Section 13-1

Radian Measure

Objectives

- to convert from degrees to radians
- to convert from radians to degrees
- to find arc length and sector area given central angles measured in radians
- to find a coterminal angle measured in radians

Radian Measure

- radian measure of an angle θ is the ratio of arc length to radius
- $1° = \frac{\pi}{180}$
- $1 \text{ radian} = \frac{180°}{\pi}$
- arc length $s = r\theta$ when θ is a radian measure
- sector area = $\frac{1}{2}r^2\theta$ or $\frac{1}{2}rs$ when r is the radius and s is the arc length and θ is a radian measure

Physics Formulas Based on Trigonometry

Speed of a Particle

$$v = \frac{s}{t}\left(cm / s\right)$$

Angular Speed

$$\omega = \frac{\theta}{t}\left(radians / s\right)$$

Relating Speed of a Particle to Angular Speed

$$v = \frac{s}{t} = r\frac{\theta}{t} = r\omega$$

Examples for 1-12

$$45°$$

$$(45)\left(\frac{\pi}{180}\right)$$

$$\frac{45\pi}{180}$$

$$\frac{\pi}{4}$$

Examples for 13-28

$$\frac{\pi}{6}$$

$$\left(\frac{\pi}{6}\right)\left(\frac{180}{\pi}\right)$$

30°

Examples for 29-36

$$10°$$

$$(10)\left(\frac{\pi}{180}\right)$$

$$\frac{(10)(3.14)}{180}$$

0.17

Examples for 37-44

$$3$$

$$(3)\left(\frac{180}{\pi}\right)$$

$$\frac{(3)(180)}{3.14}$$

171.9°

Examples for 45-56

$$r = 4,\ \theta = 1,\ s = ?,\ A = ?$$

$$s = r\theta \ \text{ and } \ A = \frac{1}{2}r^2\theta \ \text{ or } A = \frac{1}{2}rs$$

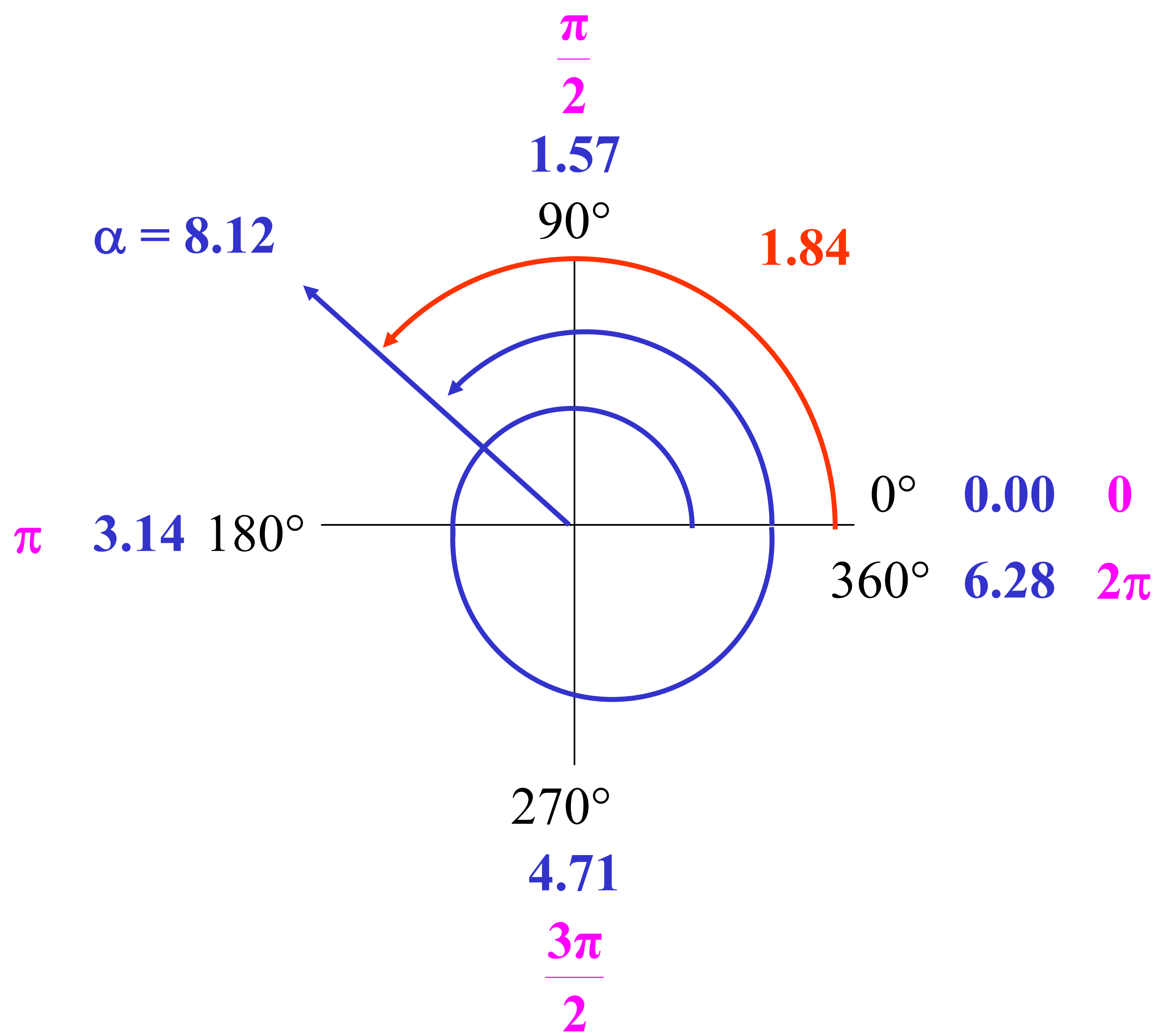
$$s = (4)(1)$$

$$\mathbf{s = 4}$$

$$A = \frac{1}{2}(4)(4)$$

$$\mathbf{A = 8}$$

Examples for 57-60



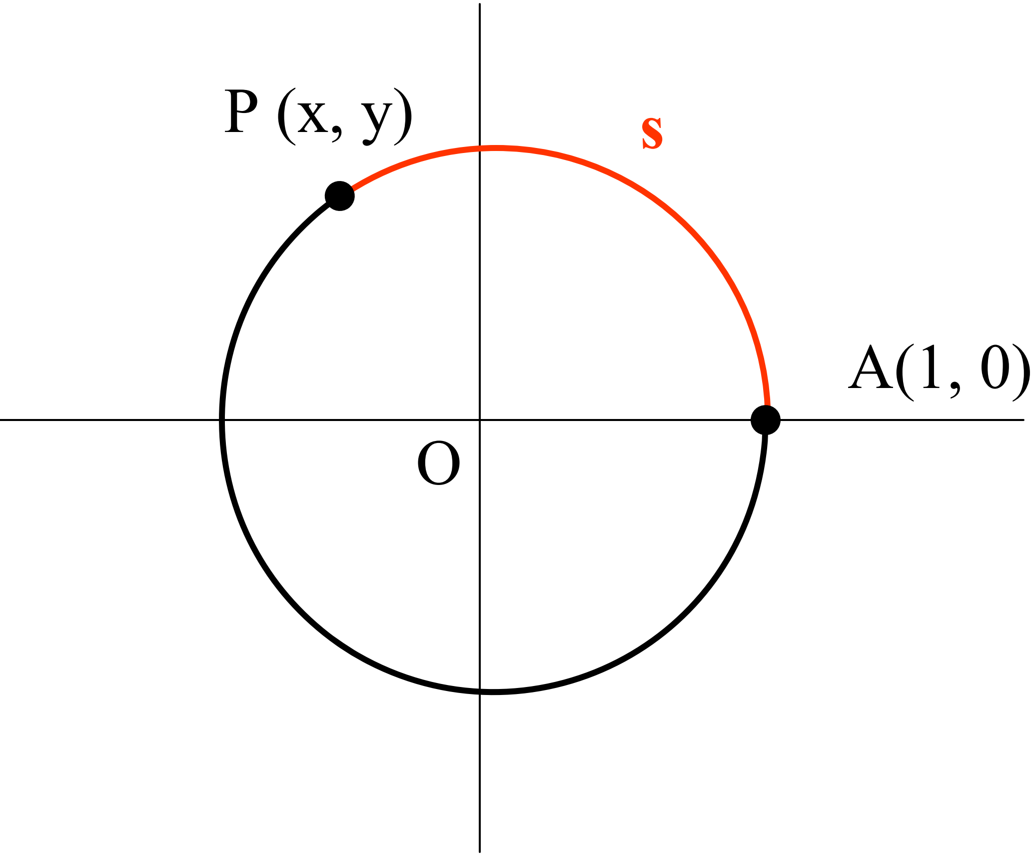
Section 13-2

Circular Functions

Objectives

- to find the exact value of the six trigonometric functions of a given angle expressed as a point on its terminal side
- to find the sine, cosine and tangent of a value
- to find the values of the six trigonometric functions for an angle expressed in radians
- to find the angle, in radians, having a specific trigonometric value
- to verify a basic trigonometric identity

