Chapter 5A. Torque

A PowerPoint Presentation by

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Torque is a twist or turn that tends to produce rotation. * * * Applications are found in many common tools around the home or industry where it is necessary to turn, tighten or loosen devices.

Objectives: After completing this module, you should be able to:

- Define and give examples of the terms torque, moment arm, axis, and line of action of a force.
- Draw, label and calculate the moment arms for a variety of applied forces given an axis of rotation.
- Calculate the resultant torque about any axis given the magnitude and locations of forces on an extended object.
- Optional: Define and apply the vector cross product to calculate torque.

Definition of Torque

Torque is defined as the tendency to produce a change in rotational motion.

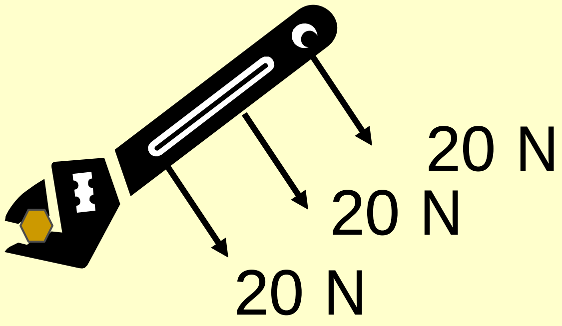
Examples:

Torque is Determined by Three Factors:

- The magnitude of the applied force.
- The direction of the applied force.
- The location of the applied force.

The forces nearer the end of the wrench have greater torques.

Location of force



Units for Torque

Torque is proportional to the magnitude of F and to the distance r from the axis. Thus, a tentative formula might be:

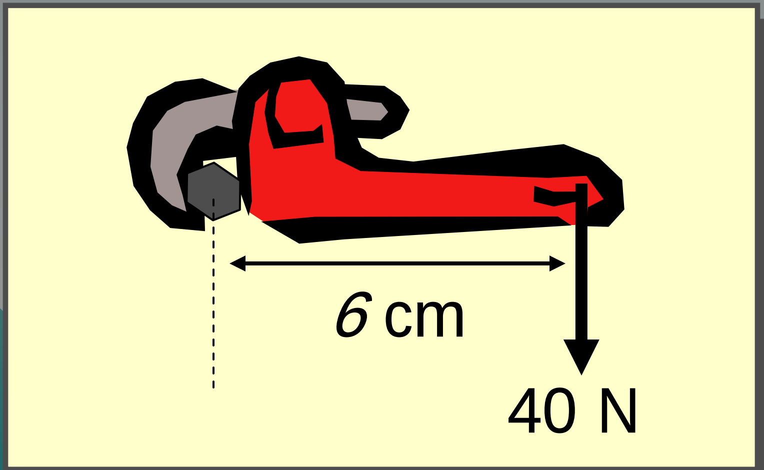
$$\tau = Fr$$

Units: N·m or lb·ft

$$\tau = (40\text{ N})(0.60\text{ m})$$

$$= 24.0\text{ N·m, cw}$$

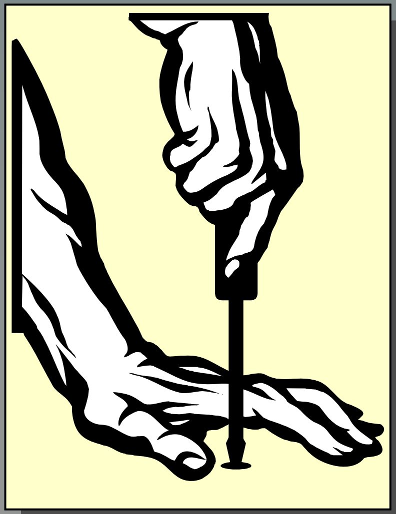
$$\tau = 24.0\text{ N·m, cw}$$



Direction of Torque

Torque is a vector quantity that has direction as well as magnitude.

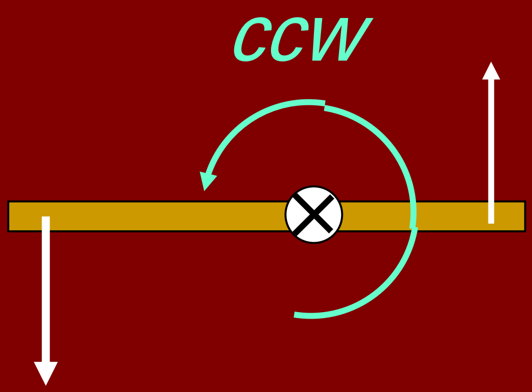
Turning the handle of a screwdriver clockwise and then counterclockwise will advance the screw first inward and then outward.

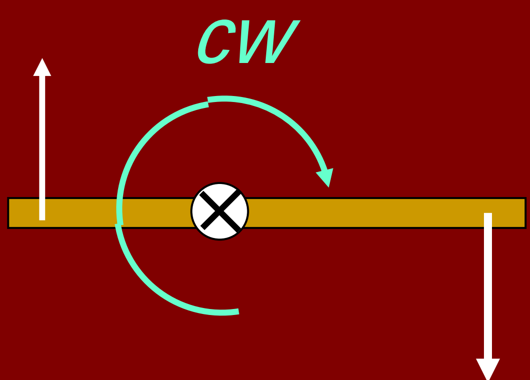


Sign Convention for Torque

By convention, counterclockwise torques are positive and clockwise torques are negative.

