Section 8-1

Sequences

Objectives for 8-1

- to find the first four terms of a defined sequence
- to determine whether a sequence is arithmetic
- to specify a sequence recursively
- to specify a sequence explicitly

Sequences

- terms: the numbers in a sequence
- finite sequences: a sequence with a last term
- infinite sequence: a sequence with no last term
- arithmetic sequence (progression): a sequence of numbers with a common difference ($d = a_{n+1} - a_n$)
- recursive rule: states the value of the first term (a_1), and states the relationship between any subsequent term (a_{n+1}) and the term(s) before it (a_n) in the sequence
  - If the first term in an arithmetic sequence is a_1, and d is the common difference, then $a_{n+1} = a_n + d$.
- explicit rule: gives the value of any term in the sequence (a_n) based on its position in the sequence (n)
  - If the first term in an arithmetic sequence is a_1, and d is the common difference, then $a_n = a_1 + (n - 1)d$.

Example for 1-12

$$a_n = 8 - 3n$$

$$a_1 = 8 - 3(1) = 5$$
$$a_2 = 8 - 3(2) = 2$$
$$a_3 = 8 - 3(3) = -1$$
$$a_4 = 8 - 3(4) = -4$$

$$2 - 5 = -3$$
$$-1 - 2 = -3$$
$$-4 - (-1) = -3$$

5, 2, - 1, - 4; arithmetic d = - 3

Example for 13-18

7, 14, 28, 56, ...

$$\mathbf{a_1 = 7,\ a_{n+1} = 2a_n}$$

Example for 19-24

$a, ar, ar^2, ar^3, \ldots$

Notice that:

$a_1 = ar^0$

$a_2 = ar^1$

$a_3 = ar^2$

$a_4 = ar^3$

so that every term has both a factor (a) and a factor (r), and that the exponent of the factor (r) is always one less than the position of the term (n) in the sequence. Since an explicit description of the sequence describes any term based on its position we have discovered the rule.

$$\mathbf{a_n = ar^{n-1}}$$

Example for 25-27

1, 4, 9, 16, 25, ...

Notice that:

$a_1 = 1^2$

$a_2 = 2^2$

$a_3 = 3^2$

$a_4 = 4^2$

$a_5 = 5^2$

every term is a perfect square and that the base is equal to the position of the term in the sequence; therefore, the explicit rule is **$a_n = n^2$**.

Notice that:

$a_2 - a_1 = 3$

$a_3 - a_2 = 5$

$a_4 - a_3 = 7$

$a_5 - a_4 = 9$

the difference between consecutive terms creates a sequence of odd numbers beginning with 3; therefore, the recursive rule is **$a_1 = 1, a_{n+1} = a_n + 2n + 1$**.

Section 8-2

Arithmetic Sequences and Arithmetic Means

Objectives for 8-2

- to find the missing value in a defined arithmetic sequence
- to insert a given number of arithmetic means
- to find the first term and the common difference of an arithmetic sequence given two terms

Arithmetic Sequences and Arithmetic Means

- If the first term in the sequence is a_1, and d is the common difference, then $a_n = a_1 + (n - 1)d$.
- arithmetic means: the terms between two given terms in an arithmetic sequence
- average: a single arithmetic means