Circular Functions



$$\sin s = y$$

$$\cos s = x$$

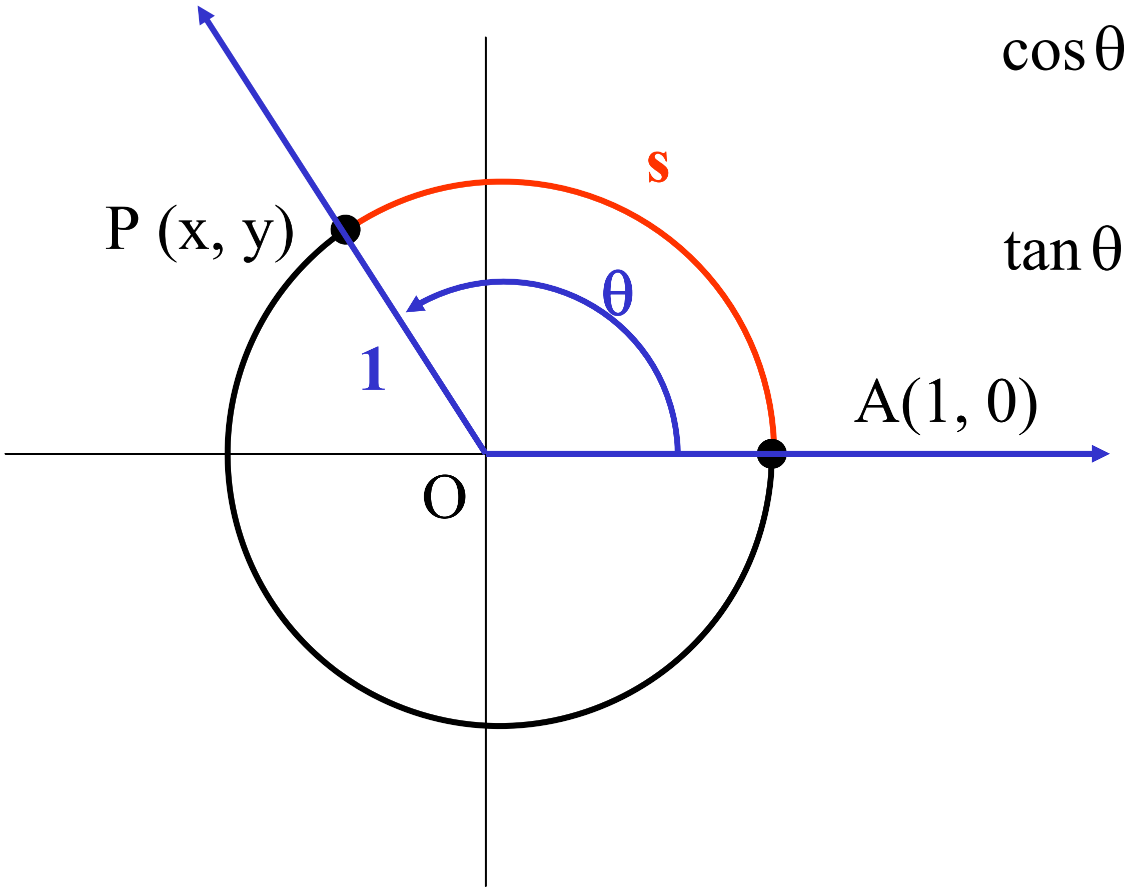
$$\tan s = \frac{\sin s}{\cos s} \quad \text{if } \cos s \neq 0$$

$$\csc s = \frac{1}{\sin s} \quad \text{if } \sin s \neq 0$$

$$\sec s = \frac{1}{\cos s} \quad \text{if } \cos s \neq 0$$

$$\cot s = \frac{\cos s}{\sin s} \quad \text{if } \sin s \neq 0$$

Circular Functions, Trigonometric Functions and the Unit Circle

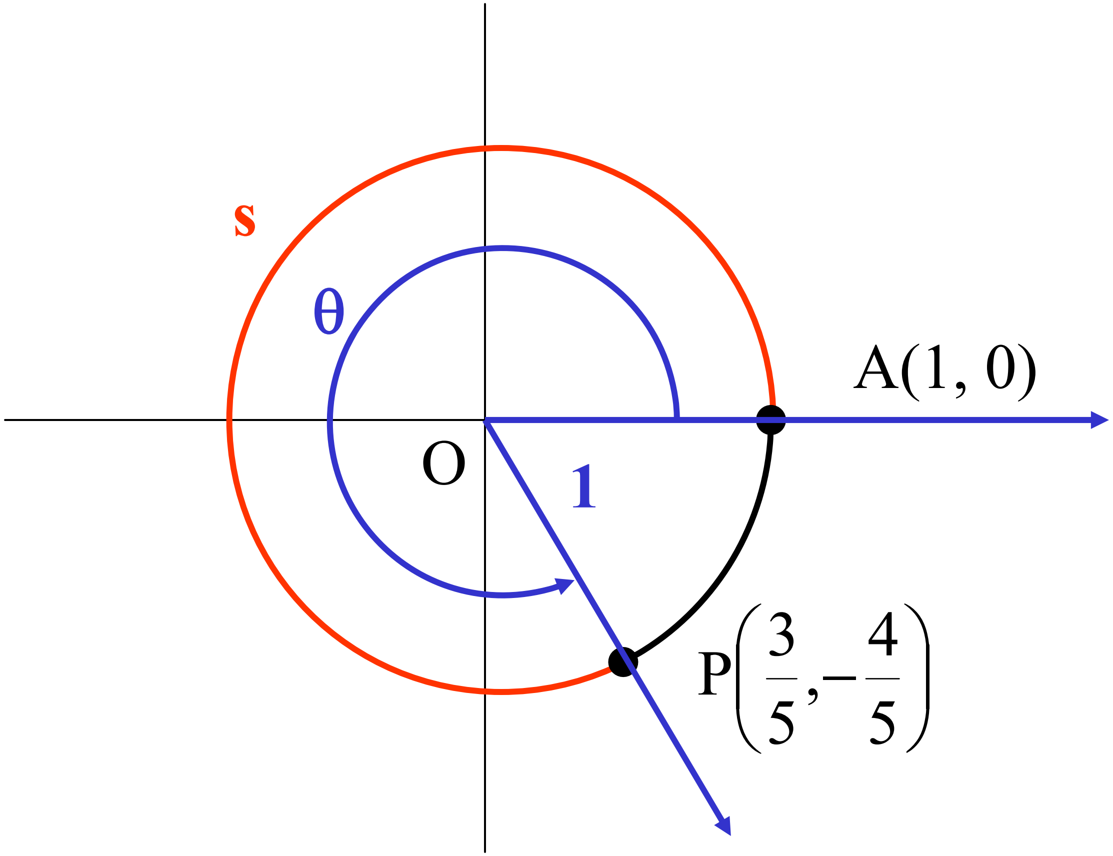


$$\sin\theta = \frac{y}{1} = y = \sin s$$

$$\cos\theta = \frac{x}{1} = x = \cos s$$

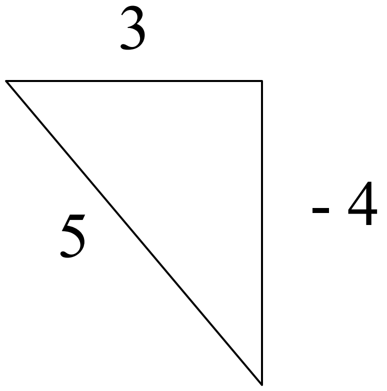
$$\tan\theta = \frac{y}{x} = \frac{\sin s}{\cos s} = \tan s$$

Examples for 1-4



$$\sin s = -\frac{4}{5} = \sin\theta$$

$$\cos s = \frac{3}{5} = \cos\theta$$



$$\tan s = -\frac{4}{3}, \csc s = -\frac{5}{4}, \sec s = \frac{5}{3}, \cot s = -\frac{3}{4}$$

Examples for 5-8

You may use your calculator to do these problems but you must change it into radians mode. If you don't you you will have to convert this radians number to a degree before you use your calculator.

sin 1.25 = 0.9490
cos 1.25 = 0.3153
tan 1.25 = 3.0096

Examples for 9-18

All of these radians values are for either an angle from a pattern right triangle or from a quadrantal angle.

$$\frac{\pi}{3} = 60°$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$$

$$\cos\frac{\pi}{3} = \frac{1}{2} \qquad \sec\frac{\pi}{3} = 2$$

$$\tan\frac{\pi}{3} = \sqrt{3} \qquad \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$$

Examples for 19-22

You may use your calculator to find the value in degrees and then convert it into radians or you may switch your calculator into radians and do the problem without converting.

