Postulates

Postulate 1 Ruler Postulate:

1. The points on a line can be paired with the real numbers in such a way that any two points can have coordinates 0 and 1.
2. Once a coordinate system has been chosen in this way, the distance between any two points equals the absolute value of the difference of their coordinates.

Postulate 2 Segment Addition Postulate: If B is between A and C, then AB + BC = AC.

Postulate 3 Protractor Postulate: On $\overleftrightarrow{AB}$ in a given plane, choose any point O between A and B. Consider $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and all rays that can be drawn from O on one side of $\overleftrightarrow{AB}$. These rays can be paired with the real numbers from 0 to 180 in such a way that:

a. $\overrightarrow{OA}$ is paired with 0, and $\overrightarrow{OB}$ is paired with 180.

b. If $\overrightarrow{OP}$ is paired with x, and $\overrightarrow{OQ}$ is paired with y, then $m \angle POQ = | x - y |$.

Postulate 4 Angle Addition Postulate: If point B lies in the interior of $\angle AOC$, then $m \angle AOB + m \angle BOC = m \angle AOC$. If $\angle AOC$ is a straight angle and B is any point not on $\overleftrightarrow{AC}$, then $m \angle AOB + m \angle BOC = 180$.

Postulate 5 A line contains at least two points; a plane contains at least three points not all in one line; space contains at least four points not all in one plane.

Postulate 6 Through any two points there is exactly one line.

Postulate 7 Through any three points there is at least one plane, and through any three noncollinear points there is exactly one plane

Postulate 8 If two points are in a plane, then the line containing those points is also in that plane.

Postulate 9 If two planes intersect, then their intersection is a line.

Postulate 10 If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Postulate 11 If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

Postulate 12 SSS Postulate: If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

Postulate 13 SAS Postulate: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Postulate 14 ASA Postulate: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Postulate 15 AA Similarity Postulate: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Postulate 16 Arc Addition Postulate: The measure of the arc formed by two adjacent arcs is the sum of the measures of these two arcs.

Postulate 17 The area of a square is the square of the length of a side. ($A = s^2$)

Postulate 18 Area Congruence Postulate: If two figures are congruent, then they have the same area.

Postulate 19 Area Addition Postulate: The area of a region is the sum of the areas of its non-overlapping parts.

Points, Lines, Planes, and Angles

1-1 If two lines intersect, then they intersect in exactly one point.

Theorems

1-2 Through a line and a point not in a line there is exactly one plane.

1-3 If two lines intersect, then exactly one plane contains the lines.

Deductive Reasoning

2-1 Midpoint Theorem: If M is the midpoint of $\overline{AB}$, then AM = ½AB and MB = ½AB.

2-2 Angle Bisector Theorem: If $\overrightarrow{BX}$ is the bisector of ∠ ABC, then m ∠ ABX = ½m ∠ ABC and m ∠ XBC = ½m ∠ ABC.

2-3 Vertical angles are congruent.

2-4 If two lines are perpendicular, then they form congruent adjacent angles.

2-5 If two lines form congruent adjacent angles, then the lines are perpendicular.

2-6 If the exterior sides of two adjacent angles are perpendicular, then the angles are complementary.

2-7 If two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.

2-8 If two angles are complements of congruent angles (or the same angle), then the two angles are congruent.

Parallel Lines and Planes

3-1 If two parallel planes are cut by a third plane, then the lines of intersection are parallel.

3-2 If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

3-3 If two parallel lines are cut by a transversal, then same side interior angles are supplementary.

3-4 If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one also.

3-5 If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

3-6 If two lines are cut by a transversal and same side interior angles are supplementary, then the lines are parallel.

3-7 In a plane two lines perpendicular to the same line are parallel.

3-8 Through a point outside a line, there is exactly one line parallel to the given line.

3-9 Through a point outside a line, there is exactly one line perpendicular to the given line.

3-10 Two lines parallel to a third line are parallel to each other.

3-11 The sum of the measures of the angles of a triangle is 180.
Corollary 1 If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.
Corollary 2 Each angle of an equiangular triangle has measure 60.
Corollary 3 In a triangle, there can be at most one right or obtuse angle.
Corollary 4 The acute angles of a right triangle are complementary.

3-12 The measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles.

3-13 The sum of the measures of the angles of any convex polygon with n sides is (n - 2)180.

3-14 The sum of the measures of the exterior angles of any convex polygon, one angle at each vertex, is 360.

Congruent Triangles

Theorems

4-1 Isosceles Triangle Theorem: If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Corollary 1 An equilateral triangle is also equiangular.

Corollary 2 An equilateral triangle has three 60° angles.

Corollary 3 The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

4-2 If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

4-3 AAS Theorem: If two angles and the non-included side of a triangle are congruent to two angles and the non-included side of another triangle, then the triangles are congruent.

