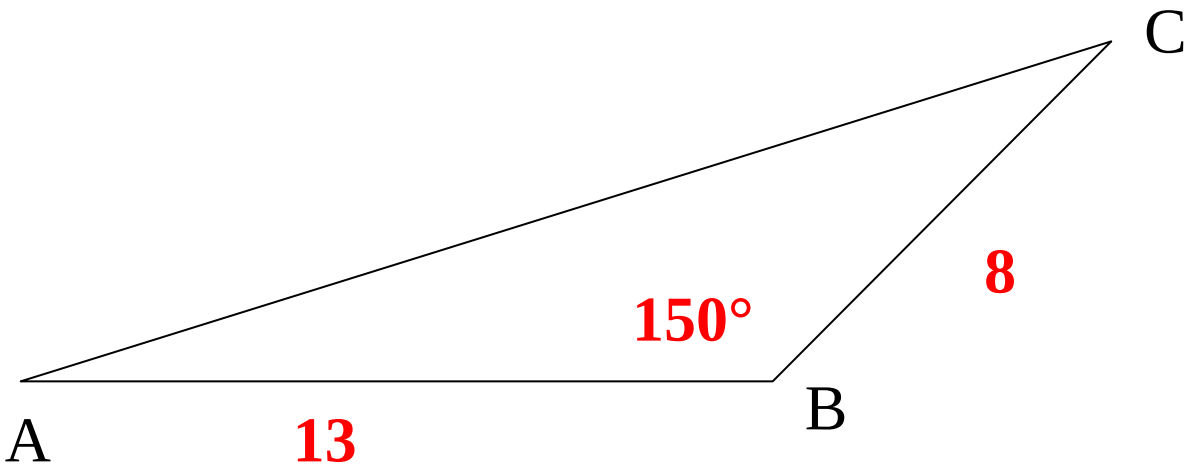
Objectives

- to use law of sines and law of cosines to solve triangle problems
- to solve triangle applications problems

Solving General Triangles

- From geometry we know that triangle congruence can be proven with SSS, SAS, ASA, AAS but not with SSA.
- SSS, SAS, ASA, and AAS all give a single solution to a triangle.
- We use the following laws to solve each pattern:
  - SSS: Law of Cosines
  - SAS: Law of Cosines
  - ASA: Law of Sines
  - AAS: Law of Sines
- SSA is called the ambiguous case because it results in six possible solutions to the triangle: four if the given angle is acute and two if the given angle is right or obtuse.
- We use Law of Sines to solve the SSA pattern.

Examples for 1-12



SAS is solved first with Law of Cosines

$$b^2 = 8^2 + 13^2 - (2)(8)(13)(\cos 150)$$

b = 20.3

$$\frac{\sin 150}{20.3} = \frac{\sin C}{13} = \frac{\sin A}{8}$$

$$\sin C = \frac{(13)(0.5)}{20.3} = 0.3202$$

$$\sin A = \frac{(8)(0.5)}{20.3} = 0.1970$$

$\angle C = 18.7°$, $\angle A = 11.3°$

Section 12-9

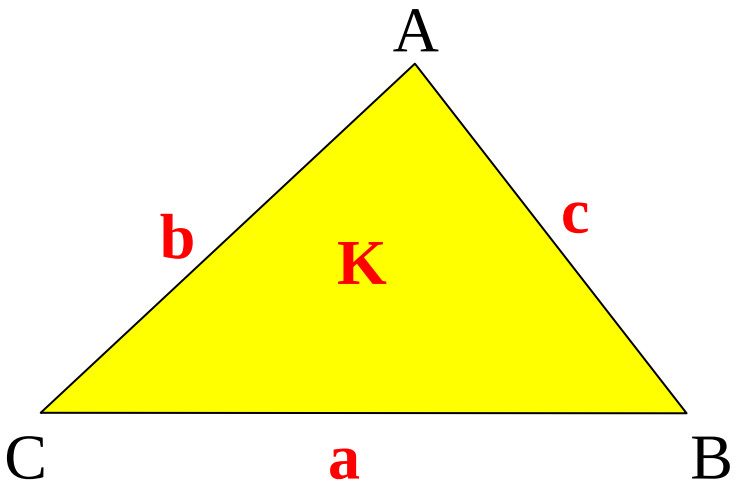
Areas of Triangles

Objectives

- to use trigonometry to solve for the area of a triangle

Areas of Triangles

The area K of a triangle ABC is given by:



$$K = \frac{1}{2}bc\sin A \qquad K = \frac{1}{2}ac\sin B \qquad K = \frac{1}{2}ab\sin C$$

$$K = \frac{1}{2}a^2\frac{\sin B\sin C}{\sin A} \qquad K = \frac{1}{2}b^2\frac{\sin A\sin C}{\sin B} \qquad K = \frac{1}{2}c^2\frac{\sin A\sin B}{\sin C}$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

Examples for 1-16

$\angle A = 25°, \angle B = 50°, b = 30$

$$K = \frac{1}{2} b^2 \frac{\sin A \sin C}{\sin B}$$

$$\angle C = 105°$$

$$K = \frac{1}{2}(30)^2 \frac{(\sin 25°)(\sin 105°)}{\sin 50°}$$

K = 240