$$\cos x = 0.7248$$

0.76

Examples for 23-27

$$\frac{3\pi}{4} = 135°$$

$$\sin 135 = \frac{\sqrt{2}}{2}$$

$$\cos 135 = -\frac{\sqrt{2}}{2}$$

$$\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$$

$$\frac{1}{2} + \frac{1}{2} = 1$$

Examples for 28&29

Show that if $0 < s < \frac{\pi}{2}$, then $\sec s = OQ$

$$\sec s = \sec \angle QOA$$

$$\sec \angle QOA = \frac{1}{\cos \angle QOA}$$

$$\cos \angle QOA = \frac{1}{OQ}$$

$$\sec s = \sec \angle QOA = OQ$$

Section 13-3

Periodicity and Symmetry

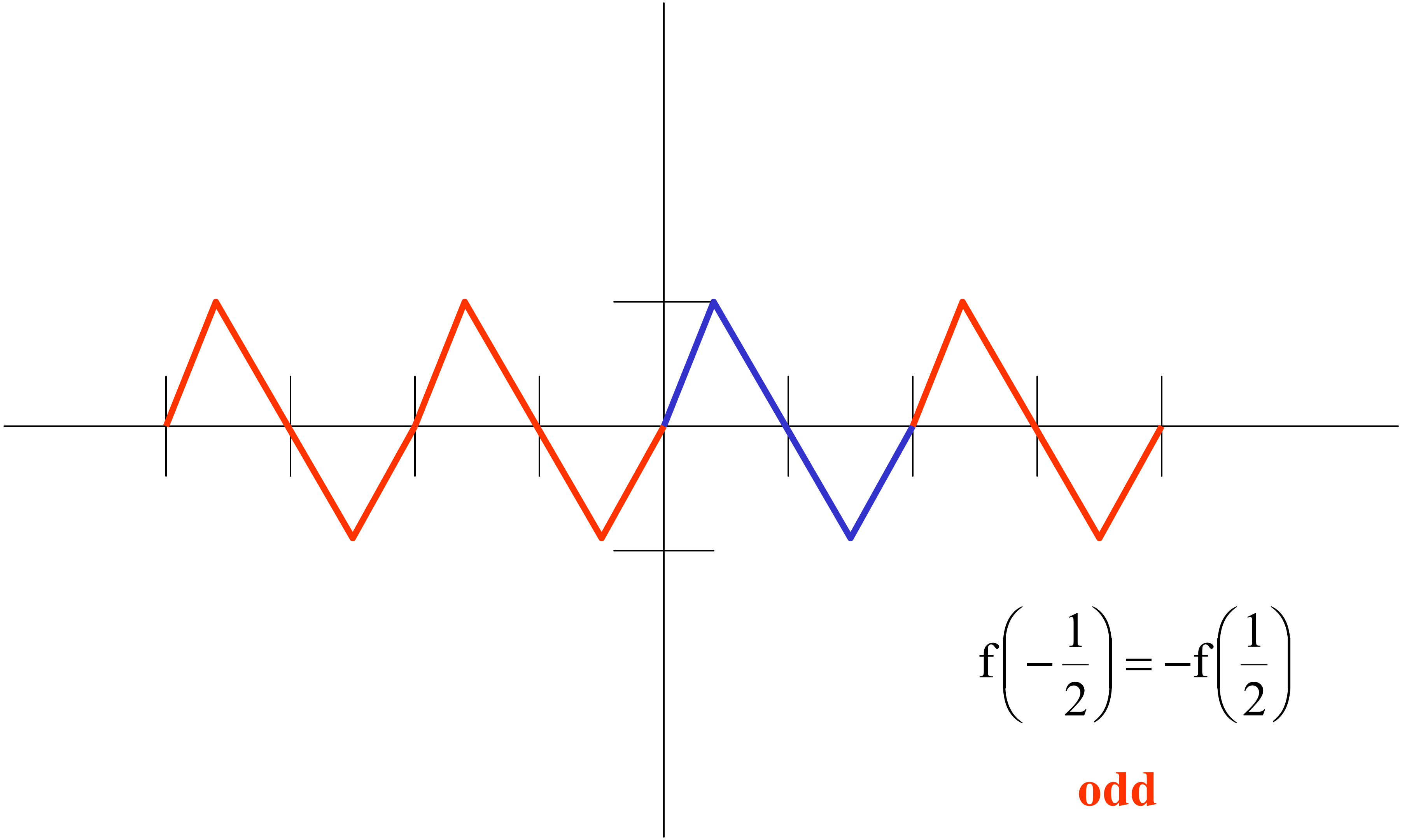
Objectives

- to identify from a graph whether a function is odd, even or neither
- to graph an even and an odd function over a given interval
- to determine from an equation whether a function is odd, even or neither
- to demonstrate basic properties of odd and even functions

Periodicity and Symmetry

- periodic: a function where for some constant p $f(x + p) = f(x)$. The smallest such p is the period of p.
- f is an odd function if $f(-x) = -f(x)$
- f is an even function if $f(-x) = f(x)$
- even functions are symmetric with respect to the y-axis
- odd functions are symmetric with respect to the origin

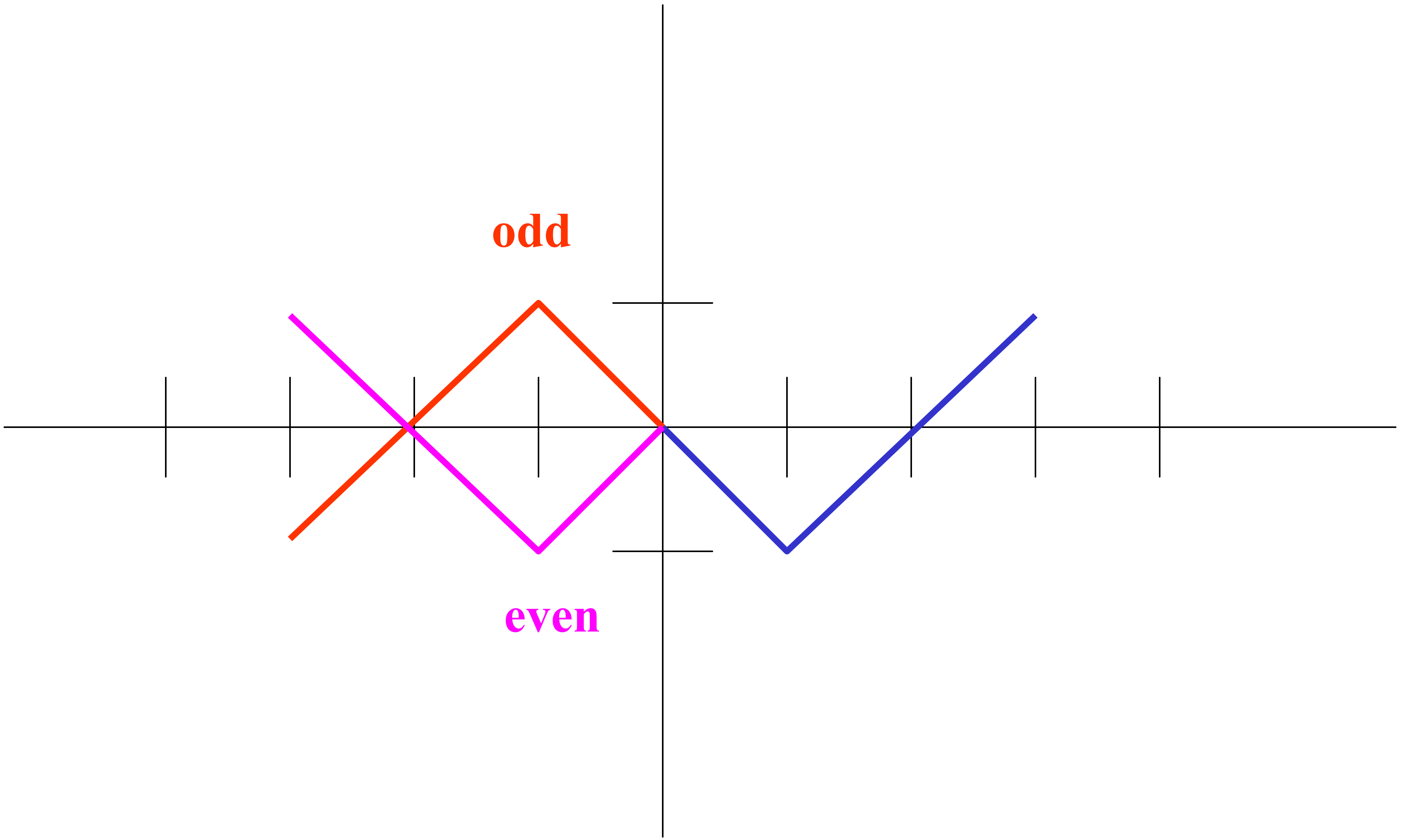
Examples for 1-6



$$f\left(-\frac{1}{2}\right) = -f\left(\frac{1}{2}\right)$$

odd

Examples for 7-10



Examples for 11-20

$$f(x) = x^3 + x$$

$$f(1) = (1)^3 + 1 = 2$$

$$f(-1) = (-1)^3 + (-1) = -2$$

odd

Examples for 21-27

- Let f(x) and g(x) be odd.
- f(- x) = - f(x) and g(- x) = - g(x)
- f(- x) • g(- x) = - f(x) • - g(x)
- f(- x) • g(- x) = f(x) • g(x)
- f(- x) • g(- x) is even

Section 13-4

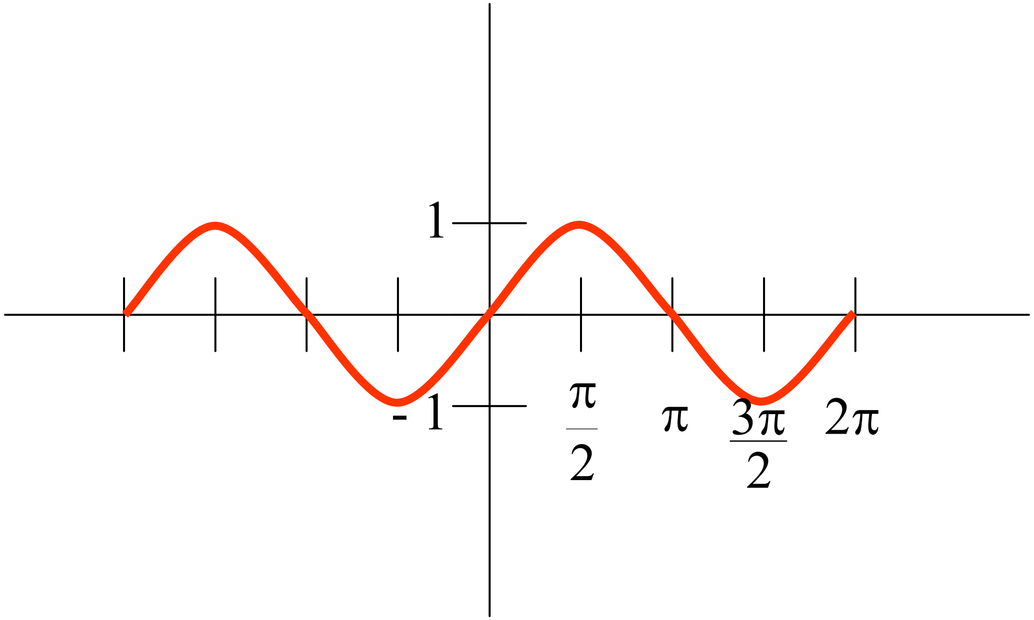
Graphs of Sine and Cosine

Objectives

- to find the amplitude, maximum, minimum and period of a trigonometric equation
- to find the trigonometric equation to describe a graph
- to find an equation of the form $y = c + a \sin bx$ that satisfies given conditions
- to graph equations of the form $y = c + a \sin bx$
- to graph both sine and cosine equations

