Section 4-1

Graphing a System of Equations

Objectives for Section 4-1

- to determine from slope and y-intercepts that a linear system in two variables is either consistent, inconsistent or dependent
- to determine the solution of a linear system in two variables from its graph

Systems of Linear Equations in Two Variables

- system of linear equations: a set of equations in the same two variables
- solution: all ordered pairs that satisfy both equations, or the intersection of the two graphs
  - a linear system may have a single point as a solution when the lines intersect
  - a linear system may have no solution if the lines are parallel
  - a linear system may have an infinite set of points in its solution when the lines coincide i.e. the two equations may look different but are really describing the same line

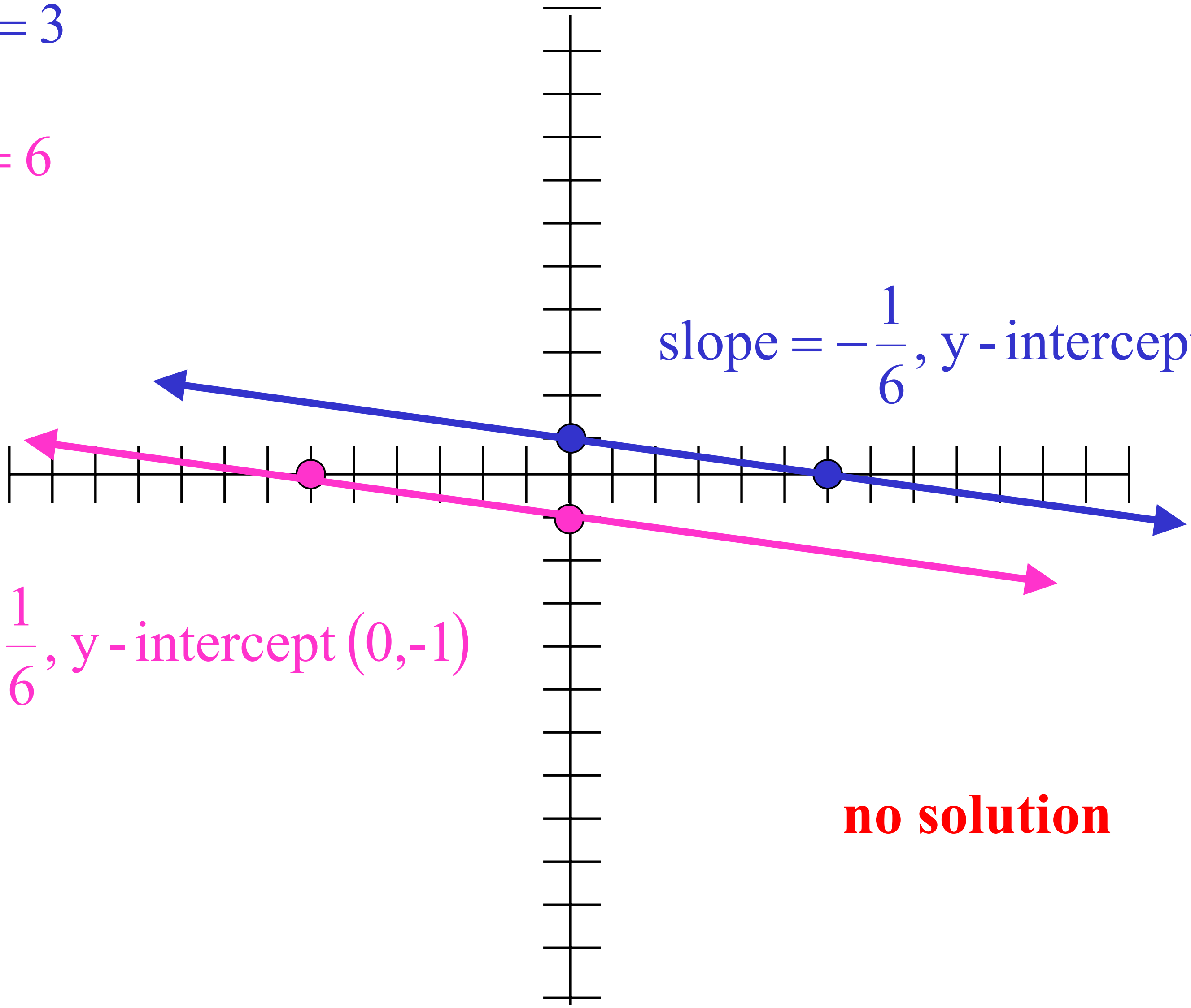
Systems of Linear Equations in Two Variables