4-4 HL Theorem: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

4-5 If a point lies on the perpendicular bisector of a segment, then that point is equidistant from the endpoints of that segment.

4-6 If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of that segment.

4-7 If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.

4-8 If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

Quadrilaterals

5-1 If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.

5-2 If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent.

5-3 Diagonals of a parallelogram bisect each other.

5-4 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

5-5 If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

5-6 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

5-7 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5-8 If two lines are parallel, then all points on one line are equidistant from the other line.

5-9 If three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

5-10 A line that contains the midpoint of one side of a triangle and is parallel to another side passes through the midpoint of the third side.

5-11 The segment that joins the midpoints of two sides of a triangle
(1) is parallel to the third side.
(2) is half as long as the third side.

5-12 The diagonals of a rectangle are congruent.

5-13 The diagonals of a rhombus are perpendicular.

5-14 Each diagonal of a rhombus bisects two angles of the rhombus.

5-15 The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

5-16 If an angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

5-17 If two consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.

5-18 Base angles of an isosceles trapezoid are congruent.

Theorems

5-19 The median of a trapezoid
(1) is parallel to the bases.
(2) has a length equal to the average of the base lengths.

Inequalities in Geometry

6-1 Exterior Angle Inequality Theorem: The measure of an exterior angle of a triangle is greater than the measure of either remote interior angles.

6-2 If one side of a triangle is longer than the other side, then the angle opposite the first side is larger than the angle opposite the second side.

6-3 If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.
Corollary 1 The perpendicular segment from a point to a line is the shortest segment from the point to the line.
Corollary 2 The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

6-4 Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

6-5 SAS Inequality Theorem: If two sides of one triangle are congruent to two sides of another triangle, but the includes angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.

6-6 SSS Inequality Theorem: If two sides of a triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

Similar Polygons

7-1 SAS Similarity Theorem: If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar.

7-2 SSS Similarity Theorem: If the sides of two triangles are in proportion, then the triangles are similar.

7-3 Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.
Corollary 1 If three parallel lines intersect two transversals, then they divide the transversals proportionally.

7-4 Triangle Angle-Bisector Theorem: If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

Right Triangles

8-1 If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.
Corollary 1 When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.
Corollary 2 When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.

8-2 Pythagorean Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Theorems

8-3 If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

8-4 If the square of the longest side of a triangle is less than the sum of the squares of the other two sides, then the triangle is an acute triangle.

8-5 If the square of the longest side of a triangle is greater than the sum of the squares of the other two sides, then the triangle is an obtuse triangle.

8-6 45°-45°-90° Theorem: In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.

8-7 30°-60°-90° Theorem: In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Circles

9-1 If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.
Corollary 1 Tangents to a circle from a point are congruent.

9-2 If a line in the plane of the circle is perpendicular to a radius at its outer endpoint, then the line is tangent to the circle.

9-3 In the same circle or congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

9-4 In the same circles or congruent circles,
(1) congruent arcs have congruent chords.
(2) congruent chords have congruent arcs.

9-5 A diameter that is perpendicular to a chord bisects the chord and its arc.

9-6 In the same circle or congruent circles,
(1) chords equally distant from the center (or centers) are congruent.
(2) congruent chords are equally distant from the center (or centers).

9-7 The measure of an inscribed angle is equal to half the measure of the intercepted arc.
Corollary 1 If two inscribed angles intercept the same arc, then the angles are congruent.
Corollary 2 An angle inscribed in a semicircle is a right angle.
Corollary 3 If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

9-8 The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.

9-9 The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.

9-10 The measure of the angle formed by two secants, two tangents, or a secant and a tangent drawn from a point outside a circle is equal to half the difference of the measures of the intercepted arcs.

9-11 When two chords intersect inside a circle, the product of the segments of one chord is equal to the product of the segments of the other chord.

9-12 When two secant segments are drawn to a circle from an external point, the product of one secant segment and its external segment equals the product of the other secant segment and its external segment.

9-13 When a secant segment and a tangent segment are drawn to a circle from an external point, the product of the secant segment and its external segment is equal to the square of the tangent segment.

Construction and Loci

Theorems

10-1 The bisectors of the angles of a triangle intersect in a point that is equidistant from the three sides of the triangle.

10-2 The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the three vertices of the triangle.

10-3 The lines that contain the altitudes of a triangle intersect in a point.

10-4 The medians of a triangle intersect in a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

Areas of Plane Figures

11-1 The area of a rectangle equals the product of its base and its height. ($A = bh$)

11-2 The area of a parallelogram equals the product of a base and the height to that base. ($A = bh$)

11-3 The area of a triangle equals half the product of a base and the height to the base. ($A = ½bh$)

11-4 The area of a rhombus equals half the product of its diagonals. ($A = ½d_1d_2$)

11-5 The area of a trapezoid equals half the product of the height and the sum of the bases.
($A = ½h(b_1 + b_2)$)

11-6 The area of a regular polygon is equal to half the product of the apothem and the perimeter.
($A = ½ap$)

11-7 If the scale factor of two similar figures is a:b, then
(1) the ratio of the perimeters is a:b.
(2) the ratio of the areas is $a^2:b^2$.

