Section 2-1

Solving Inequalities with One Variable

Objectives

- solve one variable inequalities.
- graph the solution set of an inequality on a number line.
- write the solution set of an inequality as a simplified inequality.

Solving Inequalities in One Variable

- Properties of Order:
  - comparison: there are three fundamental relationships between numbers that can be true and they are mutually exclusive. These relationships are: $a < b$, $a = b$ & $a > b$ given that a & b are real numbers.
  - transitive: the natural order of numbers can be transmitted from a first value through a second and to a third. If $a <b$ and $b < c$, then $a < c$, given a,b and c are real numbers.
  - addition: you may add the same value to both sides of an inequality without changing the relationship of order. If $a < b$, then $a + c < b + c$.

Solving Inequalities in One Variable

- multiplication:
  - You may multiply a positive value to both sides of an inequality without changing the relationship of order. If a < b & c is positive , then ac < bc.
  - You may multiply a negative value to both sides of an inequality but this reverses the relationship of order. If a < b & c is negative, then ac > bc.

Solving Inequalities in One Variable

- Transformations that Produce Equivalent Inequalities
  - Simplify either side of an inequality.
  - Add the same value or expression to each side of the inequality
  - Multiply the same positive value to both sides of the inequality
  - Multiply the same negative value to both sides of the inequality and reverse the relationship of order.
- equivalent inequalities: inequalities with the same solution set.

Solving Inequalities in One Variable

- Graphing inequalities
  - a closed dot indicates the value is included in the solution set
  - an open dot indicates the value is not included in the solution set

Example for 1-24

$7y - 2(y - 4) > 6 - (2 - y)$

$7y - 2(y) - 2(-4) > 6 - (2) - (-y)$

$7y - 2y + 8 > 6 - 2 + y$

$5y + 8 > 4 + y$

variables > constants

$5y + 8 - y > 4 + y - y$

$5y - y + 8 > 4$

$4y + 8 > 4$

$4y + 8 - 8 > 4 - 8$

$4y > -4$

$\frac{1}{4}(4y) > \frac{1}{4}(-4)$

$y > -1$

Example for 25-30

If $a < b$, then $a - c < b - c$

$a < b$

$a + (-c) < b + (-c)$ by the property of addition for order

$a - c < b - c$ by the definition of subtraction

It is true.

Example for 25-30

If $a < b$, then $a^2 < b^2$

$a < b$

Since $a^2 = a(a)$ we will say $a(a) < b(a)$ or $a^2 < ba$.

In order for $ba = b^2$ we would have to know that $b = a$.

It is false.

$-3 < 1$, but $(-3)^2 > (1)^2$ or $9 > 1$

Sections 2-2 & 2-3

Solving Combined Inequalities

Problem Solving Using Inequalities

Objectives

- solve conjunctions and graph the solution set on a number line.
- solve disjunctions and graph the solution set on a number line.
- solve word problems involving one variable inequalities.

Solving Combined Inequalities

- conjunction: is true when both sentences are true
  - uses the word "and"
  - e.g. $x > a$ and $x < b$ written as $a < x < b$
- disjunction: is true when at least one of the sentences is true
  - uses the word "or"

Problem Solving Using Inequalities

- Mathematical translations for English phrases indicating inequalities:
  - "x is at least a" or "x is no less than a" translates as $x \geq a$
  - "x is at most a" or "x is no greater than a" translates as $x \leq a$
  - "x is between a and b" or "x is between a and b, exclusive" translates as $a < x < b$
  - "x is between b and a" or "x is between b and a, exclusive" translates as $b < x < a$
  - "x is between a and b, inclusive" translates as $a \leq x \leq b$
  - "x is between b and a, inclusive" translates as $b \leq x \leq a$

Example for 1-32

$$2t + 7 \geq 13 \text{ or } 5t - 4 < 6$$

all expressions are in simplified form.

$$2t + 7 \geq 13$$

$$2t + 7 - 7 \geq 13 - 7$$

$$2t \geq 6$$

$$\tfrac{1}{2}(2t) \geq \tfrac{1}{2}(6)$$

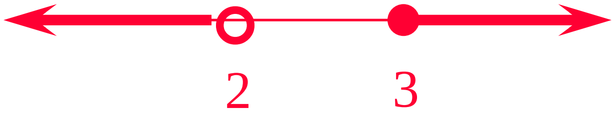
$$t \geq 3$$

$$5t - 4 + 4 < 6 + 4$$

$$5t < 10$$

$$\frac{1}{5}(5t) < \frac{1}{5}(10)$$

$$t < 2$$



Example for 1-32

$$-5 < 1 - 2k < 3$$

all expressions are in simplified form

$$-5 < 1 - 2k$$

$$-1 - 5 < -1 + 1 - 2k$$

$$-6 < -2k$$

$$-\tfrac{1}{2}(-6) > -\tfrac{1}{2}(-2k)$$

$$3 > k$$

$$1 - 2k < 3$$

$$-1 + 1 - 2k < -1 + 3$$

$$-2k < 2$$

$$-\tfrac{1}{2}(-2k) > -\tfrac{1}{2}(2)$$

$$k > -1$$

Example for 1-14

Find all sets of three consecutive odd integers whose sum is between 20 and 30.