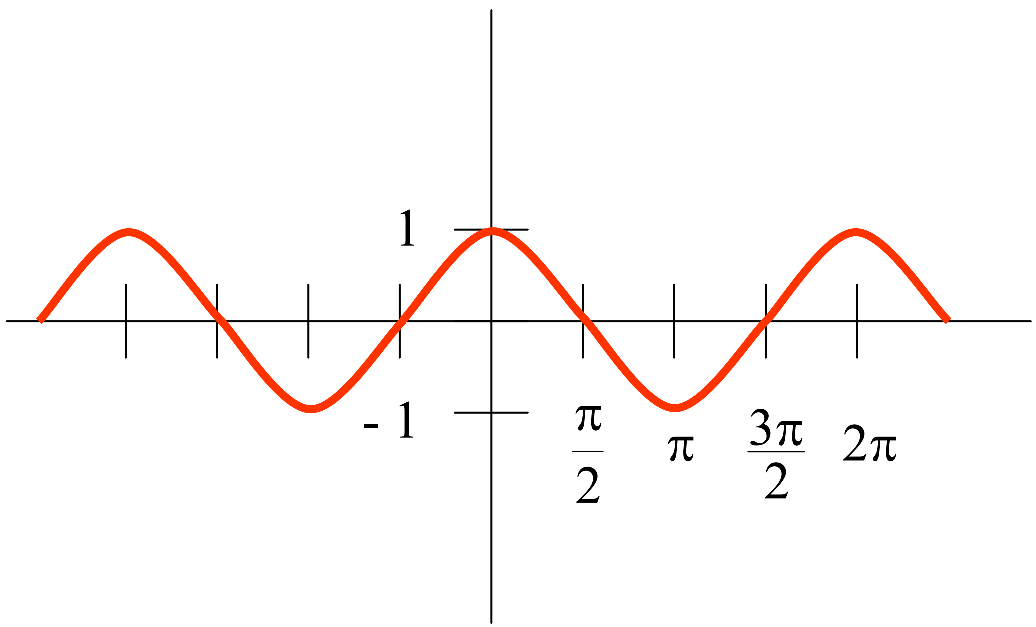
Graph of the Sine Curve

- y = sin x
  - is an odd function
  - it is symmetric with respect to the origin
  - has an amplitude of 1
  - has a period of 2π
  - has a maximum of 1 and a minimum of - 1



Graph of the Cosine Curve

- y = cos x
  - is an even function
  - is symmetric with respect to the y-axis
  - has an amplitude of 1
  - has a period of 2π
  - has a maximum of 1 and a minimum of - 1
  - If the cosine curve were shifted $\frac{\pi}{2}$ units to the right it would coincide with the sine curve



$y = c + a \sin bx$ and $y = c + a \cos bx$

- In the function $y = c + a \sin bx$ or $y = c + a \cos bx$,
  - c causes a vertical shift (up when $c > 0$ and down when $c < 0$) in the graph and is calculated from a graph by finding the average of the maximum and minimum y values
  - a affects the amplitude of the graph causing a vertical stretch or shrinking of the graph and is calculated from a graph by taking half the difference of the maximum and minimum y values
  - b affects the periodicity of the graph causing a horizontal stretch or shrinking of the graph and is calculated from a graph for the sine and cosine curves with the formula $p = \frac{2\pi}{b}$ where p is the period of the graph.

Phase Shift

- In the function $y = c + a \sin(bx - d)$ or
  $$y = c + a \cos(bx - d)$$
  - d represents the phase shift, the number of units the graph is shifted to the right if $d > 0$ or
  - the number of units the graph is shifted to the left if $d < 0$.

Graphs of the Sine and Cosine

- To look at the relationship between the unit circle and the sine curve go to: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=92
- To look at how a, b and c affect the graph of the sine and cosine curves go to: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=86

Examples for 1-12

$y = 2 \cos x$

Recall that the amplitude of $y = \cos x$ is 1, the maximum is 1, the minimum is – 1 and the period is 2π.

a affects the amplitude and therefore the maximum and minimum but not the periodicity.

2; 2, - 2; 2π

Examples for 13-20

Because the y intercept is 0 we know that it is a sine curve.

$$a = \frac{M - m}{2} = \frac{2 - (-2)}{2} = 2$$

$$p = \frac{2\pi}{b} \text{ so } 2\pi = \frac{2\pi}{b} \text{ therefore } b = \frac{2\pi}{2\pi} = 1$$

$$c = \frac{M + m}{2} = \frac{2 + (-2)}{2} = 0$$

y = 2 sin x

Examples for 21-26

$$M = 7,\ m = 1,\ \text{period} = \pi$$

$$a = \frac{M - m}{2} = \frac{7 - 1}{2} = 3$$

$$p = \frac{2\pi}{b} \ \text{ so } \pi = \frac{2\pi}{b} \ \text{ therefore } b = \frac{2\pi}{\pi} = 2$$

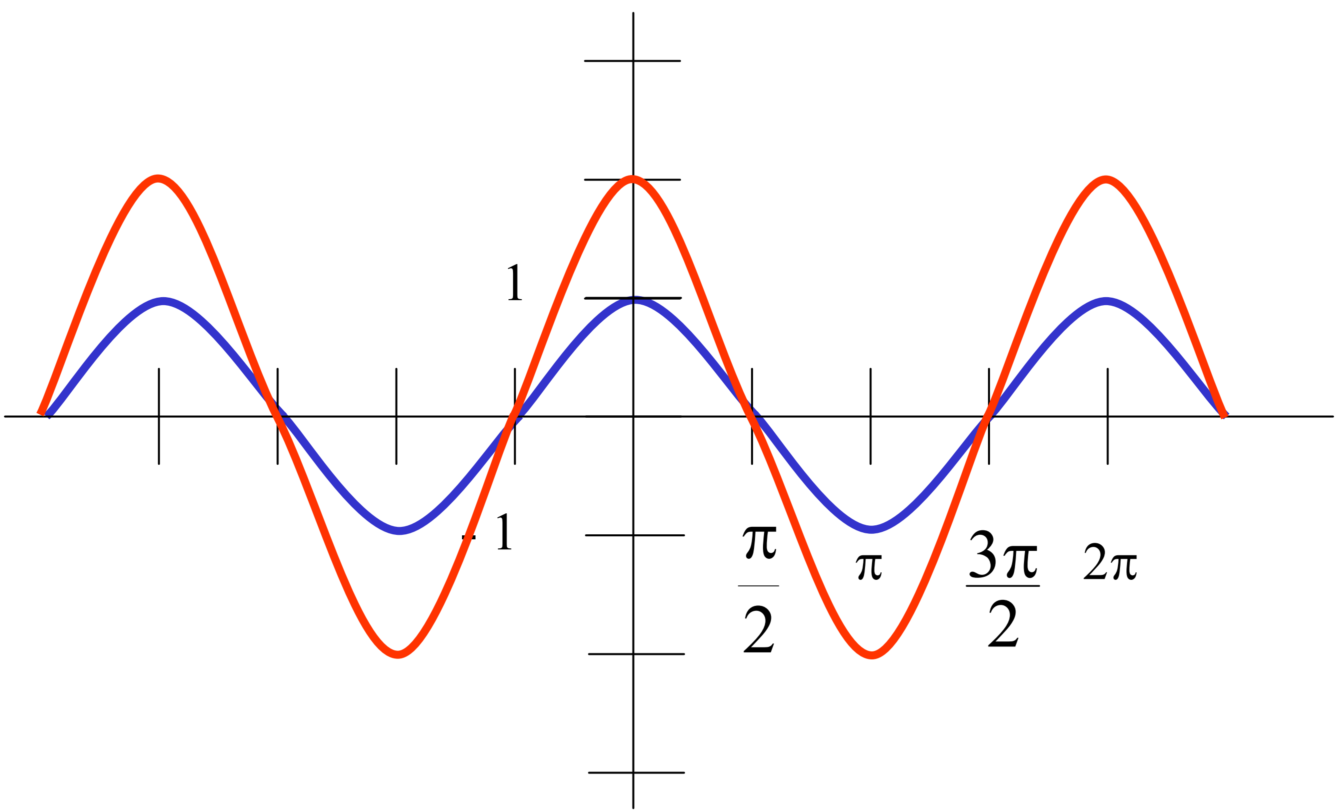
$$c = \frac{M + m}{2} = \frac{7 + 1}{2} = 4$$