Example for 1-8

$3, 53, 103, ..., a_{15}$

$a_{15} = 3 + (15 - 1)(50)$

$a_{15} = 3 + (14)(50)$

$a_{15} = 3 + 700$

$\mathbf{a_{15} = 703}$

Example for 9-18

$a_1 = 11, a_{15} = 95, d = ?$

$95 = 11 + (15 - 1)d$

$95 = 11 + 14d$

$84 = 14d$

$\mathbf{6 = d}$

Example for 21-26

five between – 5 and 37

– 5, ____, ____, ____, ____, ____, 37

$37 = -5 + (7 - 1)d$

$37 = -5 + 6d$

$42 = 6d$

$7 = d$

– 5, 2, 9, 16, 23, 30, 37

Example for 27-32

$$a_7 = 3,\ a_{16} = -15$$

$$-15 = a_1 + (16 - 1)d$$

$$3 = a_1 + (7 - 1)d$$

$$-15 = a_1 + 15d$$

$$3 = a_1 + 6d$$

$$-18 = 9d$$

$$\mathbf{-2 = d}$$

$$3 = a_1 - 12$$

$$\mathbf{15 = a_1}$$

Example for 33-36

$a_6 = 18, a_{10} = 6, a_9 = ?$

$6 = a_1 + (10 - 1)d$
$18 = a_1 + (6 - 1)d$

$6 = a_1 + 9d$
$18 = a_1 + 5d$

$-12 = 4d$
$-3 = d$
$18 = a_1 - 15$
$33 = a_1$

$a_9 = 33 + (9 - 1)(-3)$

$\mathbf{a_9 = 9}$

Example for 37-42

The first two terms of an arithmetic sequence are $x + 2$ and $3x - 2$, and the sixth term is $5x$.

$$d = 3x - 2 - (x + 2) = 2x - 4$$

$$5x = (x + 2) + (6 - 1)(2x - 4)$$

$$5x = (x + 2) + 5(2x - 4)$$

$$5x = x + 2 + 10x - 20$$

$$5x = 11x - 18$$

$$18 = 6x$$

$$\mathbf{3 = x}$$

Section 8-3

Arithmetic Series

Objectives for 8-3

- to find the sum of an arithmetic series
- to write an arithmetic series in summation notation
- to find values from an arithmetic series given information about its sum

Arithmetic Series

- series: the sum of the terms of a sequence
- Given any sequence $a_1, a_2, a_3, \ldots$ with n or more terms, the associated series of n terms, S_n, is
  $S_n = a_1 + a_2 + a_3 + \ldots + a_n$
- summation notation: If l is the lower bound of the replacement set of a series, u is the upper bound of the replacement set of a series, n is the index and the sequence the series is based upon can be described explicitly by a_n, then the set of n is the range of the summation and the series when written in summation notation takes the form

$$\sum_{n=l}^{u} a_n$$

Arithmetic Series

- If S_n is the sum of the first n terms of an arithmetic sequence whose first term is a_1, whose common difference is d, and whose nth term is a_n, then

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

Example for 1-8

$a_1 = -7,\ a_{15} = 29,\ n = 15$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{15} = \frac{15}{2}(-7 + 29)$$

$$S_{15} = \frac{15}{2}(22)$$

$$S_{15} = 165$$

Example for 9-20

$$\sum_{i=2}^{11} 9-4i$$

$$n = 10$$

$$a_1 = 9 - 4(2) = 1$$

$$a_{10} = 9 - 4(11) = -35$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{10} = \frac{10}{2}(1-35)$$

$$S_{10} = -170$$

Example for 21-26

$$1 + 8 + 15 + 22$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 1 + (n-1)(7)$$

$$a_n = 7n - 6$$

$$\sum_{n=1}^{4} 7n - 6$$

Example for 27-32

$$a_{12} = 59,\ S_{12} = 324,\ a_1 = ?$$

$$S_n = \frac{n}{2}\left(a_1 + a_n\right)$$

$$324 = \frac{12}{2}\left(a_1 + 59\right)$$

$$324 = 6\left(a_1 + 59\right)$$

$$54 = a_1 + 59$$

$$-5 = a_1$$

Example for 33-34

$$100 + 95 + 90 + \ldots + 10$$

$$a_1 = 100$$
$$d = -5$$

$$a_n = a_1 + (n-1)d$$

$$10 = 100 + (n-1)(-5)$$

$$-90 = (n-1)(-5)$$

$$18 = n - 1$$

$$19 = n$$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_{19} = \frac{19}{2}[2(100) + (19-1)(-5)]$$

$$S_{19} = 1045$$

Section 8-4

Geometric Sequences

Objectives for 8-4

- to find the first four terms of a geometric sequence given its first terms and its common ratio
- to find a specified term in a geometric sequence
- to determine whether a sequence is arithmetic or geometric

Geometric Sequences

- geometric sequence (progression): any sequence in which each term after the first is the product of the preceding term and a fixed number, common ratio ($r = \frac{a_{n+1}}{a_n}$)
- recursive rule for a geometric sequence: $a_{n+1} = (a_n)(r)$
- explicit rule for a geometric sequence whose first term is a_1 and whose common ratio is a nonzero number r is $a_n = a_1 r^{n-1}$

Compound Interest

- compound interest is a geometric progression defined by the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ where A is the amount at the end of the term, P is the principal investment, r is the interest rate and n is the number of times per year that interest is compounded.