- consistent equations: a system with at least one solution
- dependent equations: a system with infinite solutions
- inconsistent equations: a system with no solutions
- equivalent systems: systems with the same solution set

Example for 1-18

$$\frac{1}{2}x + 3y = 3$$

$$-x - 6y = 6$$



$$\text{slope} = -\frac{1}{6}, \text{ y-intercept } (0,1)$$

$$\text{slope} = -\frac{1}{6}, \text{ y-intercept } (0,\text{-}1)$$

no solution

Example for 19-24

x + y = 4

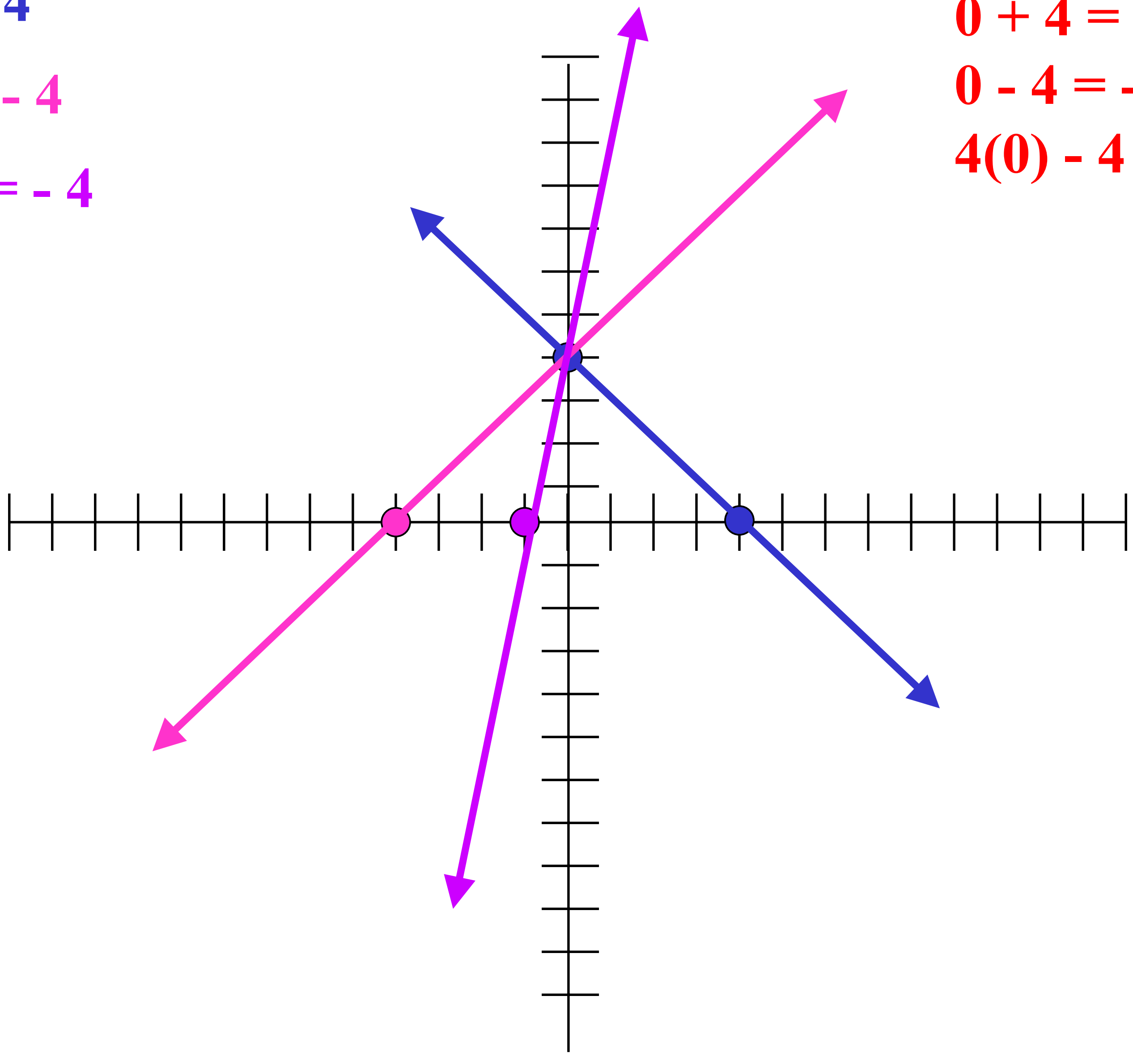
x - y = - 4

4x - y = - 4

0 + 4 = 4

0 - 4 = - 4

4(0) - 4 = - 4



Section 4-2

Solving a System of Equations

Objectives for Section 4-2

- to solve systems by linear combination
- to solve systems by substitution
- to use substitution to solve a system that isn’t linear

Systems of Linear Equations in Two Variables

- the goal of the solution process is to eliminate one of the variables.
  - graph the equations and identify the intersection: not a very reliable method since your answer is only as accurate as your graph
  - add the two equations together: works only if the coefficients of the same variable are opposites
  - subtract the two equations: works only if the coefficients of the same variable are equal
  - multiply then add/subtract: works with reliably with any system. You may pick any constant to multiply through either equation to create opposite /equal coefficients
  - substitution: works reliably with any system but is easiest when one of the coefficients is either 1 or - 1. You must rearrange one of the equations into "x =" or "y ="

Systems of Linear Equations in Two Variables

- solve for the remaining unknown variable
- substitute the value back into one of the original equations and solve for the second variable
- check to make sure your point works in both of the original equations.
- Remember that if both x and y cancel and you are left with a true statement then the answer is infinite; if both variables cancel and you are left with nonsense the answer is null set.