Positive torque: Counter-clockwise, out of page

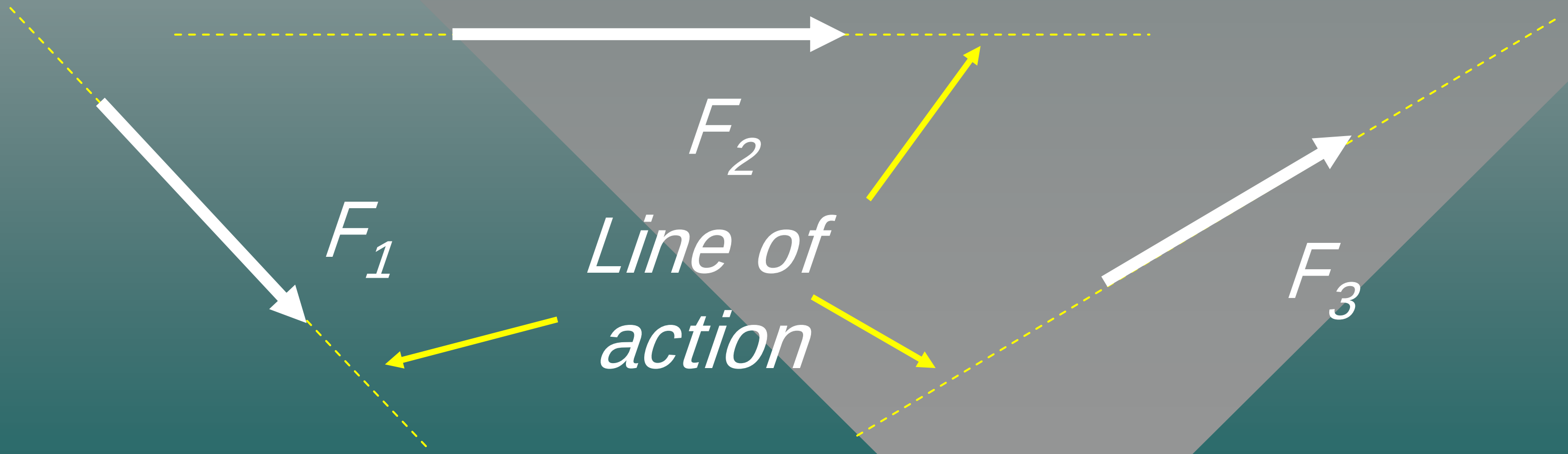




Negative torque: clockwise, into page

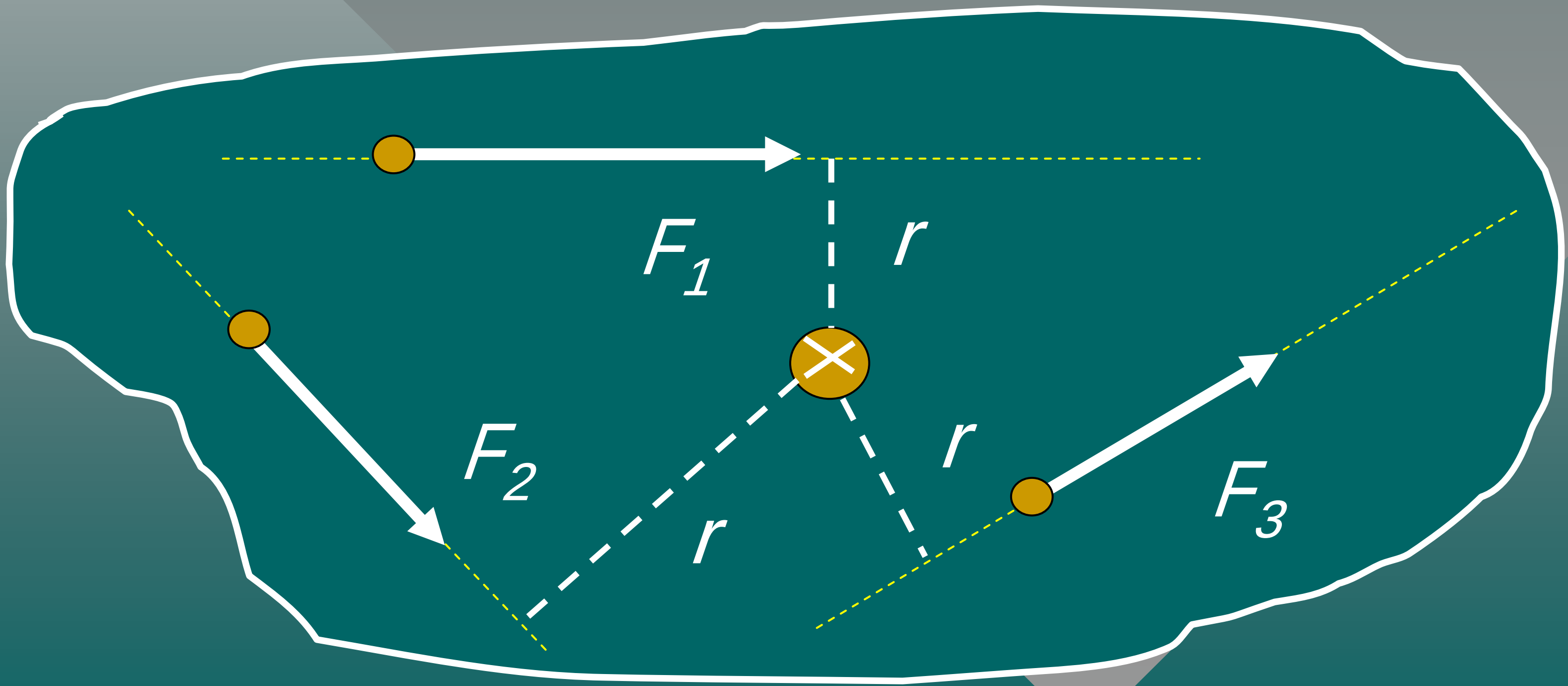
Line of Action of a Force

*The **line of action** of a force is an imaginary line of indefinite length drawn along the direction of the force.*



The Moment Arm

The ***moment arm*** *of a force is the perpendicular distance from the line of action of a force to the axis of rotation.*