Exponential Formulas

- Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

  - (A)mount $ coming out of the account at the maturity date of the investment
  - (P)rincipal $ put into account at the beginning of the investment period
  - (r)ate of annual interest
  - (n)umber of times in a year the interest is calculated
  - (t)ime the investment takes to mature

Exponential Formulas

- Compound Depreciation

$$A = P\left(1 - \frac{r}{n}\right)^{nt}$$

  - (A)mount $ coming out of the account at the maturity date of the investment
  - (P)rincipal $ put into account at the beginning of the investment period
  - (r)ate of annual interest
  - (n)umber of times in a year the interest is calculated
  - (t)ime the investment takes to mature

Exponential Formulas

- Monthly Loan Payments & Total Cost of a Loan

$$\text{Payment} = P \div \left[\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \right]$$

  - (P)rincipal $ put into account at the beginning of the investment period
  - (r)ate of annual interest
  - (n)umber of times in a year the interest is calculated
  - (t)ime the investment takes to mature
- The cost of a loan is the product of the monthly payment, the number of times in a year the interest is calculated and the total time of the loan

Example for Compound Interest

\$1000 is invested at 12% interest compounded quarterly. Determine how much the investment is worth after 3 years.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = what we are asked to find

$P = 1000$

$r = .12$

$n = 4$

$t = 3$

$$A = 1000\left(1 + \frac{.12}{4}\right)^{(4)(3)}$$

$$A = 1000(1.03)^{12}$$

A = \$1425.76

Example for Compound Depreciation

The value of a new $12,500 automobile decreases 20% per year. Find its value after 3 years.

$$A = P\left(1 - \frac{r}{n}\right)^{nt}$$

A = what we are asked to find

$P = 12{,}500$

$r = .20$

$n = 1$

$t = 3$

$$A = 12500\left(1 - \frac{.20}{1}\right)^{(1)(3)}$$

$$A = 12500(.8)^3$$

A = $6400

Example for Monthly Payments & Total Cost of a Loan

You get a 10 year student loan at 8% to pay for $20,000 of your college tuition that is not covered by any grants,scholarships or your parents. How much is your monthly payment going to be and what will be the ultimate cost of your $20,000 of tuition?

$$\text{Payment} = 20000 \div \left[\frac{1-\left(1+\frac{.08}{12}\right)^{-(12)(10)}}{\frac{.08}{12}} \right]$$

payment = $242.66 **cost = $29,118.62**

The majority of your first 37.58 (3+ years of) payments will go towards paying the $9,118.66 in interest off and not the $20,000 principal that you borrowed.