$y = 4 + 3 \sin 2x$

Examples for 27-32

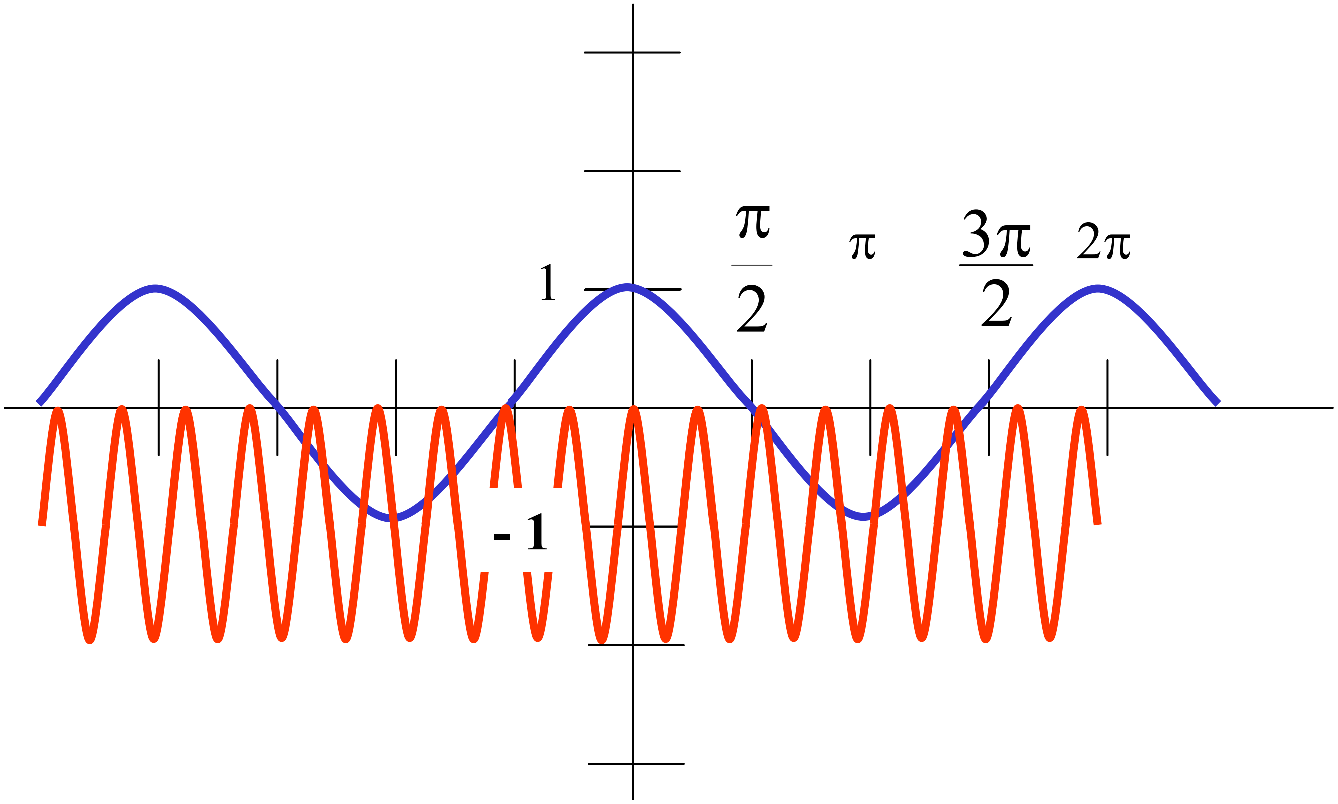
$y = 2 \cos x$

Remember that in problem #1 we said that this had an amplitude of 2, a maximum of 2, a minimum of – 2 and a period of 2π



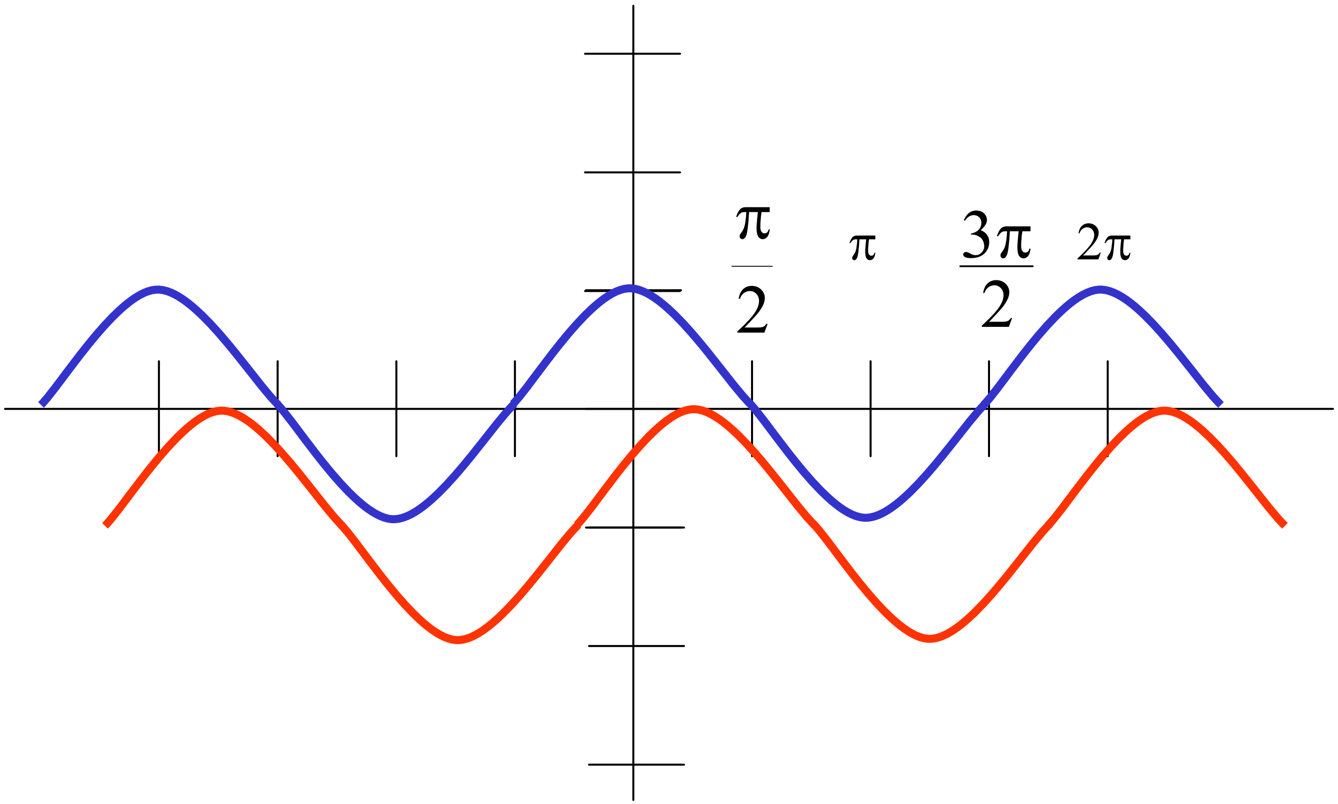
Examples for 33-38

$y = \cos 2\pi x - 1$



Examples for 39-41

$$y = \cos\left(x - \frac{\pi}{4}\right) - 1$$



Section 13-5

Graphs of Other Functions

Objectives

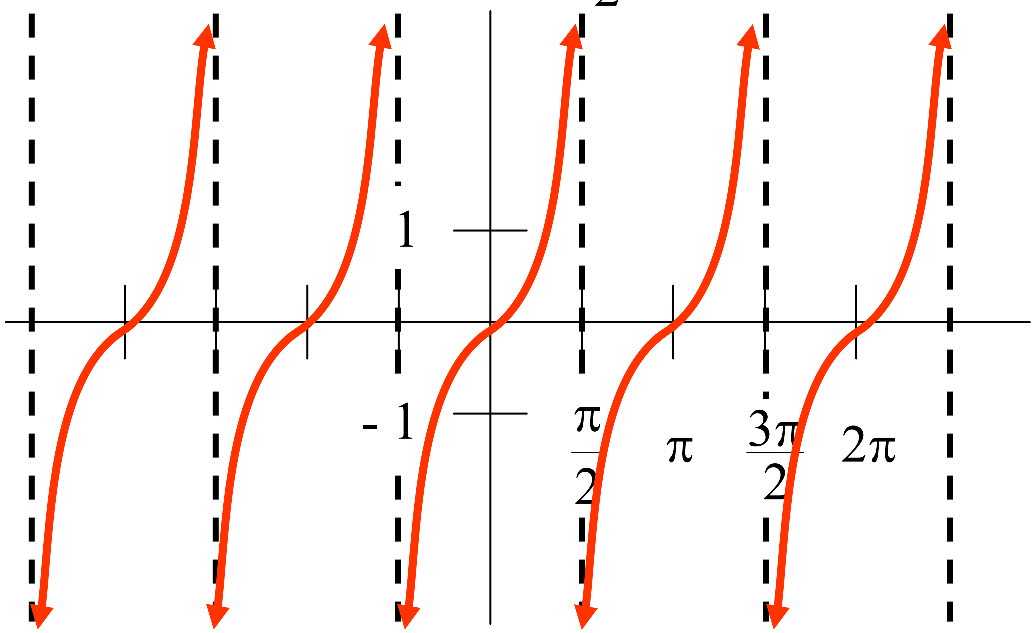
- to determine whether a function is odd, even or neither
- to sketch a graph of a trigonometric equation

Operations on Odd and Even Functions

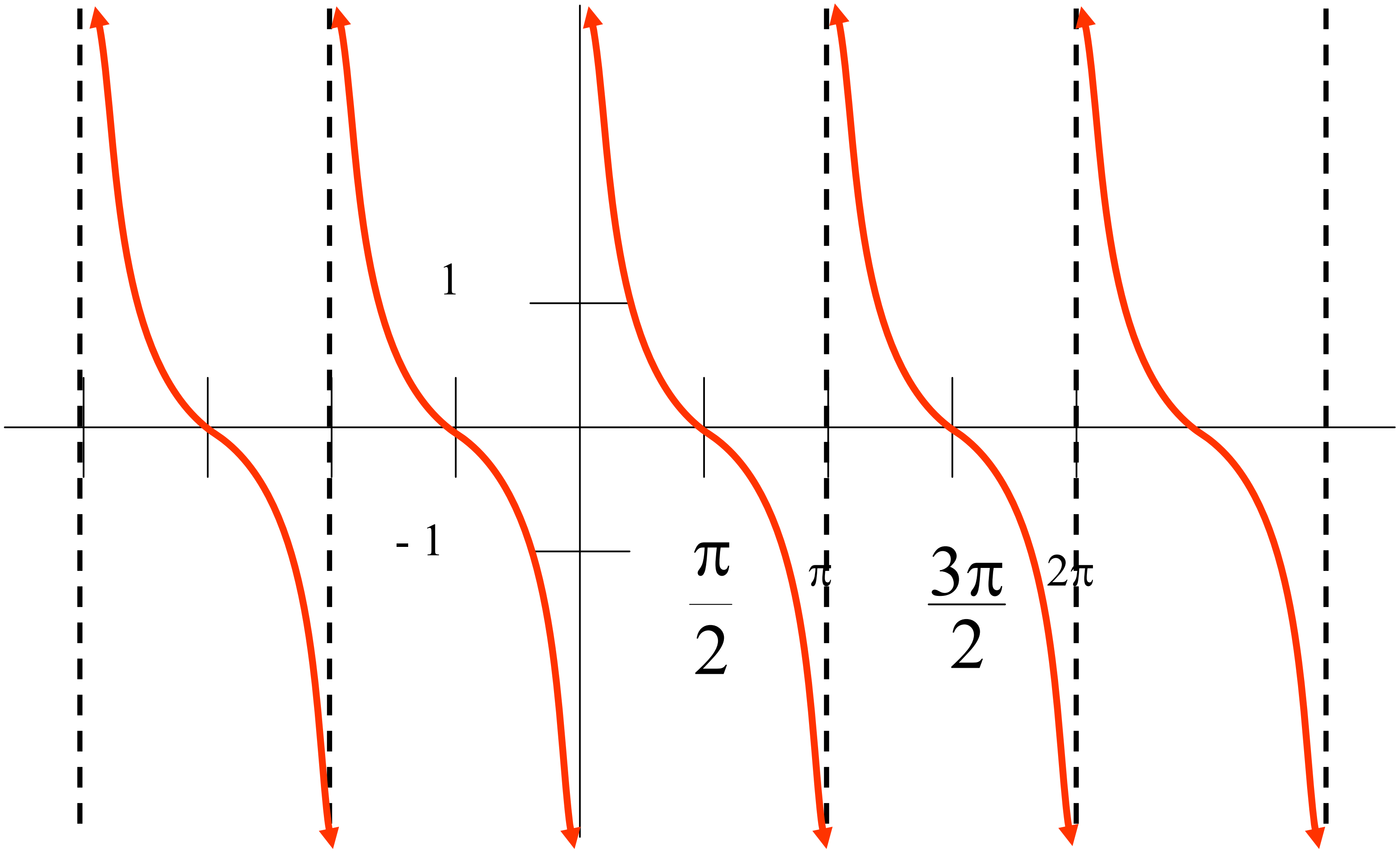
- The product or the quotient of two odd functions is even.
- The product or the quotient of two even functions is even.
- The product or the quotient of one odd and one even function is odd.
- The reciprocal of an odd function is odd.
- The reciprocal of an even function is even.
- A negative number times an odd function is even .
- A negative number times an even function is odd.
- Sums and differences of odds and evens are neither.