Example for 1-12, 19-30

$$8x - 5y = -5$$
$$3x + 2y = 2$$

$$2[8x - 5y = -5]$$
$$5[3x + 2y = 2]$$

$$16x - 10y = -10$$
$$\underline{15x + 10y = 10}$$

$$31x = 0$$
$$x = 0$$

$$3(0) + 2y = 2$$
$$y = 1$$

(0, 1)

Example for 13-18, 19 - 30

$$x - 15y = 3$$
$$7x - 9y = -11$$

$$x = 15y + 3$$

$$7(15y + 3) - 9y = -11$$

$$105y + 21 - 9y = -11$$

$$96y = -32$$

$$y = -\frac{1}{3}$$

$$7x - 9\left(-\frac{1}{3}\right) = -11$$

$$7x + 3 = -11$$

$$x = -2$$

$$\left(-2, -\frac{1}{3}\right)$$

Example for 31 & 32

$$\frac{5}{x} - \frac{2}{y} = 5$$

$$\frac{4}{x} - \frac{3}{y} = -3$$

$$3\left[\frac{5}{x} - \frac{2}{y} = 5\right]$$

$$2\left[\frac{4}{x} - \frac{3}{y} = -3\right]$$

$$\frac{15}{x} - \frac{6}{y} = 15$$

$$\frac{8}{x} - \frac{6}{y} = -6$$

$$\frac{7}{x} = 21$$

$$x = \frac{1}{3}$$

$$15 - \frac{2}{y} = 5$$

$$-\frac{2}{y} = -10$$

$$y = \frac{1}{5}$$

$$\left(\frac{1}{3}, \frac{1}{5}\right)$$

Section 4-3

Determinants

Objectives for Section 4-3

- to evaluate second order determinants
- to use Cramer's Rule to solve a system

Determinants

- determinant: a real number associated with a square matrix
- order of a square matrix: either the number of rows or the number of columns
- Determinant of a 2x2 matrix (2^{nd} order determinant):

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Cramer's Rule

- The solution of a system of n linear equations in n variables is given by

$$x = \frac{D_x}{D}, \; y = \frac{D_y}{D}, \ldots$$

  where D is the determinant of the matrix of coefficients of the variables ($D \neq 0$) and D_x, D_y, ... are derived from D by replacing the coefficients of x, y, ... respectively, by the constants.
- When $D = 0$ the system may have no solution or it may have infinitely many solutions.
- If $D = 0$ and $D_y \neq 0$, then the equations are inconsistent and the equations are parallel.
- If $D = 0$ and $D_y = 0$, then the equations are dependent and their graphs coincide.