x = 1st of three consecutive odd integers

The sum of the numbers is between 20 & 30.

$$20 < x + (x + 2) + (x + 4) < 30$$

$$20 < x + x + x + 2 + 4 < 30$$

$$20 < 3x + 6 < 30$$

$$20 < 3x + 6 \ \& \ 3x + 6 < 30$$

$$20 - 6 < 3x + 6 - 6 \ \& \ 3x + 6 - 6 < 30 - 6$$

$$14 < 3x \ \& \ 3x < 24$$

$$\frac{1}{3}(14) < \frac{1}{3}(3x) \ \& \ \frac{1}{3}(3x) < \frac{1}{3}(24)$$

$$\frac{14}{3} < x < 8 \quad x = 5 \text{ or } x = 7$$

{5, 7, 9} or {7, 9, 11}

Sections 2-4 & 2-5

Absolute Value in Open Sentences

Solving Absolute Value Sentences Graphically

Objectives

- solve open sentences using absolute value.
- graph the solution set on a number line.
- solve open sentences involving absolute value graphically.
- solve for a specified variable in an open sentence using absolute value.

Absolute value in Open Sentences

- For every set of absolute value symbols in an open sentence you have two possible expressions to solve.
  - you must solve the open sentence where the value inside the absolute value bars is positive.
  - you must solve the open sentence where the value inside the absolute value bars is negative.
- When the graphs of all of the possible expressions are connected you have a conjunction.
- When the graphs of all of the possible expressions are disconnected you have a disjunction.

Solving Absolute Value Sentences Graphically

- The distance on a number line between two numbers a & b is equal to the absolute value of their difference.
- All open sentences involving absolute value can be thought of as finding the distance between two coordinates on a number line.

Example for 1-28

$$|s+3| = 3$$

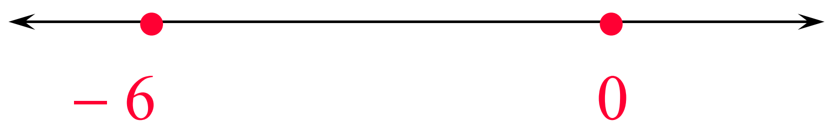
$$s + 3 = 3 \quad \& \quad -(s + 3) = 3$$

$$s + 3 = 3 \quad \& \quad -s - 3 = 3$$

$$s + 3 - 3 = 3 - 3 \quad \& \quad -s - 3 + 3 = 3 + 3$$

$$s = 0 \quad \& \quad -s = 6 \text{ or } s = -6$$

$$\{0, -6\}$$



Example for 1-28

$$|2t + 5| < 3$$

$$2t + 5 < 3 \;\;\&\;\; -(2t + 5) < 3$$

$$2t + 5 < 3 \;\;\&\; -2t - 5 < 3$$

$$2t + 5 - 5 < 3 - 5 \;\&\;\; -2t - 5 + 5 < 3 + 5$$

$$2t < -2 \;\;\&\;\; -2t < 8$$

$$\tfrac{1}{2}(2t) < \tfrac{1}{2}(-2) \;\;\&\;\; -\tfrac{1}{2}(-2t) > -\tfrac{1}{2}(8)$$

$$t < -1 \;\;\&\; t > -4$$

$$\{t: -4 < t < -1\}$$

Example for 1-24

$$3 - |2x - 3| > 1$$

$$-|2x - 3| > -2$$

$$-1(-|2x - 3|) < -1(-2)$$

$$|2x - 3| < 2$$

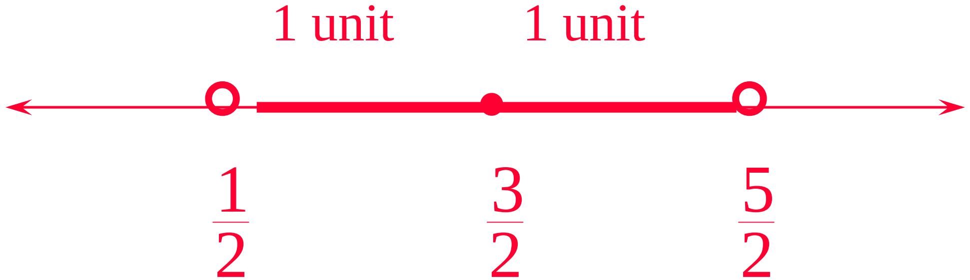
the variable is already first

$$|2| \bullet \left|x - \frac{3}{2}\right| < 2$$

the statement is already written as subtraction

$$\frac{1}{2}(2) \bullet \left|x - \frac{3}{2}\right| < \frac{1}{2}(2) \qquad \left|x - \frac{3}{2}\right| < 1$$

