# Sections 1-1 & 1-2

Real Numbers and Their Graphs

Simplifying Expressions

# Objectives

- graph real numbers on a number line,
- compare numbers,
- find their absolute values,
- given a number, identify which subset(s) of the set of real numbers that it is an element of.
- understand and use correctly the basic terminology of algebra.
- simplify numerical and algebraic expressions by following the correct order of operations

## Real Numbers & Their Graphs

- Subsets of the set of real numbers:
  - natural numbers {1, 2, 3, … }
  - whole numbers {0, 1, 2, 3, … }
  - integers { … , - 3, - 2, - 1, 0, 1, 2, 3, … }
  - rational numbers: {all fractions, repeating & terminating decimals}
  - irrational numbers: { non-repeating or non-terminating decimals }
- Number lines and graphs of real numbers
  - Each point on a number line is paired with exactly one real number, called the coordinate of the point.
  - Each real number is paired with exactly one point on a number line, called the graph of the number.

# Real Numbers & Their Graphs

- Comparing points and their coordinates on a number line.
  - On a number line coordinates increase in value as you move from left to right so that :
    - coordinates of points to the left of a given point are less then (<) the given point’s coordinate.
    - coordinates of points to the right of a given point are greater than (>) the given point’s coordinate.
  - The origin is the point with the coordinate of 0.
  - The opposite of a given coordinate has the opposite sign and is the same distance from and on the other side of the origin.
  - The absolute value of a number is defined as the distance of the coordinate from the origin;  therefore, absolute value is always positive because distance is always positive.

# Simplifying Expressions

- Basic terms of algebra:
  - term:  any symbol or grouping of symbols used to represent a single value in a sum.
  - factor:  any symbol or group of symbols used to represent a single value in a product.
  - operation:  addition, multiplication, subtraction or division
  - numerical expression:  a symbol or group of symbols used to represent a number.
  - value:  the number represented by an expression.
  - equation:  a mathematical sentence stating an equality relationship between two expressions

# Simplifying Expressions

- inequality:  a mathematical sentence stating a relationship of order between two expressions.  This relationship can be one of the following:  $<, >, \leq, \geq$
- power: product of equal factors
- base:  the repeated factor in a power
- exponent:  the positive number of times the base occurs as a factor in a power.
- grouping symbols:  including but not limited to ( ), [ ], { }, absolute value bars, vincula, radical symbols, etc.
- variable:  any symbol, usually a letter, used to represent the elements of a given set called the domain of the variable.
- values:  the elements of the domain

# Simplifying Expressions

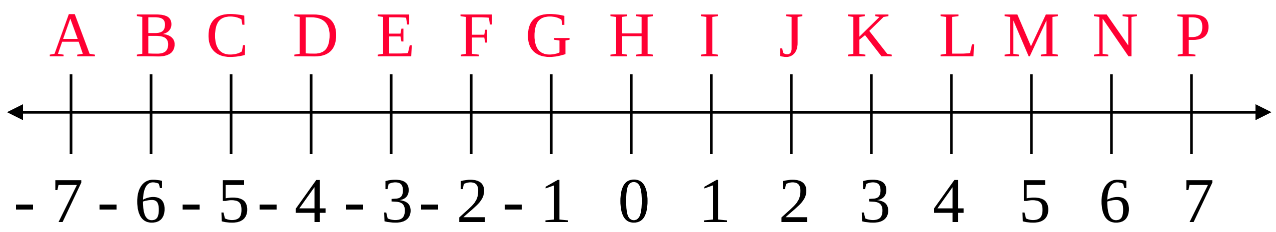
- algebraic expression:  a symbol or group of symbols including some variables used to represent a number
- simplify:  a process by which either a numerical or algebraic expression is transformed into as few terms and operations as possible
- evaluating:  the process of replacing the variables in an expression with their associated values from their domains and simplifying the resulting numerical expression.
- substitution:  the process of replacing an expression with a known logical equivalent for that expression.