Example for 1-8

$$\begin{vmatrix} 8 & -2 \\ -3 & 7 \end{vmatrix}$$

(8)(7) - (- 3)(- 2)

56 - 6

50

Example for 9-22

$$4x + 5y = 3$$
$$5x + 6y = 2$$

$$D = \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1$$

$$D_x = \begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} = 18 - 10 = 8$$

$$D_y = \begin{vmatrix} 4 & 3 \\ 5 & 2 \end{vmatrix} = 8 - 15 = -7$$

$$x = \frac{8}{-1}, y = \frac{-7}{-1}$$

(- 8, 7)

Section 4-4

Using Two Variables to Solve Problems

Objectives for Section 4-4

- to use systems to solve two variable word problems

Problem Solving Using Systems

- First, you should follow the same steps to solve these word problems as you should have been using in the past.
- Second, for these problems you will have two variables to work with.
- Third, when you begin translating you will have to have two different equations to work with i.e. you must set up a system of equations.
- Fourth, you may use whichever method is appropriate to solve your system of equations.
- Fifth, remember to check your answer not only in your system but also to make sure that it is a plausible answer for the word problem.

Example for 1-16

Let L = fill rate of a large pipe

Let S = fill rate of a small pipe

$$(3L)(2) + (2S)(2) = 1000$$

$$\frac{5}{6}(3L) + 5(2S) = 1000$$

$$3L + 2S = 500$$

$$\frac{1}{2}L + 2S = 200$$

$$\frac{5}{2}L = 300$$

L = 120 L/h, S = 70 L/h

Example for 17-20

$$y = A x^2 + Bx; \ (4, 28)\ (3, 12)$$

$$28 = 16A + 4B$$
$$12 = 9A + 3B$$

$$7 = 4A + B$$
$$4 = 3A + B$$

$$7 - 4A = 4 - 3A$$

3 = A

- 5 = B

Section 4-5

Graphing a System of Linear Inequalities

Objectives for Section 4-5

- to graph the solution to a system of linear inequalities

Linear Inequalities in Two Variables

- When you replace the equals sign in a linear equation with two variables with an inequality symbol you create a linear inequality.
  - the solution of a linear inequality in two variables is still any ordered pair of numbers that makes a true statement of order.
  - all linear inequalities have an infinite solution set since the graph of a linear inequality is either an open or closed half-plane.

Linear Inequalities in Two Variables

- To graph a linear inequality you must:
  - first graph its boundary which is the linear equation associated to the inequality (the equation you get when you replace the inequality symbol with an equals sign).
  - second this line will be solid if the inequality was either $\geq$ or $\leq$. It will be a dashed line if the inequality was either $<$ or $>$.
  - third choose a point from one side of the boundary line and test it in the inequality if it makes a true statement of order then shade that side of the boundary line; if it makes a false statement of order then shade the other side of the boundary line.

Linear Inequalities in Two Variables

- Systems of linear inequalities: you must solve a system of linear inequalities by graphing
  - first, graph each inequality in the system on the same coordinate plane.
  - second, the solution to the system occurs everywhere that the shaded regions overlap.