Areas and Volumes of Solids

12-1 The lateral area of a right prism equals the perimeter of a base times the height of the prism.
(L.A. = ph)

12-2 The volume of a right prism equals the area of a base times the height of the prism. ($V = Bh$)

12-3 The lateral area of a regular pyramid equals half the perimeter of the base times the slant height.
(L.A. = ½pl)

12-4 The volume of a pyramid equals one third the area of the base times the height of the pyramid.
($V = \frac{1}{3}Bh$)

12-5 The lateral area of a cylinder equals the circumference of the base times the height of the cylinder
(L.A. = $2\pi rh$)

12-6 The volume of a cylinder is equal to the area of a base times the height of the cylinder.
($V = \pi r^2h$)

12-7 The lateral area of a cone is equals half the circumference of the base times the slant height.
(L.A. = $½\pi rl$)

12-8 The volume of a cone equals one third the area of the base times the height of the cone.
($V = \frac{1}{3}\pi r^2h$)

12-9 The area of a sphere equals 4π times the square of the radius. ($A = 4\pi r^2$)

12-10 The volume of a sphere equals $\frac{4}{3}\pi$ times the cube of the radius. ($V = \frac{4}{3}\pi r^3$)

12-11 If the scale factor of two similar solids is a:b, then
(1) the ratio of corresponding perimeters is a:b.
(2) the ratio of the base areas, of the lateral areas, and of the total areas is $a^2:b^2$.

Theorems

(3) the ratio of the volumes is $a^3:b^3$.

Coordinate Geometry

13-1 Distance Formula: The distance d between points (x_1, y_1) and (x_2, y_2) is given by
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

13-2 An equation of the circle with center (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$.

13-3 Two nonvertical lines are parallel if and only if their slopes are equal.

13-4 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.
$m_1 \bullet m_2 = -1$ or $m_1 = -\frac{1}{m_2}$

13-5 Midpoint Formula: The midpoint of the segment that joins points (x_1, y_1) and (x_2, y_2) is the point
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

13-6 Standard Form: The graph of any equation can be written in the form Ax + By = C, with A and B not both zero, is a line.

13-7 Slope-Intercept Form: A line with the equation y = mx + b has slope m and y-intercept b.

13-8 Point-Slope Form: An equation of the line that passes through the point (x_1, y_1) and has slope m is $y - y_1 = m(x - x_1)$.

Transformations

14-1 An isometry maps a triangle to a congruent triangle.
Corollary 1 An isometry maps an angle to a congruent angle.
Corollary 2 An isometry maps a polygon to a polygon with the same area.

14-2 A reflection in a line is an isometry.

14-3 A translation is an isometry.

14-4 A rotation is an isometry.

14-5 A dilation maps any triangle to a similar triangle.
Corollary 1 A dilation maps an angle to a congruent angle.
Corollary 2 A dilation $D_{O, k}$ maps any segment to a parallel segment |k| times as long.
Corollary 3 A dilation $D_{O, k}$ maps any polygon to a similar polygon whose area is k^2 times as large.

14-6 The composite of two isometries is an isometry.

14-7 A composite of reflections in two parallel lines is a translation. The translation glides all points through twice the distance from the first line of reflection to the second.

14-8 A composite of reflections in two intersecting lines is a rotation about the point of intersection of the two lines. The measure of the angle of rotation is twice the measure of the angle from the first line of reflection to the second.
Corollary 1 A composite of reflections in perpendicular lines is a half-turn about the point where the lines intersect.

Constructions

Construction 1 Given a segment, construct a segment congruent to the given segment.

Construction 2 Given an angle, construct an angle congruent to the given angle.

Theorems

Construction 3	Given an angle, construct the bisector of the angle.
Construction 4	Given a segment, construct the perpendicular bisector of the segment.
Construction 5	Given a point on a line, construct the perpendicular to the line at the given point.
Construction 6	Given a point outside a line, construct the perpendicular to the line from the given point.
Construction 7	Given a point outside a line, construct the parallel to the given line through the given point.
Construction 8	Given a point on a circle, construct the tangent to the circle at the given point.
Construction 9	Given a point outside a circle, construct a tangent to the circle from the given point.
Construction 10	Given a triangle, circumscribe a triangle about the triangle.
Construction 11	Given a triangle, inscribe a circle in the triangle.
Construction 12	Given a segment, divide the segment into a given number of congruent parts.
Construction 13	Given three segments, construct a fourth segment so that the four segments are in proportion.
Construction 14	Given two segments, construct their geometric mean.