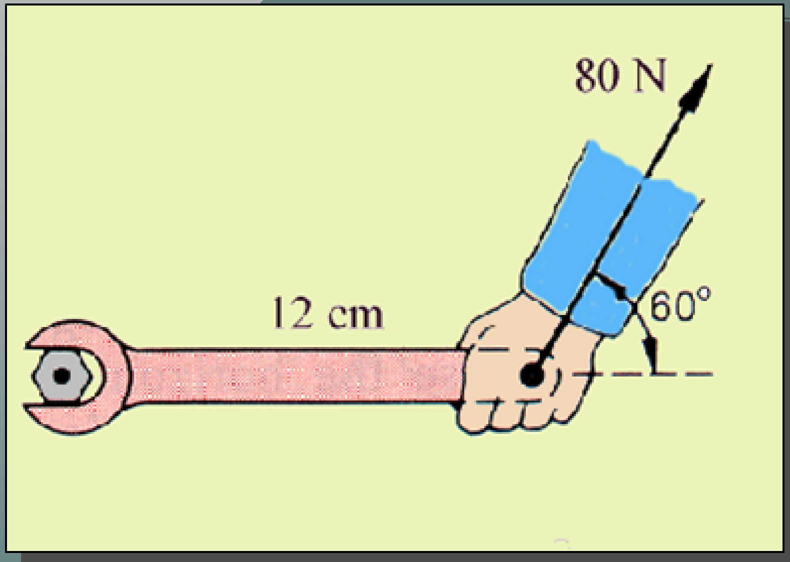
Calculating Torque

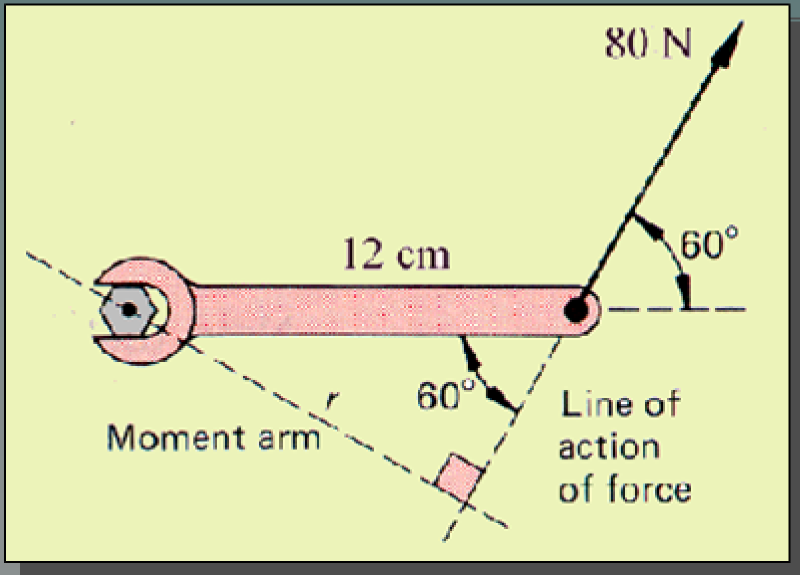
- Read problem and draw a rough figure.
- Extend line of action of the force.
- Draw and label moment arm.
- Calculate the moment arm if necessary.
- Apply definition of torque:

$$\tau = Fr$$

Torque = force x moment arm

Example 1: An 80-N force acts at the end of a 12-cm wrench as shown. Find the torque.