# Simplifying Expressions

- Order of Operations:
  - Simplify the expressions within each grouping symbol working outward from the innermost grouping. If the grouping symbol has a mathematical operation associated with it such as absolute value or roots, this operation cannot be performed until the expression within the symbol has been simplified to one term.
  - Perform any types of multiplication which include powers, multiplication of factors or division in order from left to right.
  - Perform any types of addition including addition of negatives and subtraction in order from left to right.

Examples for 1-4

Identify the number associated with the given letter on the number line.




The number below the letter B is - 6.

# Example for 5-10

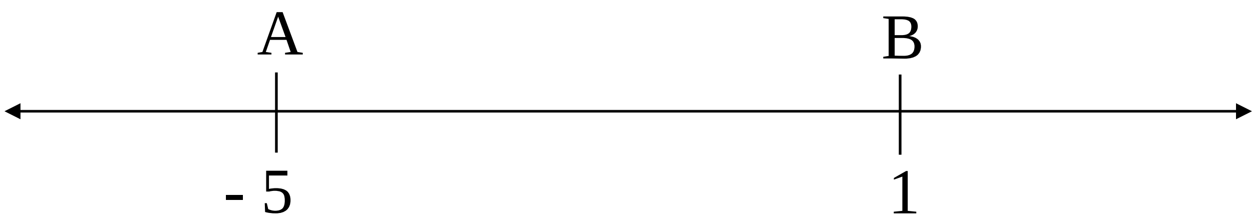
Find the coordinate of the point halfway between D & J.

The coordinates of D & J are - 4 & 2 respectively. The distance between D & J is $|-4-2| = |-6| = 6$.

The question says "halfway" so (½)(6) = 3.

In #5 point D is furthest to the left and has a coordinate of - 4 so - 4 + 3 = - 1. - 1 is our answer.

Example for 33-40




Find the coordinate of the point 2 units to the left of B.

“left” means to subtract the 2 from the coordinate of B, 1. 1 - 2 = - 1

# Example for 1-10

(9 - 3) - 2 _____ 9 - (3 - 2)

The left side simplifies to: (9 - 3) - 2 = 6 - 2 = 4

The right side simplifies to:  9 - (3 - 2) = 9 - 1 = 8

4 < 8

# Example for 11-24

$$64 \div 4^2 + 3(3^2 - 1)$$

$$64 \div 4^2 + 3\left(3^2 - 1\right) = 64 \div 4^2 + 3(9 - 1)$$

$$64 \div 4^2 + 3(9 - 1) = 64 \div 4^2 + 3(8)$$

$$64 \div 4^2 + 3(8) = 64 \div 16 + 3(8)$$

$$64 \div 16 + 3(8) = 4 + 24 = 28$$

# Example for 25-36

x = 3, y = 2, z = 5

$$\frac{z^2 - \left(x^2 - y^2\right)}{3y^2 z} = \frac{5^2 - \left(3^2 - 2^2\right)}{3\left(2^2\right)5}$$

$$\frac{5^2 - \left(3^2 - 2^2\right)}{3\left(2^2\right)5} = \frac{5^2 - (9 - 4)}{3\left(2^2\right)5} = \frac{5^2 - (5)}{3\left(2^2\right)5} = \frac{25 - (5)}{3(4)5} = \frac{20}{60} = \frac{1}{3}$$

# Example for 37-40

$$2|a| + |b|$$

a = 6, b = - 2

$$2|a| + |b| = 2|6| + |-2|$$

$$2|6| + |-2| = 2(6) + 2 = 12 + 2 = 14$$

# Example for 41-44

$$\frac{3u^2 - 2(v-3)^2}{2\left(u^2 - 1\right) - v^2}; u = 4, v = 5$$

$$\frac{3u^2 - 2(v-3)^2}{2\left(u^2 - 1\right) - v^2} = \frac{3\left(4^2\right) - 2(5-3)^2}{2\left(4^2 - 1\right) - 5^2}$$

$$\frac{3\left(4^2\right) - 2(5-3)^2}{2\left(4^2 - 1\right) - 5^2} = \frac{3\left(4^2\right) - 2(2)^2}{2(16-1) - 5^2} = \frac{3(16) - 2(4)}{2(15) - 25} = \frac{48 - 8}{30 - 25} = \frac{40}{5} = 8$$