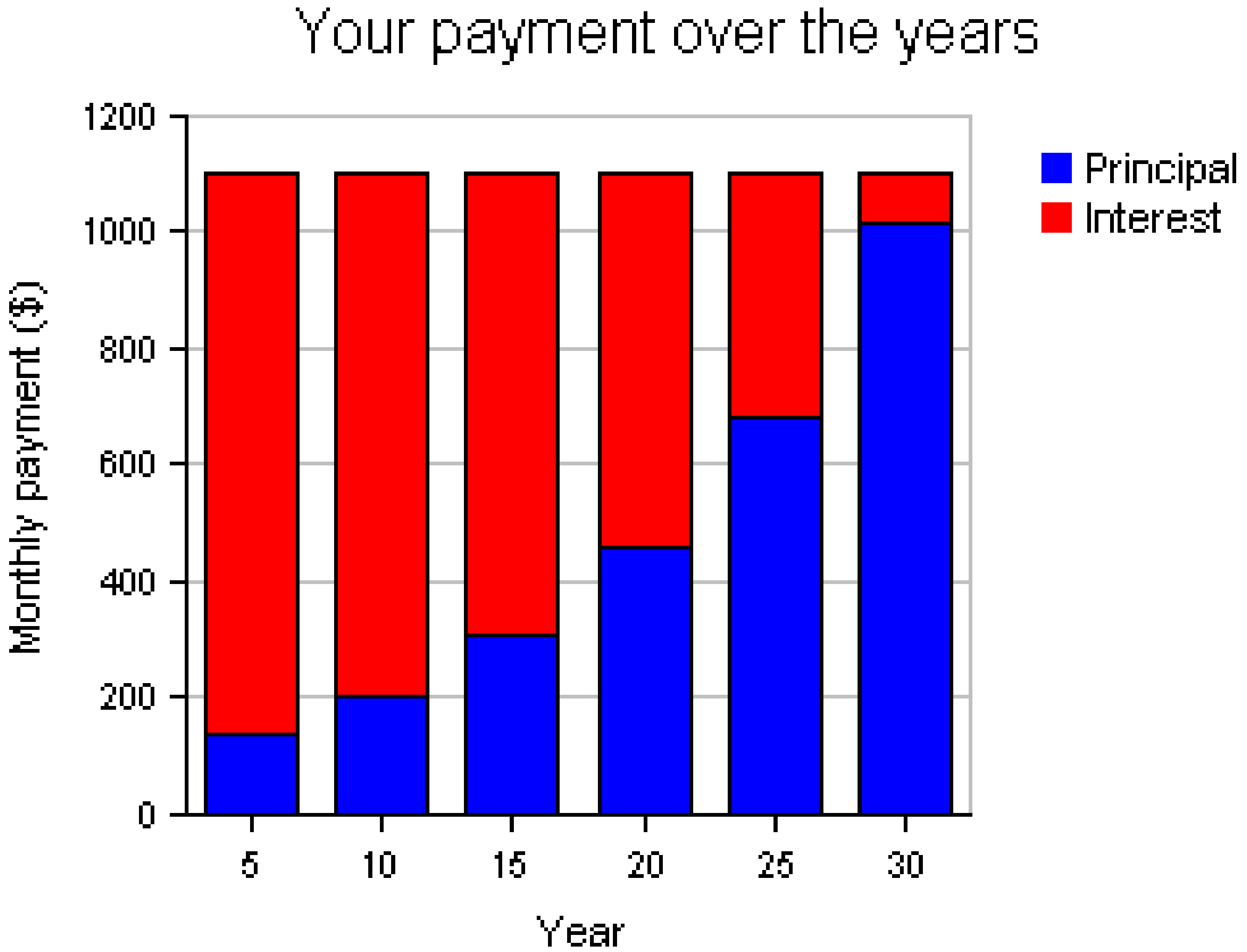
Example for Monthly Payments & Total Cost of a Loan

You get a 30 year fixed mortgage at 8% to pay for a \$150,000 home. How much is your monthly payment going to be and what will be the ultimate cost of your \$150,000 home?

$$\text{Payment} = 150000 \div \left[\frac{1 - \left(1 + \frac{.08}{12}\right)^{-(12)(30)}}{\frac{.08}{12}} \right]$$

payment = \$1100.65   cost = \$396,234

The majority of your first 223.72 (18+ years of) payments will go towards paying the \$246,234 in interest off and not the \$150,000 principal that you borrowed.



Your payment over the years
Monthly payment ($)
1200
1000
800
600
400
200
0
5
10
15
20
25
30
Year
Principal
Interest

Your annual schedule of payments

Month	Monthly payment	Remaining amount owed	Principal paid	Interest paid	Cumulative interest paid
1	$1,100	$149,899	$100	$1,000	$1,000
2	$1,100	$149,798	$101	$999	$1,999
3	$1,100	$149,696	$101	$998	$2,997
4	$1,100	$149,593	$102	$997	$3,995
5	$1,100	$149,490	$103	$997	$4,993
6	$1,100	$149,385	$104	$996	$5,989
7	$1,100	$149,281	$104	$995	$6,985
8	$1,100	$149,175	$105	$995	$7,980
9	$1,100	$149,069	$106	$994	$8,975
10	$1,100	$148,962	$106	$993	$9,969
11	$1,100	$148,855	$107	$993	$10,962
12	$1,100	$148,746	$108	$992	$11,954

Your annual schedule of payments					
Month	Monthly payment	Remaining amount owed	Principal paid	Interest paid	Cumulative interest paid
349	$1,100	$11,636	$1,016	$84	$245,762
350	$1,100	$10,613	$1,023	$77	$245,839
351	$1,100	$9,583	$1,029	$70	$245,910
352	$1,100	$8,546	$1,036	$63	$245,974
353	$1,100	$7,503	$1,043	$56	$246,031
354	$1,100	$6,452	$1,050	$50	$246,081
355	$1,100	$5,394	$1,057	$43	$246,124
356	$1,100	$4,330	$1,064	$35	$246,160
357	$1,100	$3,258	$1,071	$28	$246,189
358	$1,100	$2,179	$1,078	$21	$246,211
359	$1,100	$1,093	$1,086	$14	$246,225
360	$1,100	$0	$1,093	$7	$246,232