Tangent Curve

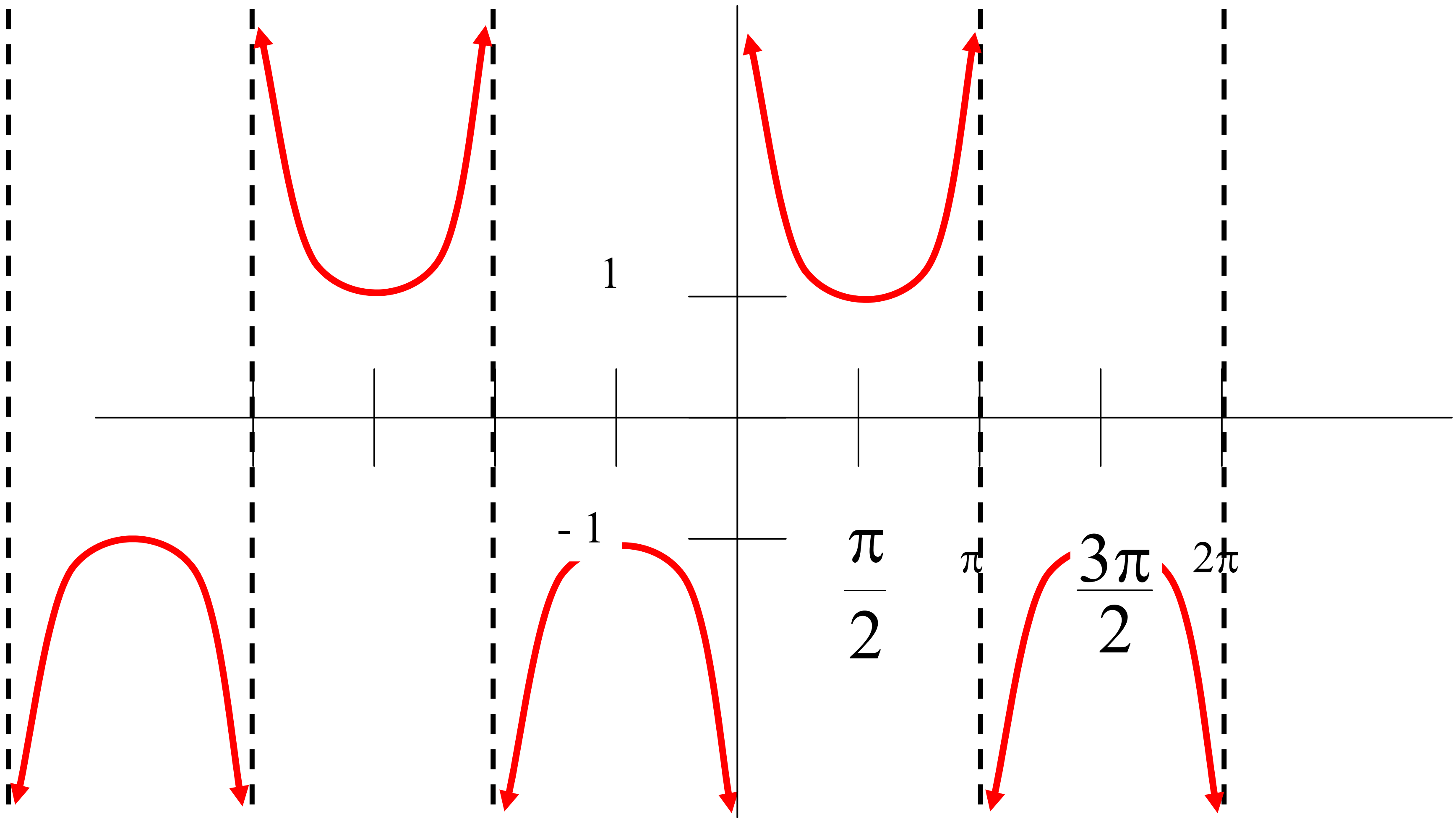
- y = tan x
  - tangent is the quotient of sine (odd) and cosine (even) therefore tangent is an odd function
  - tangent is symmetric with respect to the origin
  - tangent has no amplitude, no maximum and no minimum values
  - tangent has a period of π
  - tangent has asymptotes at $\pm\frac{\pi}{2}$ and every π units after that.