Example for 1-36

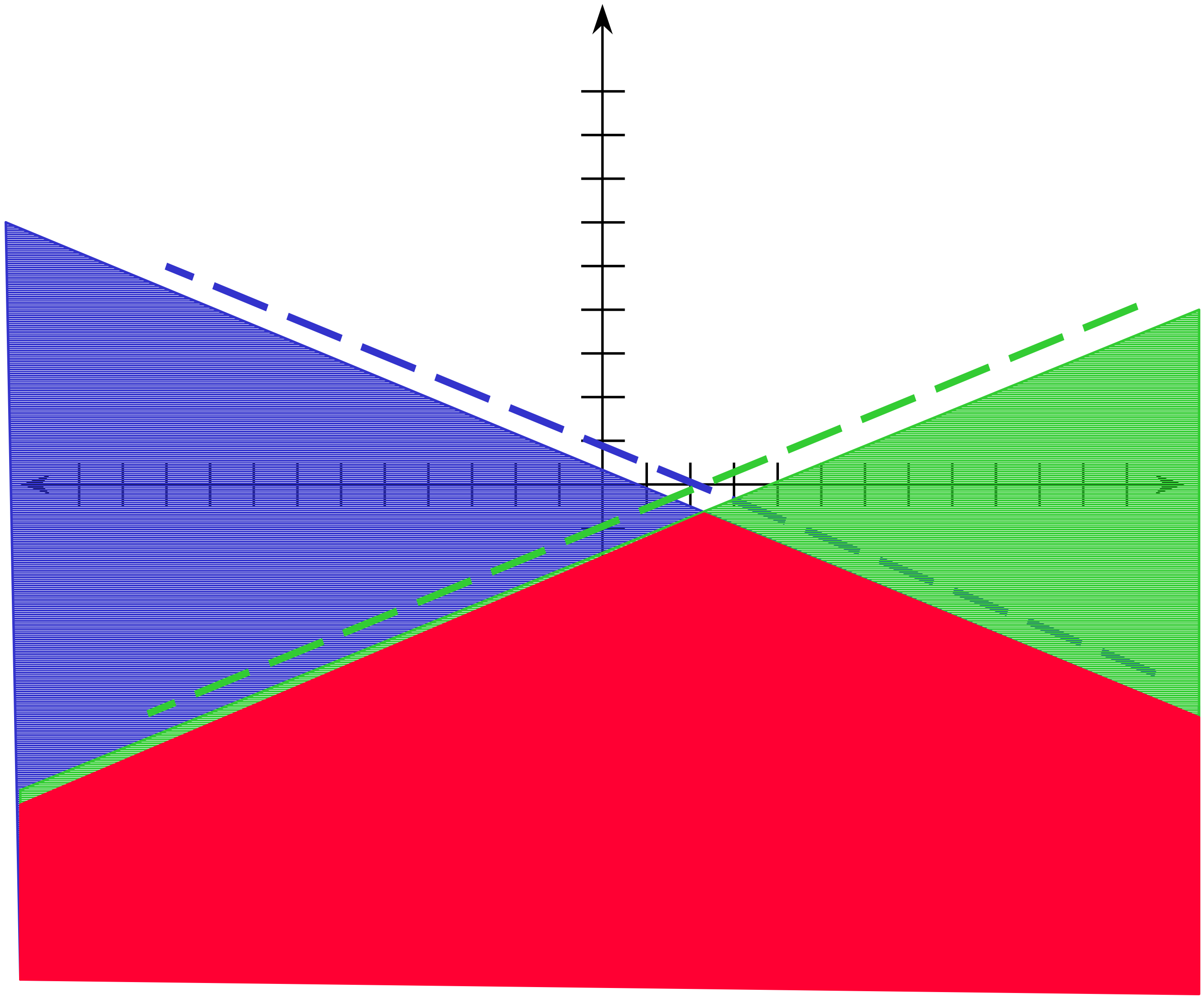
$x + 2y < 2$ & $x - 2y > 2$

boundary for $x + 2y < 2$ is $x + 2y = 2$ x-int (2, 0) y-int (0, 1)

boundary for $x - 2y > 2$ is $x - 2y = 2$ x-int (2, 0) y-int (0, - 1)

see graph on next slide

Graph Example for 1-36



Section 4-6

Linear Programming

Objectives for Section 4-6

- to graph a system of linear inequalities that represents constraints on a word problem
- to identify the corner points of the solution set to the system
- to find the value of an expression for the corner points
- to state the maximum and minimum values of the expression within the constraints

Linear Programming

- constraints: a set of conditions usually represented by linear inequalities
- feasibility region: the solution set to the system of constraints
- corner points: the points of intersection among the boundary lines representing the constraints
- If a and b are any real numbers, and if the linear expression $ax + by$ has a maximum value over a feasibility region that is the intersection of a finite number of closed half-planes and that has corner points, then the maximum occurs for the coordinates of some corner point. The minimum, if one exists, will also at the coordinates of a corner point.

Example for 1-19

$0 \le x \le 8$
$x + y \le 9$
$x + 4y \le 24$
$3x + y$

(0, 6)
(4, 5)
(8, 1)

$3(0) + 6 = 6$
$3(4) + 5 = 17$
$3(8) + 1 = 25$

max = 25
no min