Example for 1-6

$$a_1 = 3,\ r = -3$$

$$a_n = a_1 r^{n-1}$$

$$a_1 = 3$$

$$a_2 = (3)(-3)^{2-1} = -9$$

$$a_3 = (3)(-3)^{3-1} = 27$$

$$a_4 = (3)(-3)^{4-1} = -81$$

3, - 9, 27, - 81

Example for 7-12

$$a_1 = -\frac{1}{9}, r = -3, a_5 = ?$$

$$a_n = a_1 r^{n-1}$$

$$a_5 = \left(-\frac{1}{9}\right)(-3)^{5-1} = -9$$

$$a_5 = -9$$

Example for 13-20

$$\frac{1}{20}, \frac{1}{10}, \frac{1}{5}, \ldots, a_7 = ?$$

$$r = \frac{\frac{1}{10}}{\frac{1}{20}} = \left(\frac{1}{10}\right)\left(\frac{20}{1}\right) = 2$$

$$a_n = a_1 r^{n-1}$$

$$a_7 = \left(\frac{1}{20}\right)2^{7-1} = \frac{2^6}{\left(2^2\right)(5)} = \frac{2^4}{5}$$

$$a_7 = \frac{16}{5}$$

Example for 21-26

$$-4\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{2}, \ldots, a_6$$

$$-\frac{9}{2}, -\frac{3}{2}, -\frac{1}{2}, \ldots, a_6$$

$$-\frac{3}{2} - \left(-\frac{9}{2}\right) = 3 \qquad -\frac{1}{2} - \left(-\frac{3}{2}\right) = 1$$

no common difference; therefore, it is not arithmetic

$$\frac{-\frac{3}{2}}{-\frac{9}{2}} = \frac{1}{3} \qquad \frac{-\frac{1}{2}}{-\frac{3}{2}} = \frac{1}{3}$$

common ratio; therefore, it is **geometric**

$$a_n = a_1 r^{n-1}$$

$$a_6 = \left(-\frac{9}{2}\right)\left(\frac{1}{3}\right)^{6-1} = -\frac{3^2}{(2)\left(3^5\right)} = -\frac{1}{(2)\left(3^3\right)} = -\frac{1}{54}$$

Section 8-5

Geometric Means

Objectives for 8-5

- to find the common ration of a geometric sequence
- to insert a given umber of geometric means
- to find the specified value from a geometric sequence

Geometric Means

- geometric means: the terms between two given terms in a geometric sequence
- a single geometric mean inserted between two numbers is called the geometric mean or the mean proportional
  - if both numbers, a and b, are positive then the mean proportional is considered to be $\sqrt{ab}$
  - if both numbers, a and b, are negative then the mean proportional is considered to be $-\sqrt{ab}$

Example for 1-6

$$a_1 = -2, a_4 = 250$$

$$a_n = a_1 r^{n-1}$$

$$250 = (-2) r^{4-1}$$

$$-125 = r^3$$

r = - 5

Example for 7-12

$$a_3 = -2, a_6 = 54$$

$$a_n = a_1 r^{n-1}$$

$$54 = a_1 r^5$$

$$-2 = a_1 r^2$$

$$\frac{-2}{r^2} = a_1$$

$$54 = \left(-\frac{2}{r^2}\right) r^5$$

$$54 = -2r^3$$

$$-27 = r^3$$

$$-3 = r$$

$$\frac{-2}{r^2} = a_1$$

$$\frac{-2}{(-3)^2} = a_1$$

$$-\frac{2}{9} = a_1$$

$$\frac{2}{3} = a_2$$

Example for 13-20

three between $-\frac{25}{3}$ and $-\frac{27}{25}$

$$-\frac{25}{3}, \_\_\_\_, \_\_\_\_, \_\_\_\_, -\frac{27}{25}$$

$$a_n = a_1 r^{n-1}$$

$$-\frac{27}{25} = \left(-\frac{25}{3}\right) r^4$$

$$\left(-\frac{27}{25}\right)\left(-\frac{3}{25}\right) = r^4$$

$$\frac{3^4}{5^4} = r^4 \qquad \pm\frac{3}{5} = r$$

$$-\frac{25}{3}, 5, -3, \frac{9}{5}, -\frac{27}{25} \qquad -\frac{25}{3}, -5, -3, -\frac{9}{5}, -\frac{27}{25}$$