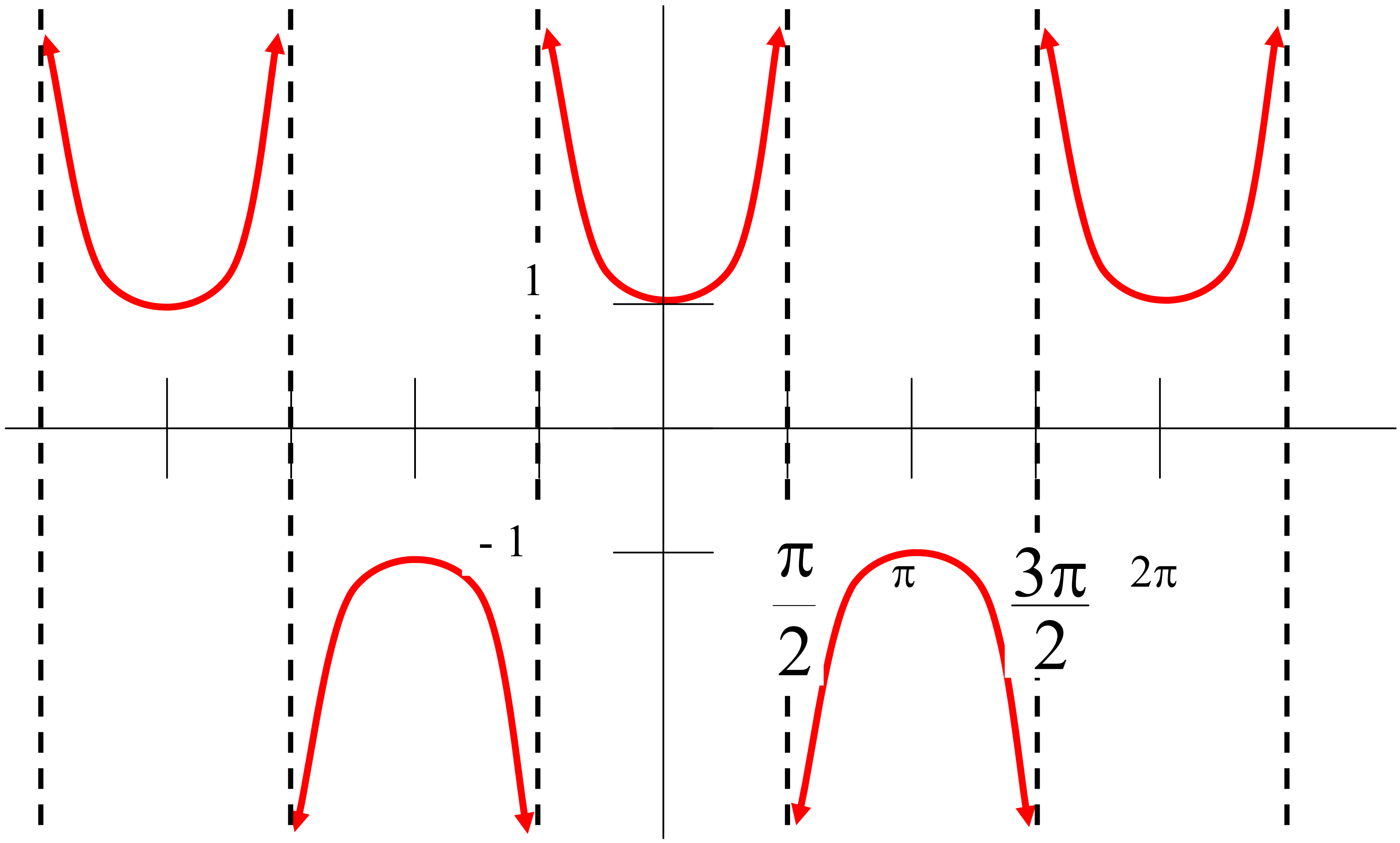
Cotangent Curve: y = cot x



Cosecant Curve: y = csc x



Secant Curve: y = sec x



Examples for 1-8

$x^2 \cot x$

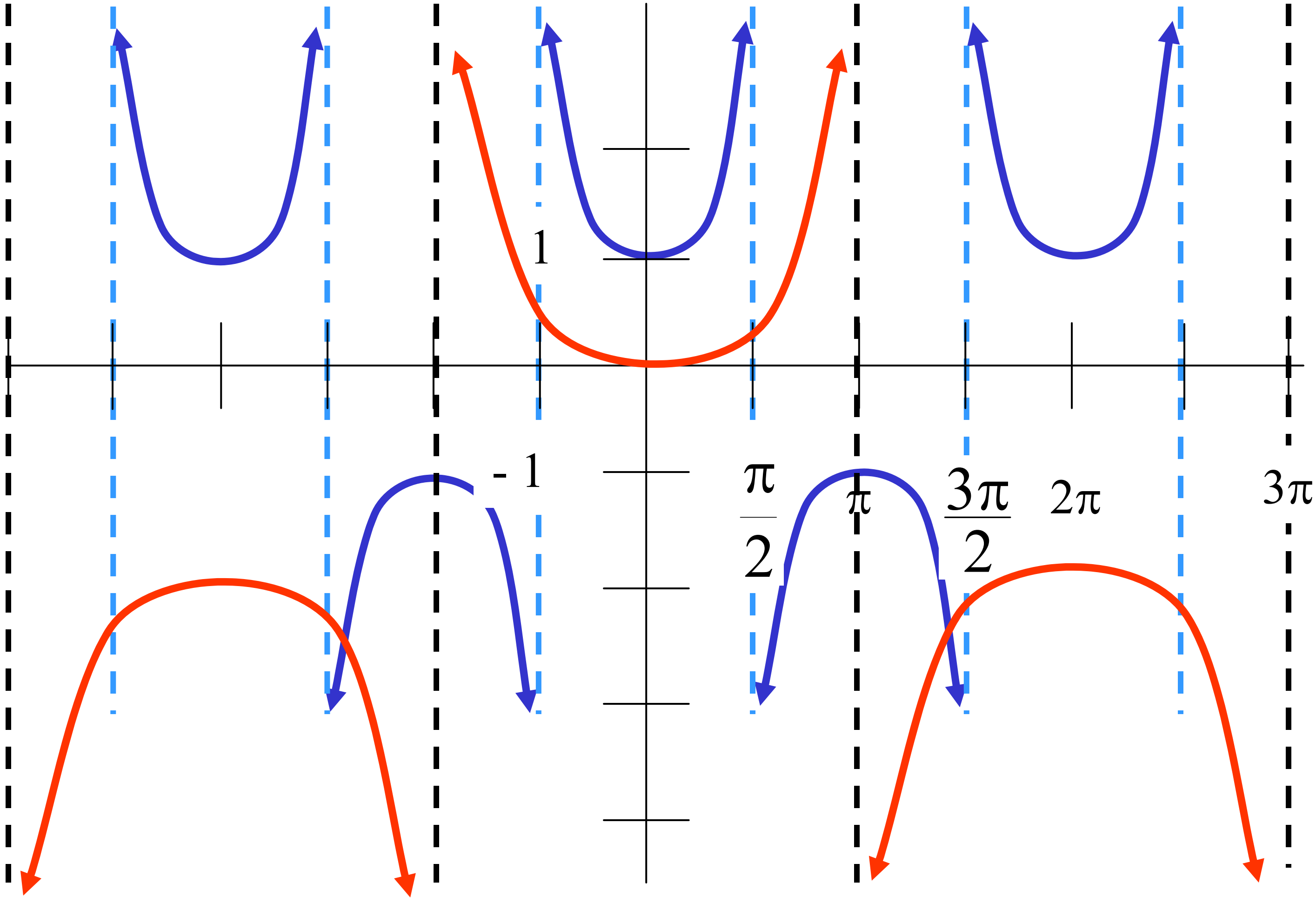
x^2 will always be positive so it will not affect the sign of the answer

cot x is odd because tan x is odd

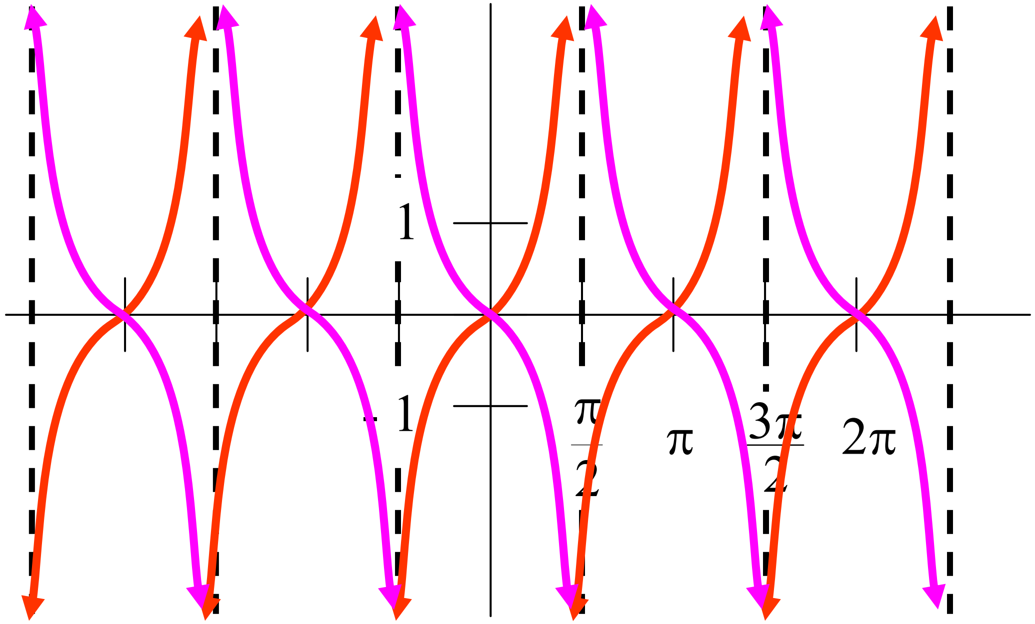
odd

Examples for 9-18

$$y = \sec\frac{1}{2}x - 1$$



Examples for 19-22

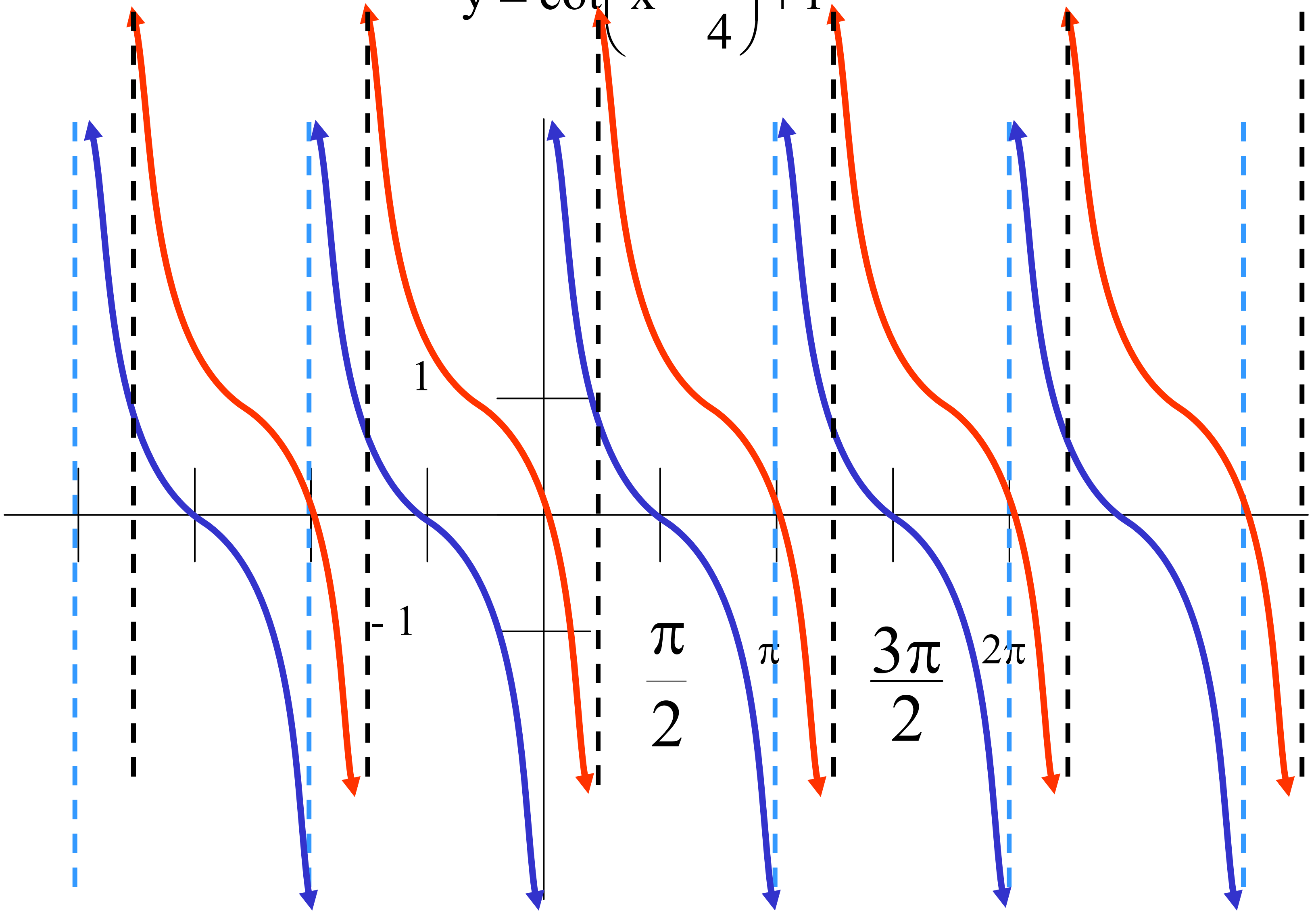


y = tan x

y = - tan x

Examples for 23-28

$$y = \cot\left(x - \frac{\pi}{4}\right) + 1$$



Section 13-6

The Fundamental Identities

Objectives

- to simplify trigonometric statements
- to write one trigonometric function in terms of another
- to prove basic trigonometric identities