# Sections 1-3, 1-4 & 1-5

Basic Properties of Real Numbers

Sums and Differences

Products

# Objectives

- express the properties of equality, addition and subtraction in your own words.
- understand which property of equality, addition or subtraction is being used when named.
- identify which property is being used at each stage of a problem's solution.
- use the properties correctly to solve multi-step problems. add positive and negative real numbers **without the use of a calculator.**
- subtract positive and negative numbers **without the use of a calculator.**
- multiply positive and negative real numbers **without a calculator**

# Basic Properties of Real Numbers

- Properties of Equality
  - reflexive: a number equals itself. a = a
  - symmetric: you may exchange the expressions on either side of the equals sign. If a = b, then b = a.
  - transitive: value can be transferred from a first term through a second an into a third. If a = b and b = c, then a = c.
  - addition: you may add equal values to both sides of an equation and maintain the equality. If a = b, then a + c = b + c or c + a = c + b.
  - multiplication: you may multiply both sides of an equation by the same value and maintain the equality. If a = b, then ac = bc or ca = cb.

# Basic Properties of Real Numbers

- Field Properties of Real Numbers:
  - closure for addition: when two real numbers are added the sum is a real number.
  - closure for multiplication: when two real numbers are multiplied the product is a real number.
  - commutative for addition: the order of the terms in addition does not effect the value of the sum. $a + b = b + a$.
  - commutative for multiplication: the order of the factors in multiplication does not effect the value of the product. $ab = ba$.
  - associative for addition: the way terms are grouped in addition does not effect the value of the sum. $(a + b) + c = a + (b + c)$.

# Basic Properties of Real Numbers

- associative for multiplication: the way the factors are grouped in multiplication does not effect the product. (ab)c = a(bc)
- identity properties
  - identity for addition: for every real number a there exists a unique real number 0 such that a + 0 = a.
  - identity for multiplication: for every real number a there exists a unique real number 1 such that (a)(1) = a

# Basic Properties of Real Numbers

- inverse properties
  - opposite or additive inverse: for every real number a there exists a real number - a such that a + (- a) = 0
  - reciprocals: for every real number a there exists a real number $\frac{1}{a}$ such that $\left(\frac{a}{1}\right)\left(\frac{1}{a}\right) = 1$
- distributive: real numbers can be multiplied into the terms of a sum or real numbers can be divided out of the terms of a sum. a(b + c) = ab + ac.

# Sums & Differences

- subtraction is the same as addition of a negative.
  a - b = a + (-b)
- If two numbers have the same sign, then add their absolute values and the sign of the sum is the same as the sign of the numbers.
- If two numbers have opposite signs, then subtract the lesser absolute value from the greater absolute value and the sign of the sum is the sign of the number with the greater absolute value.
- When simplifying an expression by addition or subtraction only like (similar) terms can be combined.
  - like terms: must both be constants or must both contain the same variable(s) raised to the same exponent(s).

# Products

- Properties of Multiplication:
  - zero: for every real number a there exists a unique real number 0 such that a(0) = 0(a) = 0.
  - - 1: for every real number a there exists a unique real number - 1 such that a(- 1) = - 1(a) = - a.
  - opposite of a product: for all real numbers a and b, - ab = (- a)b = a(- b).
  - opposite of a sum: for all real numbers a and b, - (a + b) = (- a) + (- b).

# Products

- Rules for Multiplication:
  - The product of two numbers with the same sign is positive.
  - The product of two numbers with opposite signs is negative.
  - The absolute value of a product is equal to the product of the absolute values.
  - The product of an even number of negative factors is positive.
  - The product of an odd number of negative factors is negative.
  - A product is 0 if and only if one of its factors is 0.