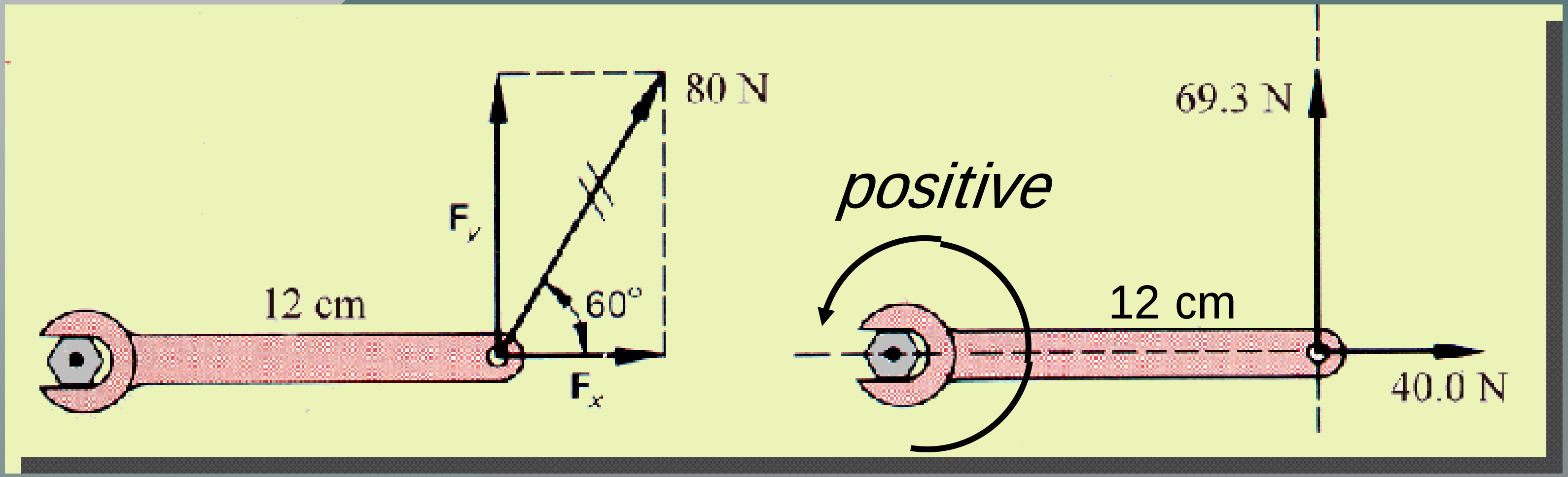


- *Extend line of action, draw, calculate r.*

r = 12 cm sin 60^0
= 10.4 cm

τ = (80 N)(0.104 m)
= 8.31 N m

Alternate: An 80-N force acts at the end of a 12-cm wrench as shown. Find the torque.



Resolve 80-N force into components as shown.

Note from figure: $r_x = 0$ *and* $r_y = 12$ cm

$\tau = (69.3\text{ N})(0.12\text{ m})$

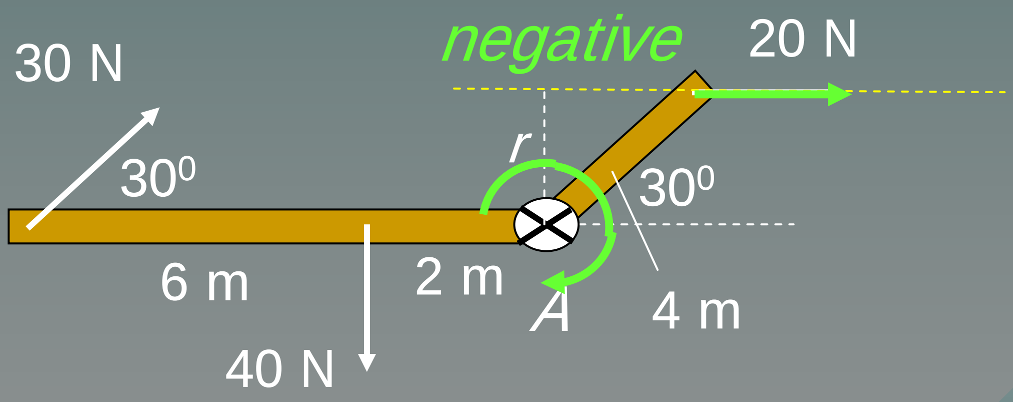
$\tau = 8.31$ N m as before

Calculating Resultant Torque

- Read, draw, and label a rough figure.
- Draw free-body diagram showing all forces, distances, and axis of rotation.
- Extend lines of action for each force.
- Calculate moment arms if necessary.
- Calculate torques due to EACH individual force affixing proper sign. CCW (+) and CW (-).
- Resultant torque is sum of individual torques.

Example 2: Find resultant torque about axis A for the arrangement shown below:

Find τ due to each force. Consider 20-N force first:



r = (4 m) sin 30^0
= 2.00 m

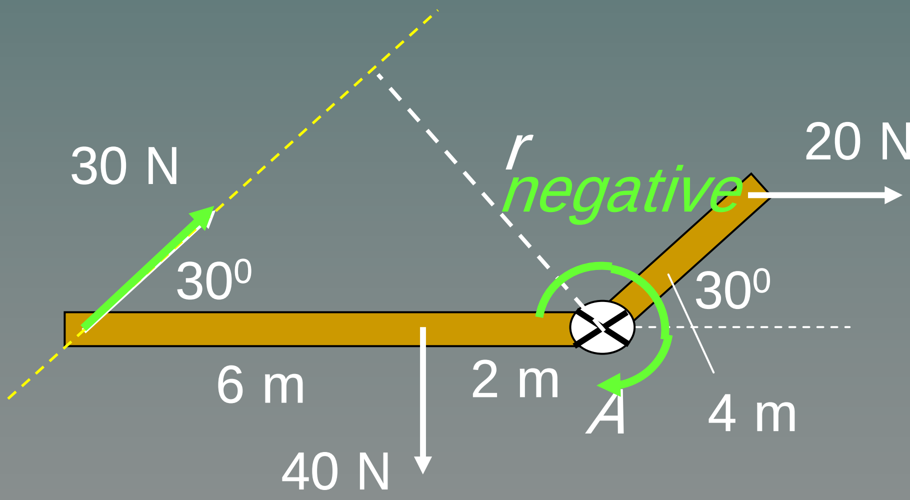
$\tau = Fr$ = (20 N)(2 m)
= 40 N m, cw

The torque about A is clockwise and negative.

τ_{20} = -40 N m

Example 2 (Cont.): Next we find torque due to 30-N force about same axis A.

Find τ due to each force. Consider 30-N force next.



r = (8 m) sin 30^0
= 4.00 m

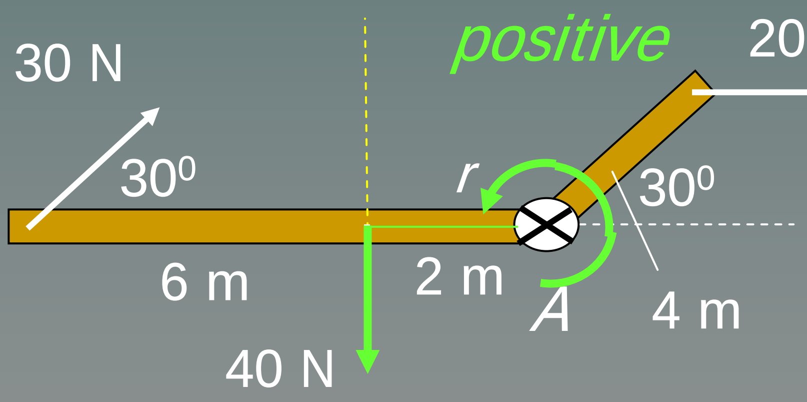
$\tau = Fr$ = (30 N)(4 m)
= 120 N m, cw

The torque about A is clockwise and negative.

τ_{30} = -120 N m

Example 2 (Cont.): Finally, we consider the torque due to the 40-N force.

Find τ due to each force. Consider 40-N force next:



r = (2 m) sin 90^0
= 2.00 m

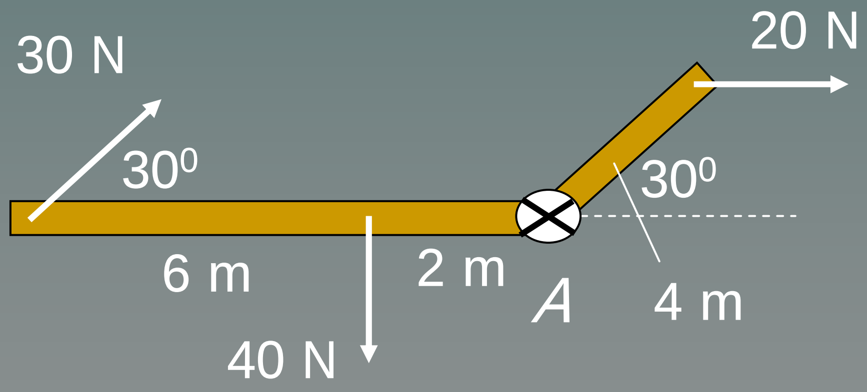
$\tau = Fr$ = (40 N)(2 m)
= 80 N m, ccw

The torque about A is CCW and positive.

τ_{40} = +80 N m

Example 2 (Conclusion): Find resultant torque about axis A for the arrangement shown below:

Resultant torque is the sum of individual torques.



$$\tau_R = \tau_{20} + \tau_{20} + \tau_{20} = -40 \text{ N m} - 120 \text{ N m} + 80 \text{ N m}$$

$$\tau_R = -80 \text{ N m}$$

Clockwise

Part II: Torque and the Cross Product or Vector Product.

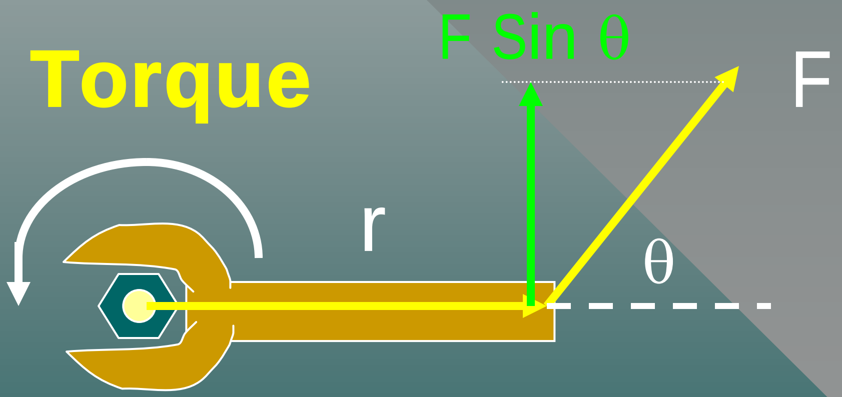
Optional Discussion

This concludes the general treatment of torque. Part II details the use of the vector product in calculating resultant torque. Check with your instructor before studying this section.