Example for 21-26

$$a_6 = 250, a_8 = 6250, a_3 = ?$$

$$a_n = a_1 r^{n-1}$$

$$6250 = a_1 r^7$$

$$250 = a_1 r^5$$

$$\frac{250}{r^5} = a_1$$

$$6250 = \left(\frac{250}{r^5}\right) r^7$$

$$6250 = 250 r^2$$

$$25 = r^2$$

$$\pm 5 = r$$

$$\frac{250}{r^5} = a_1$$

$$\frac{250}{5^5} = a_1 \qquad \frac{250}{(-5)^5} = a_1$$

$$\frac{(5^3)(2)}{5^5} = a_1 \qquad -\frac{(5^3)(2)}{(5)^5} = a_1$$

$$\frac{2}{25} = a_1 \qquad -\frac{2}{25} = a_1$$

$$a_n = a_1 r^{n-1}$$

$$a_3 = \left(\frac{2}{25}\right) 5^2 \qquad a_3 = \left(-\frac{2}{25}\right) 5^2$$

$$\mathbf{a_3 = 2, -2}$$

Section 8-6

Geometric Series

Objectives for 8-6

- to find the sum of a finite geometric series
- to find a specified value from a geometric series if given information about its sum

Geometric Series

- If S_n is the sum of the first n terms of a geometric sequence whose first term is a_1, whose common ratio is r, and whose n^{th} term is a_n, then

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad r \neq 1$$

$$S_n = \frac{a_1 - r a_n}{1 - r} \quad r \neq 1$$

Example for 1-6

$$a_1 = 1, r = -3, n = 6$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad r \neq 1$$

$$S_6 = \frac{1 - (1)(-3)^6}{1 - (-3)}$$

$$S_6 = \frac{1 - 729}{4}$$

$$\mathbf{S_n = - 182}$$

Example for 7-18

$$\sum_{n=1}^{5} 8\left(\frac{3^{n-1}}{2^{2n-2}}\right)$$

$$a_1 = 8, a_2 = 8\left(\frac{3}{4}\right) = 6, r = \frac{3}{4}, n = 5$$

$$S_n = \frac{a_1 - a_1 r^n}{1-r} \quad r \neq 1$$

$$S_5 = \frac{8-(8)\left(\frac{3}{4}\right)^5}{1-\frac{3}{4}}$$

$$S_5 = \frac{781}{32}$$

Example for 19-28

$$a_1 = 4, r = 3, S_n = 484, n = ?$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad r \neq 1$$

$$484 = \frac{4 - (4)(3)^n}{1 - 3}$$

$$484 = \frac{4 - (4)(3)^n}{-2}$$

$$-968 = 4 - (4)(3)^n$$

$$243 = (3)^n$$

$$3^5 = (3)^n$$

$$\mathbf{n = 5}$$

Section 8-7

Limit of a Sequence

Objectives for 8-7

- to find the limit of a sequence
- to calculate the error of approximation and to write its general formula

Limit of a Sequence

- limit: a value that an expression approaches but never equals written as $\lim_{n \to \infty} [a_n] = L$ and read as " the limit of an as n increases without bound is L"

- error of approximation: the absolute value of the difference between a given limit and the actual value of the sequence
- convergent sequence: an infinite sequence with a limit
- An infinite sequence has a limit L if you make the error of approximation, $|L - a_n|$, less than any positive number, however small, by choosing n great enough.

Limit of a Sequence

- nondecreasing sequence: any sequence in which each term is less than or equal to the following term
- nonincreasing sequence: any sequence in which each term is greater than or equal to the following term
- bounded sequence: there exists a number that equals or exceeds the absolute value of every term of a sequence
- Axiom of Completeness: Every bounded, nondecreasing (or nonincreasing) sequence of real numbers converges, and its limit is a real number.