# Example for 1-10

$$2(a + 4) + (-8) = 2(a) + 2(4) + (-8)$$

$$2(a) + 2(4) + (-8) = 2a + 8 + (-8)$$

$$2a + 8 + (-8) = 2a + 0 =$$

$$2a$$

# Example for 11-16

$$5(2n + 1) + (-5) = 10n$$

$$5(2n) + 5(1) + (-5) = 10n$$

$$10n + 5 + (-5) = 10n$$

$$10n + 0 = 10n$$

$10n = 10n$, true

# Example for 17-22

a. $\frac{1}{2}(1+2t)=\frac{1}{2}(1)+\frac{1}{2}(2t)$

b. $=\frac{1}{2}(1)+\left(\frac{1}{2}\bullet 2\right)(t)$

c. $=\frac{1}{2}(1)+1(t)$

d. $=\frac{1}{2}+t$

a. distributive

b. associative of multiplication

c. reciprocals or inverse of multiplication

d. identity property of multiplication

# Example for 23&24

| | |
|---|---|
| a. $3x + (-12) = 0$ | a. given |
| b. $[3x + (-12)] + 12 = 0 + 12$ | b. |
| c. $3x + [(-12) + 12] = 0 + 12$ | c. |
| d. $3x + 0 = 0 + 12$ | d. |
| e. $3x = 12$ | e. |
| f. $\frac{1}{3}(3x) = \frac{1}{3}(12)$ | f. |
| g. $\frac{1}{3}(3x) = 4$ | g. substitution |
| h. $\left(\frac{1}{3} \bullet 3\right)x = 4$ | h. |
| i. $1(x) = 4$ | i. |
| j. $x = 4$ | j. |

# Example for 25-33

- {2, 4, 6, ...} All the elements of this set can be written as 2n where n∈{natural numbers} so that all we must show is that every sum will 2 times a natural number.
- For different values of n, namely $n_1$ and $n_2$ we would have $2n_1 + 2n_2 = 2(n_1 + n_2)$ and since the natural numbers are closed for addition we know that $(n_1 + n_2) \in$ {natural numbers} so we have 2 times a natural number for the sum and the set is closed.
- Likewise, $(2n_1)(2n_2) = 2(2n_1n_2)$ and since the natural numbers are closed for multiplication we know that $(2n_1n_2) \in$ {natural numbers} so we have the product of 2 and a natural number and the set is closed.
- Thus the set of positive even numbers is closed for addition and multiplication.

# Example for 1-22

$$|6 - 13| - |22 - (-6)| =$$

$$|-7| - |22 + 6| =$$

$$7 - 28 =$$

$$-21$$

# Example for 23-26

$$4(p - 2q) =$$

$$4(p) - 4(2q) =$$

$$4p - 8q.$$

# Example for 27-34

$$7(x + 2) + 4(x - 4) =$$

$$7(x) + 7(2) + 4(x) - 4(4)$$

$$7x + 14 + 4x - 16 =$$

$$(7x + 4x) + [14 + (-16)] =$$

$$11x - 2$$

# Example for 35-42

$-7 + \_\_\_ = 2$ is the same as $-7 + x = 2$

$$-7 + x = 2$$

$$7 + (-7) + x = 7 + 2$$

$$0 + x = 9$$

$$x = 9$$

## Example for 43-50

Lake Baikal, in Siberia, is 5315 ft deep, and its surface is 1493 ft above sea level. How many feet below sea level is the bottom of the lake at its deepest point?

let sea level = 0 then measurements above sea level will be positive and measurements below sea level will be negative

1493 - 5315 =

1493 + (-5315) =

- 3822 =

3822 feet below sea level

# Example for 1-30

$$5(-3a^3 - 2) + 3(-2 - a^3) = 5(-3a^3) + 5(-2) + 3(-2) + 3(-a^3)$$

$$= -15a^3 + (-10) + (-6) + (-3a^3)$$

$$= -15a^3 + (-3a^3) + (-10) + (-6)$$

$$= -18a^3 + (-16)$$

$$= -18a^3 - 16$$

# Example for 31-34

$$x = 1$$

$$x(x - 2)(x - 4) =$$

$$1(1 - 2)(1 - 4) =$$

$$1[1 + (-2)][1 + (-4)] =$$

$$1(-1)(-3) =$$

$$3$$

# Sections 1-6 & 1-7

Quotients

Solving Equations in One Variable

# Objectives

- divide real numbers ***without using a calculator***!
- identify the three processes of transformation.
- identify the five steps to solve a single variable linear equation.
- solve one variable equations by using the processes of simplification and transformation.
- determine whether a value is an element of an open sentence's solution set.
- substitute values into a variable and solve for an unknown variable.
- isolate a single variable in a formula by using the processes of simplification and transformation.