The Vector Product

Torque can also be found by using the vector product of force F and position vector r. For example, consider the figure below.

Torque



The effect of the force F at angle θ (torque) is to advance the bolt out of the page.

Magnitude:

$(F \sin \theta) r$

Direction = Out of page (+).

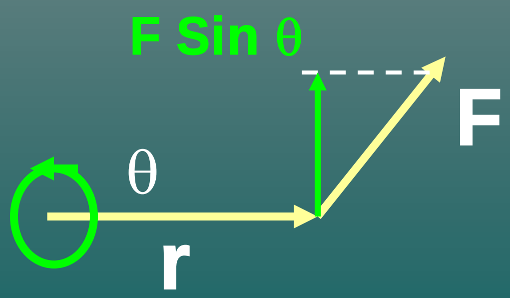
Definition of a Vector Product

The magnitude of the vector (cross) product of two vectors **A** *and* **B** *is defined as follows:*

$$\mathbf{A} \times \mathbf{B} = | A | \, | B | \sin \theta$$

In our example, the cross product of F and r is:

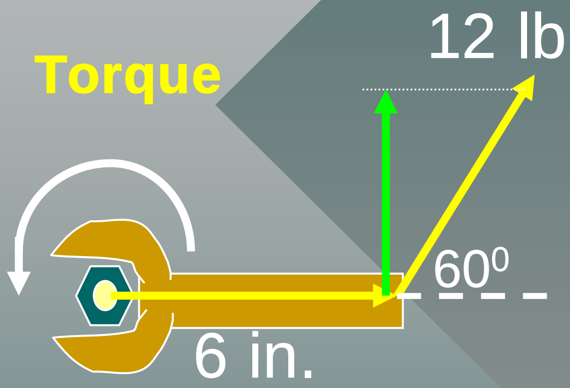
$$\mathbf{F} \times \mathbf{r} = | F | \, | r | \sin \theta$$ *Magnitude only*



In effect, this becomes simply:

$(F \sin \theta)\, r$ *or* $F\,(r \sin \theta)$

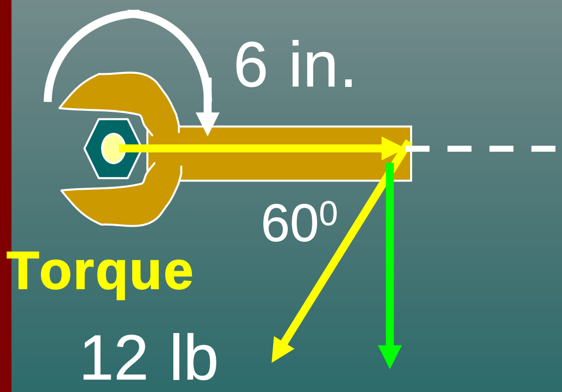
Example: Find the magnitude of the cross product of the vectors r and F drawn below:



r x **F** = | r | | F | Sin θ

r x **F** = (6 in.)(12 lb) Sin 60^0

|r x F|= 62.4 lb in.



r x **F** = | r | | F | Sin θ

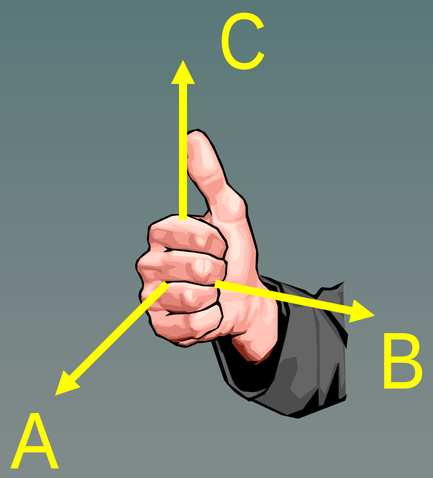
r x **F** = (6 in.)(12 lb) Sin 120^0

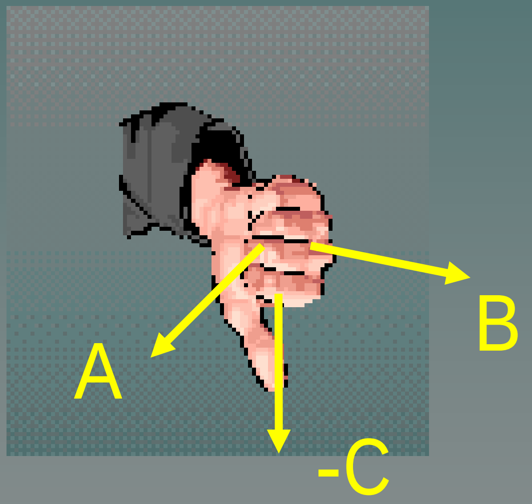
|r x F| = 62.4 lb in.

Explain difference. Also, what about **F** x **r**?