- divergent sequence: an infinite sequence that does not have a limit

Example for 1-6

$$a_n = 5\left(1 - \frac{1}{n}\right)$$

$$0, \frac{5}{2}, \frac{10}{3}, \frac{15}{4}, \ldots$$

As n goes to infinity the fraction $\frac{1}{n}$ becomes infinitely small and approaches 0; therefore, the value of a_n approaches **5**.

Example for 7-12

$$a_n = \frac{6n^2+1}{2n^2-1}; L = 3$$

$$7, \frac{25}{7}, \frac{55}{17}, \frac{97}{31}$$

$$|L - a_1| = |3 - 7| = 4$$

$$|L - a_2| = \left|3 - \frac{25}{7}\right| = \frac{4}{7}$$

$$|L - a_3| = \left|3 - \frac{55}{17}\right| = \frac{4}{17}$$

$$|L - a_4| = \left|3 - \frac{97}{31}\right| = \frac{4}{31}$$

$$|L - a_n| = \left|3 - \frac{6n^2+1}{2n^2-1}\right| = \frac{4}{2n^2-1}$$

Example for 13-18

$$\left|L - a_n\right| = \left|3 - \frac{6n^2+1}{2n^2-1}\right| = \frac{4}{2n^2-1}$$

$$\frac{4}{2n^2-1} < \frac{1}{10}$$

$$40 < 2n^2 - 1$$

$$41 < 2n^2$$

$$\frac{41}{2} < n^2$$

$$\sqrt{\frac{41}{2}} < n$$

$$4.5 < n$$

Because n is a position it must be an integer; therefore n = 5, $\frac{4}{49}$

Section 8-8

Infinite Geometric Series

Objectives for 8-8

- to find the sum of infinite geometric series
- to find a specified value from an infinite geometric series given its sum
- to rewrite a non-terminating repeating decimal as an infinite geometric series and then calculating the limit of the series to find the fractional form of the decimal

Infinite Geometric Series

- In general, for any infinite series $a_1 + a_2 + a_3 + ... + a_n + ...$,

  $S_n = \sum_{i=1}^{n} a_i$ is called a partial sum.

- If the sequence $S_1, S_2, S_3, ..., S_n, ...$ of partial sums

  converges and if $\lim_{n \to \infty} S_n = S$, then the sum of the infinite

  series is defined to be S.

- This means that $\sum_{k=1}^{\infty} a_k = S$ converges; otherwise, the series

  of partial sums diverges and the sum is undefined.

Infinite Geometric Series

- The infinite geometric series $a_1 + a_1r + a_1r^2 + \ldots$converges and has the sum $S = \frac{a_1}{1-r}$ if $|r| < 1$. If $a_1 = 0$, then the series converges and has the sum 0. If $|r| \geq 1$ and $a_1 \neq 0$, then the series diverges.

Example for 1-10

$$250 - 50 + 10 - \ldots$$

$$a_1 = 250, r = -\frac{1}{5}, \left|-\frac{1}{5}\right| < 1$$

The series converges.

$$S = \frac{a_1}{1 - r}$$

$$S = \frac{250}{1 - \left(-\frac{1}{5}\right)}$$

$$S = \frac{250}{\frac{6}{5}}$$

$$S = \frac{625}{3}$$

Example for 11-14

$$\sum_{n=1}^{\infty} \frac{1}{10}\left(\frac{6}{5}\right)^{n-1}$$

$$a_1 = \frac{1}{10}, r = \frac{6}{5}, \left|\frac{6}{5}\right| \geq 1$$

The series diverges and the sum is undefined.

Example for 15-20

$$r = -\frac{5}{6}, S = 9, a_1 = ?$$

$$S = \frac{a_1}{1 - r}$$

$$9 = \frac{a_1}{1 - \left(-\frac{5}{6}\right)}$$

$$9 = \frac{a_1}{\frac{11}{6}}$$

$$\frac{33}{2} = a_1$$

Example for 21-28

$$0.\overline{36}$$

$$0.\overline{36} = \frac{36}{100} + \frac{36}{10000} + \frac{36}{1000000} + \ldots$$

$$a_1 = \frac{36}{100} = \frac{9}{25}, r = \frac{1}{100}$$

$$S = \frac{\frac{9}{25}}{1 - \frac{1}{100}}$$

$$S = \left(\frac{9}{25}\right)\left(\frac{100}{99}\right)$$

$$S = \frac{4}{11}$$