# Quotients

- Division by a number is equivalent to multiplying by the reciprocal of that number. $\frac{a}{b} = a \div b = \left(\frac{a}{1}\right)\left(\frac{1}{b}\right)$
- Rules for Division
  - The quotient of two numbers with the same sign is positive.
  - The quotient of two numbers with opposite signs is negative.
  - Any number divided by 0 is undefined.
  - Zero divided by any number is 0.
  - $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ or $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$

# Solving Equations in One Variable

- open sentence: an equation the contains a variable.
- solution (root):  the value(s) of the variable(s) that makes the equation true.
- solution set:  the set of all roots for an open sentence.
- equivalent equations:  equations that have the same solution set over a given domain.
- solve:  the process of transforming an equation to discover the solution set.

# Solving Equations in One Variable

- null set (empty set): the set with no elements.
- identity: an equation whose solution set contains all values.
- formula: an equation that states a constant relationship between two or more variables which represent physical or geometric quantities.
- linear equation: all variables have exponents of one.

# Solving Equations in One Variable

- Transformations that Produce Equivalent Equations
  - Simplify either side of the equation.
  - Add (or subtract) the same value to both sides of the equation.
  - Multiply (or divide) the same value to both sides of the equation.

# Solving Equations in One Variable

- Solving Single Variable Linear Equations
  - Simplify both the left and the right side of the equation.
  - Choose one side for the variables and one side for the constants.
  - Using the appropriate transformations, move all variables to the same side and combine similar terms.
  - Using the appropriate transformations, move all constants to the other side and combine all numbers.
  - Use the appropriate transformation to change the coefficient of the variable into one.

# Example for 1-24

$$\frac{36c^2 - 24c - 6}{6} = \frac{6\left(6c^2 - 4c - 1\right)}{6} = 6c^2 - 4c - 1$$

# Example for 25-28

$$y = 1$$

$$\frac{y(y-3)}{y-2} = \frac{1(1-3)}{1-2} = \frac{1(-2)}{-1} = \frac{-2}{-1} = 2$$

# Example for 1-24

$$3(x - 2) - x = 2(2x + 1)$$

$$3(x) + 3(- 2) - x = 2(2x) + 2(1)$$

$$3x - 6 - x = 4x + 2$$

$$3x - x - 6 = 4x + 2$$

$$2x - 6 = 4x + 2$$

constants = variables

$$- 2x + 2x - 6 = - 2x + 4x + 2$$

$$- 6 = 2x + 2$$

$$- 6 - 2 = 2x + 2 - 2$$

$$- 8 = 2x$$

$$\frac{1}{2}(- 8) = \frac{1}{2}(2x)$$

$$- 4 = x$$

# Example for 25-30

$$x(x - 3)(x + 2) = 0; \quad x = -2$$

$$(-2)(-2 - 3)(-2 + 2) = 0$$

$$(-2)(-5)(0) = 0$$

$0 = 0$; yes

$$x(x - 3)(x + 2) = 0; \quad x = -3$$

$$-3(-3 - 3)(-3 + 2)$$

$$(-3)(-6)(-1)$$

$-18 \neq 0$; no

# Example for 31-42

$P = 2l + 2w$, solve for w

$P = 2l + 2w$ is already simplified

treat P and l as constants

constants = variables

all w's are already on left side of equation

$$- 2l + P = -2l + 2l + 2w$$
$$- 2l + P = 2w$$

$$\frac{1}{2}(- 2l + P) = \frac{1}{2}(2w)$$
$$\frac{1}{2}(-2l) + \frac{1}{2}(P) = w$$
$$- l + \frac{1}{2} P = w$$

# Example for 43-50

$A = P(1 + rt)$; $A = 168$, $P = 150$, $r = 0.08$

$168 = 150\,(1 + 0.08t)$

$168 = 150(1) + 150(0.08t)$

$168 = 150 + 12t$

$-150 + 168 = -150 + 150 + 12t$

$18 = 12t$

$1.5 = t$

# Sections 1-8 & 1-9

Words Into Symbols

Problem Solving with Equations

# Objectives

- translate an English sentence or phrase into an algebraic expression.
- translate an English sentence or phrase into an open sentence and solve for a specified variable.
- identify and perform the five steps in solving a word problem.
- identify unnecessary information in a word problem.
- recognize when a word problem has no solution.