Direction of the Vector Product.

The direction of a vector product is determined by the right hand rule.





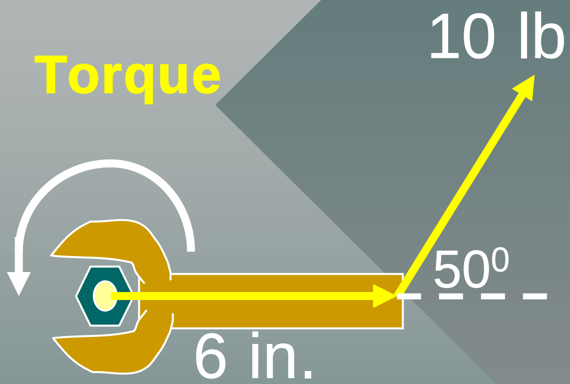
A x B = C (up)

B x A = -C (Down)

What is direction of A x C?

Curl fingers of right hand in direction of cross product (A to B) or (B to A). Thumb will point in the direction of product C.

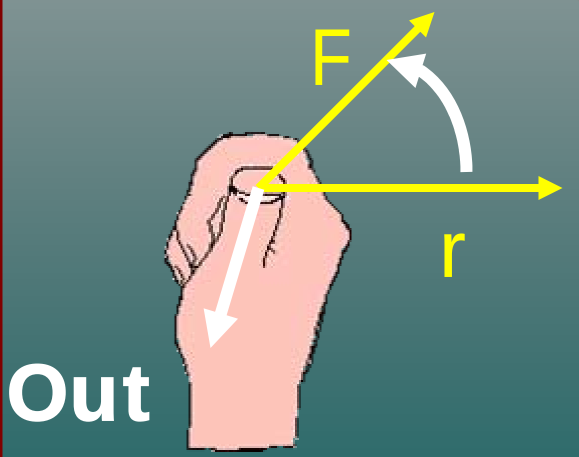
Example: What are the magnitude and direction of the cross product, r x F?



r x **F** = | r | | F | Sin θ

r x **F** = (6 in.)(10 lb) Sin 50^0

|**r** x **F**|= 38.3 lb in. *Magnitude*



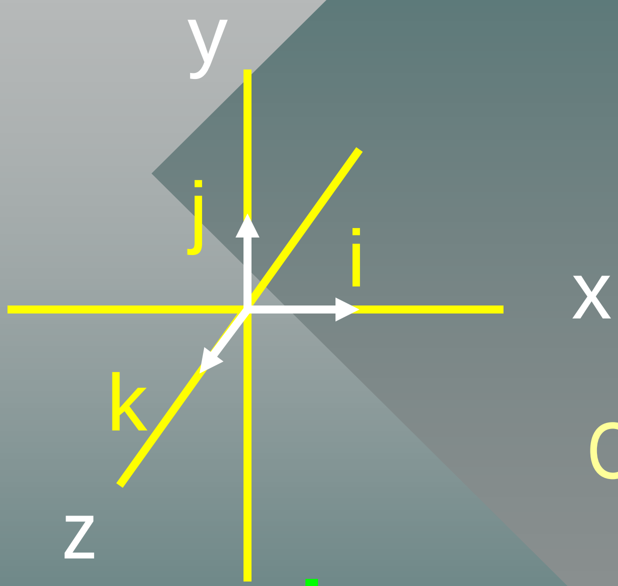
Direction by right hand rule:

Out of paper (thumb) or +k

r x **F** = (38.3 lb in.) **k**

What are magnitude and direction of F x r?

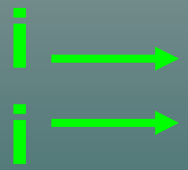
Cross Products Using (i,j,k)



Consider 3D axes (x, y, z)

Define unit vectors, i, j, k

Consider cross product: **i x i**



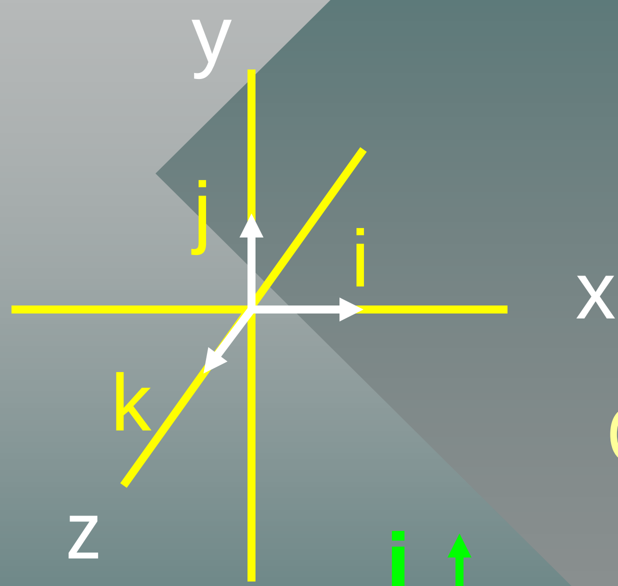
Magnitudes are zero for parallel vector products.