Reciprocal Identities

$$\sin\alpha = \frac{1}{\csc\alpha} \qquad \sin\alpha\csc\alpha = 1 \qquad \csc\alpha = \frac{1}{\sin\alpha}$$

$$\cos\alpha = \frac{1}{\sec\alpha} \qquad \cos\alpha\sec\alpha = 1 \qquad \sec\alpha = \frac{1}{\cos\alpha}$$

$$\tan\alpha = \begin{cases} \dfrac{1}{\cot\alpha} \\ \dfrac{\sin\alpha}{\cos\alpha} \end{cases} \qquad \tan\alpha\cot\alpha = 1 \qquad \cot\alpha = \begin{cases} \dfrac{1}{\tan\alpha} \\ \dfrac{\cos\alpha}{\sin\alpha} \end{cases}$$

Cofunction Identities

$\sin\theta = \cos(90° - \theta)$

$\tan\theta = \cot(90° - \theta)$

$\sec\theta = \csc(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$

$\cot\theta = \tan(90° - \theta)$

$\csc\theta = \sec(90° - \theta)$

Pythagorean Identities

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$1 + \tan^2\alpha = \sec^2\alpha$$

$$1 + \cot^2\alpha = \csc^2\alpha$$

General Strategies for Proving Identities

- Simplify the more complicated side of the identity until it is identical to the other side.
- Transform both sides of the identity into the same expression.

Special Strategies for Proving Identities

- Express functions in terms of sines and cosines.
- Look for expressions to which the Pythagorean identities can be applied.
- Use factoring.
- Combine terms on each side of the identity into a single fraction.
- Multiply one side of the equation by an expression equal to one.

Examples for 1-24

$$\cos \alpha \csc \alpha \tan \alpha$$

$$\cos\alpha\left(\frac{1}{\sin\alpha}\right)\left(\frac{\sin\alpha}{\cos\alpha}\right)$$

1

Examples for 25-30

$\tan\alpha \sec\alpha$; $\sin\alpha$

$$\left(\frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{1}{\cos\alpha}\right)$$

$$\frac{\sin\alpha}{\cos^2\alpha}$$

$$\frac{\sin\alpha}{1-\sin^2\alpha}$$

Examples for 31-38

$$\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$\left(1 - \sin^2\theta\right) - \sin^2\theta = 1 - 2\sin^2\theta$$

$$1 - 2\sin^2\theta = 1 - 2\sin^2\theta$$

Examples for 39-48

$$\frac{\tan\theta}{1+\sec\theta}+\frac{1+\sec\theta}{\tan\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{1}{\cos\theta}}+\frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta+1}{\cos\theta}}+\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\cos\theta+1}\right)+\left(\frac{\cos\theta+1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$\frac{\sin\theta}{\cos\theta+1}+\frac{\cos\theta+1}{\sin\theta}$$

$$\frac{\sin^2\theta+\cos^2\theta+2\cos\theta+1}{\sin\theta(\cos\theta+1)}$$

$$\frac{1+2\cos\theta+1}{\sin\theta(\cos\theta+1)}$$

$$\frac{2+2\cos\theta}{\sin\theta(\cos\theta+1)}$$

$$\frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$\frac{2}{\sin\theta}$$

$$2\csc\theta$$

Section 13-7

Trigonometric Addition Formulas

Objectives

- to find the exact value of trigonometric sums and differences
- to simplify trigonometric sums and differences
- to prove basic trigonometric identities
- to simplify trigonometric statements to a function of a single angle

Addition Formulas for Sine and Cosine

- $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
- $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$
- $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
- $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

Examples for 1-6

$$\cos 105°$$

$$\cos (60° + 45°)$$

$$\cos 60° \cos 45° – \sin 60° \sin 45°$$

$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2}-\sqrt{6}}{4}$$

Examples for 7-14

$$\cos 20° \cos 40° - \sin 20° \sin 40°$$

$$\cos (20° + 40°)$$

$$\cos 60°$$

$$\frac{1}{2}$$

Examples for 15-20

$$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$$

$$\sin\frac{\pi}{2}\cos\theta+\cos\frac{\pi}{2}\sin\theta$$

$$\mathbf{(1)(\cos\theta) + (0)(\sin\theta)}$$

$$\mathbf{\cos\theta}$$

Examples for 21-24

$$\sin\left(\frac{\pi}{6}+\theta\right)+\sin\left(\frac{\pi}{6}-\theta\right)$$

$$\sin\frac{\pi}{6}\cos\theta+\cos\frac{\pi}{6}\sin\theta+\sin\frac{\pi}{6}\cos\theta-\cos\frac{\pi}{6}\sin\theta$$

$$2\sin\frac{\pi}{6}\cos\theta$$

$$2\left(\frac{1}{2}\right)\cos\theta$$

$$\mathbf{\cos\theta}$$

Examples for 25-30

$$\cos\theta\cos\phi\,(\tan\theta + \tan\phi) = \sin(\theta + \phi)$$

$$\cos\theta\cos\phi\left(\frac{\sin\theta}{\cos\theta} + \frac{\sin\phi}{\cos\phi}\right)$$

$$\sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\sin(\theta + \phi)$$

Examples for 31-35

$$\sin\alpha = -\frac{3}{5} \quad \cos\beta = \frac{8}{17}$$

$$\sin(\alpha+\beta) = \left(-\frac{3}{5}\right)\left(\frac{8}{17}\right) + \left(-\frac{4}{5}\right)\left(-\frac{15}{17}\right)$$

$$\sin(\alpha+\beta) = \frac{36}{85}$$

$$\cos(\alpha+\beta) = \left(-\frac{4}{5}\right)\left(\frac{8}{17}\right) - \left(-\frac{3}{5}\right)\left(-\frac{15}{17}\right)$$

$$\cos(\alpha+\beta) = -\frac{77}{85}$$

Quadrant II

Section 13-8

Double Angle and Half Angle Formulas

Objectives

- to simplify a trigonometric function of a single angle
- to find the exact value of a function
- to find either a double angle or half angle value given a quadrant and one trigonometric function value for the angle
- to prove trigonometric identities

Double-Angle Formulas for Sine and Cosine

- $\sin 2\alpha = 2\sin \alpha \cos \alpha$
- $\cos 2\alpha = \cos^2 \alpha - \sin^2\alpha$
- $\cos 2\alpha = 1 – 2\sin^2\alpha$
- $\cos 2\alpha = 2\cos^2\alpha - 1$

Half-Angle Formulas for Sine and Cosine

- $$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

- $$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Examples for 1-12

$$1-2\sin^2\frac{\theta}{2}$$

$$\cos 2\left(\frac{\theta}{2}\right)$$

$$\mathbf{\cos\theta}$$

Examples for 13-24

$$2 \sin 15° \cos 15°$$

$$\sin 2(15)$$

$$\sin 30$$

$$\frac{1}{2}$$

Examples for 25-28

$$0 < \alpha < 180° \qquad \cos\alpha = -\frac{4}{5}$$

$$\cos 2\alpha$$

$$2\cos^2\alpha - 1$$

$$2\left(-\frac{4}{5}\right)^2 - 1$$

$$2\left(\frac{16}{25}\right) - 1$$

$$\frac{7}{25}$$

Examples for 29-32

$$180° < \alpha < 360° \qquad \cos\alpha = \frac{8}{17}$$

$$\sin\alpha = -\frac{15}{17}$$

$$\sin 2\alpha$$

$$2\sin\alpha\cos\alpha$$

$$2\left(-\frac{15}{17}\right)\left(\frac{8}{17}\right)$$

$$-\frac{240}{289}$$

Examples for 33-40

$\cot\alpha + \tan\alpha = 2\csc 2\alpha$

$$2\left(\frac{1}{\sin 2\alpha}\right)$$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$$

$$2\left(\frac{1}{2\sin\alpha\cos\alpha}\right)$$

$\cot\alpha + \tan\alpha$

$$\frac{1}{\sin\alpha\cos\alpha}$$

$$\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}$$

$$\frac{\sin^2\alpha}{\sin\alpha\cos\alpha} + \frac{\cos^2\alpha}{\sin\alpha\cos\alpha}$$

Section 13-9

Formulas for Tangents

Objectives

- to simplify trigonometric functions of a single angle
- to find the exact value of non pattern angles
- to find tangent sums, differences, half angles and double angles given the quadrant and one trigonometric value for each angle
- to prove trigonometric identities

Tangent Addition Formulas

- $$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

- $$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Formula for Tangent

- $$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

Half-Angle Formulas for Tangent

- $$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$
- $$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$
- $$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

Examples for 1-8

$$\frac{\tan 80° - \tan 20°}{1 + \tan 80° \tan 20°}$$

$\tan (80° - 20°)$

$\tan 60°$

Examples for 9-20

$$\tan 75°$$

$$\tan (45° + 30°)$$

$$\frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$\frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)}$$

$$2 + \sqrt{3}$$

Examples for 21-24

$$\tan\alpha = -\frac{8}{15} \qquad \tan\beta = \frac{3}{4}$$

α is a second quadrant angle

$$\tan(\alpha + \beta)$$

$$\frac{-\frac{8}{15} + \frac{3}{4}}{1 - \left(-\frac{8}{15}\right)\left(\frac{3}{4}\right)}$$

$$\frac{13}{84}$$

Examples for 25-32

$$\tan\left(\frac{\pi}{4}-\alpha\right)=\frac{1-\tan\alpha}{1+\tan\alpha}$$

$$\frac{\tan\frac{\pi}{4}-\tan\alpha}{1+\left(\tan\frac{\pi}{4}\right)\left(\tan\alpha\right)}=$$

$$\frac{1-\tan\alpha}{1+(1)\left(\tan\alpha\right)}=$$

$$\frac{1-\tan\alpha}{1+\tan\alpha}=$$