# English Into Math

- When translating from the English language into the mathematical language you should treat verbs as equals signs.
- Words such as “more than”, “greater than” and “sum” translate into the operation addition.
- Words such as “decreased by”, “less than” and “difference” translate into the operation subtraction.
- Values such as “twice” or “one-third” followed by “of” translate into multiplication where the value is one of the factors and the word(s) following the “of” is the other factor.

# English Into Math

- Phrases such as “at least” or “at most” indicate that the open sentence is an inequality **not** an equation.
- Common word problem elements:
  - One of the most common formulas used in word problems is the equation describing uniform physical motion: rate x time = distance or r(t) = d.
  - integers are represented by:
    … , n - 1, n, n + 1, n + 2, n + 3, …
  - consecutive odd or even integers are represented by:
    … , n - 2, n, n + 2, n + 4, …

# Problem Solving With Equations

- Steps for Solving Word Problems:
  - Read the problem carefully. Identify the question part of the problem and determine what number(s) is asked for as an answer(s). Set your variable(s) equal to this number(s).
  - Re-read the problem carefully and make a sketch if the problem describes an object or physical relationship. Write down and label any other numerical information given in the problem.
  - Slowly re-read the problem and translate the English phrases and sentences into algebraic expressions and open sentences.
  - Solve the equation(s) and find the required number(s).
  - Check your answer with the original statement of the problem and give your answer with the appropriate units of measurement.

## Problem Solving With Equations

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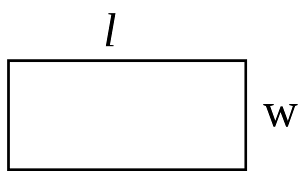
# Problem Solving With Equations

- Keep in mind:
  - some problems will give you more information than you need.
  - some problems do not have a solution.

Example for 1-28

The length and width of a rectangle are consecutive even integers, and the length is ***l*** cm. Find the area and the perimeter.

length = ***l***




width is a consecutive even integer: width = ***l*** + 2. Area of a rectangle equals length x width ( A = ***l***w). Perimeter of a rectangle equals twice the length plus twice of the width

(P = 2***l*** + 2w).

$A = l(l + 2) = l(l) + l(2) = (l^2 + 2l)\text{cm}^2$

$P = 2l + 2(l + 2) = 2l + 2(l) + 2(2) = 2l + 2l + 4 = (4l + 4)\text{cm}$

This step is not necessary for this problem, but if we had found a numerical answer we would have known that a negative number would not have been reasonable.

Example for 1-22

A jar contains 40 coins consisting of dimes and quarters and having a total value of $4.90. How many of each kind of coin are there?

The problem asks you to find "How many of each kind of coin (dimes & quarters) are there?" So we will set d = # of dimes and q = # of quarters.

total # of both dimes & quarters is 40; total value of the dimes & quarters is $4.90.

$d + q = 40$ or $d = 40 - q$; $.10d + .25q = 4.90$

Continued on the next slide.

substitute 40 - q into the .10d + .25 q = 4.90 for d and you get

$.10(40 - q) + .25q = 4.90$ which you can solve for q:

$$.10(40) + .10(- q) + .25q = 4.90$$

$$4 + (- .10q) + .25q = 4.90$$

$$4 + .15q = 4.90$$

$$- 4 + 4 + .15q = - 4 + 4.90$$

$$.15q = .90$$

$q = 6$ which we can substitute back into either equation to

solve for d: $d + 6 = 40$

$$d + 6 - 6 = 40 - 6$$

$$d = 34$$

34 dimes & 6 quarters