$\mathbf{i} \times \mathbf{i} = (1)(1)\ \mathrm{Sin}\ 0^0 = 0$

$\mathbf{j} \times \mathbf{j} = (1)(1)\ \mathrm{Sin}\ 0^0 = 0$

$\mathbf{k} \times \mathbf{k} = (1)(1)\mathrm{Sin}\ 0^0 = 0$

Vector Products Using (i,j,k)



Consider 3D axes (x, y, z)

Define unit vectors, i, j, k

Consider dot product: **i** x **j**

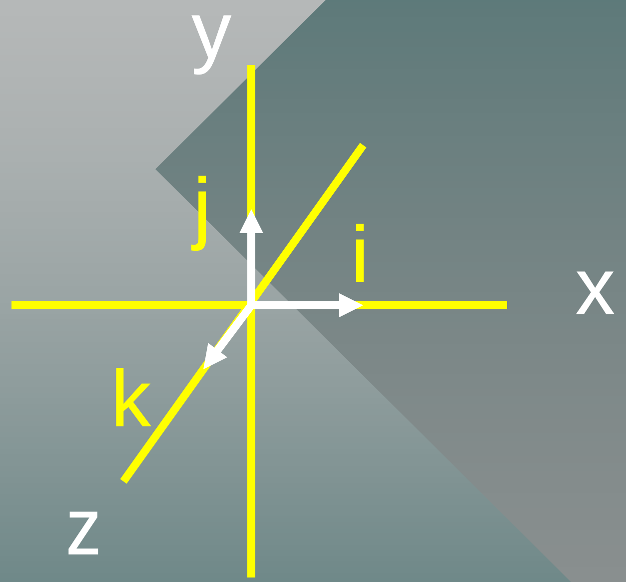
i x j = (1)(1) Sin 90^0 = 1

j x k = (1)(1) Sin 90^0 = 1

k x i = (1)(1) Sin 90^0 = 1

Magnitudes are “1” for perpendicular vector products.

Vector Product (Directions)



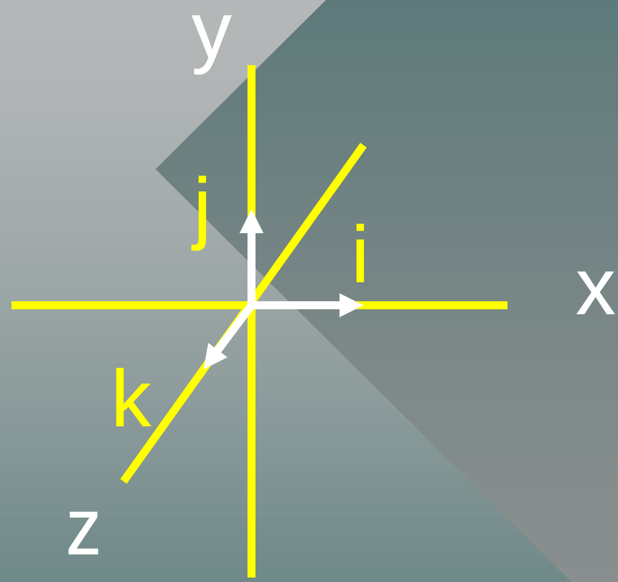
Directions are given by the right hand rule. Rotating first vector into second.

$\mathbf{i} \times \mathbf{j} = (1)(1)\ \mathrm{Sin}\ 90^0 = +1\ \mathbf{k}$

$\mathbf{j} \times \mathbf{k} = (1)(1)\ \mathrm{Sin}\ 90^0 = +1\ \mathbf{i}$

$\mathbf{k} \times \mathbf{i} = (1)(1)\ \mathrm{Sin}\ 90^0 = +1\ \mathbf{j}$

Vector Products Practice (i,j,k)



Directions are given by the right hand rule. Rotating first vector into second.

i x **k** = ? **- j** (down)

k x **j** = ? **- i** (left)

j x **-i** = ? **+ k** (out)

2 **i** x -3 **k** = ? + 6 **j** (up)

Using i,j Notation - Vector Products

Consider: A = 2 i - 4 j and B = 3 i + 5 j

A x **B** = (2 i - 4 j) x (3 i + 5 j) =

(2)(3) ixi + (2)(5) ixj + (-4)(3) jxi + (-4)(5) jxj

(ixi = 0, ixj = k, jxi = -k, jxj = 0)

A x **B** = (2)(5) k + (-4)(3)(-k) = +22 **k**

Alternative: A = 2 i - 4 j

B = 3 i + 5 j

Evaluate determinant

A x **B** = 10 - (-12) = +22 **k**

Summary

Torque is the product of a force and its moment arm as defined below:

*The **moment arm** of a force is the perpendicular distance from the line of action of a force to the axis of rotation.*

*The **line of action** of a force is an imaginary line of indefinite length drawn along the direction of the force.*

$$\tau = Fr$$

Torque = force x moment arm

Summary: Resultant Torque

- Read, draw, and label a rough figure.
- Draw free-body diagram showing all forces, distances, and axis of rotation.
- Extend lines of action for each force.
- Calculate moment arms if necessary.
- Calculate torques due to EACH individual force affixing proper sign. CCW (+) and CW (-).
- Resultant torque is sum of individual torques.

CONCLUSION: Chapter 5A Torque