find all values that are less than one unit from $\frac{3}{2}$



Sections 2-6 & 2-7

Theorems and Proofs

Theorems About Order and Absolute Value

Objectives

- give counterexamples which prove a statement false.
- identify the reasons for steps in an algebraic proof.
- prove the properties of real numbers by using axioms, definitions and theorems.
- identify the reasons for steps in the algebraic proofs about inequality and absolute values.
- prove the theorems about inequalities and absolute value.

Theorems & Proofs

- Mathematics is the language of science and technology. It is the means by which we analyze and describe our experiences in the universe.
- Mathematics and therefore science is not an absolute.
- The structure of mathematics and science is identical to that of religion--it is based on unproven assumptions that require a leap of faith (belief) guided by an informed mind (conscience).
- The primary difference between science and religion is that we can observe the subject of our study and conjecture in science and be reassured in our beliefs by what we experience with our senses.

Theorems & Proofs

- The unproven assumptions for mathematics are its axioms (postulates). The axioms we cannot prove but must accept if mathematics and science are to work are:
    - substitution (page 8)
    - properties of equality (page 14)
    - field properties of real numbers (page 15)
    - properties of order (page 59)
- The complex and powerful structures of mathematics have been developed through the rigorous process of logical proof.

Theorems & Proofs

- Proofs are the method of demonstrating the truth of a statement. All statements have two components:
  - A hypothesis or set of prerequisites that must be met before the statement is considered applicable.
  - A conclusion or result of the prerequisites having been met.
  - Interchanging the hypothesis and conclusion of a statement results in a new statement called the converse. Statements and their converses are not logical equivalents. The truth of one does not effect the truth of the other.

Theorems & Proofs

- Once a statement has been proven to be true it can be considered to be a new “law” or theorem of mathematics.
  - These theorems remain true so long as their underlying structures are true. If our assumptions are incorrect then any theorems we have created by applying those assumptions will also be incorrect.
  - To be true a statement must be true for all cases. If there is even one instance when the statement does not hold true then we say that it is always false.
  - To prove a statement false it is necessary to find a counterexample. A case which agrees with the hypothesis of the statement but disagrees with the conclusion of the statement.

Theorems & Proofs

- Theorems of Real Numbers:
  - cancellation property of addition: the same term on opposite sides of an equation will cancel each other out.
    - If $a + c = b + c$, then $a = b$.
    - If $c + a = c + b$, then $a = b$.
  - cancellation property of multiplication: the same factor on opposite sides of an equation will cancel each other out.
    - If $ac = bc$, then $a = b$.
    - If $ca = cb$, then $a = b$.
  - zero-product property: for any product to be zero one of the factors must be zero.
    - $ab = 0$ if and only if $a = 0$ or $b = 0$.

Theorems about Order & Absolute Value
Method for Writing an Algebraic Proof:

- Read the statement and identify the hypothesis and the conclusion.
  - In an if-then statement the hypothesis is everything after the "if" and before the "then"
  - In an if-then statement the conclusion is everything after the "then"
- Set up a two column grid with the statement column on the left and the reason column on the right.
- Make your first statement the hypothesis of your statement, list its reason as given.
- Use your axioms, definitions & previously proven theorems to transform the hypothesis into the conclusion.
- Do not skip steps and provide the name of the axiom, definition or theorem for the reason of each transformation.

Example for 1-4

If $a^2 = b^2$, then $a = b$.

hypothesis is $a^2 = b^2$ & conclusion is $a = b$

$(3)^2 = (-3)^2$ or $9 = 9$ but $3 \neq -3$

$a = 3$ & $b = -3$ is your counterexample

Example for 5-14

1. $-ab = -ba$	1. commutative of (x)
2. $-ab = (-b)a$	2. associative of (x)
3. $-ab = a(-b)$	3. commutative of (x)

Example for 15-21

I.	a is a real number $\neq 0$	Given
II.	$\left(\frac{1}{a}\right)a = 1$	Property of reciprocals (existence of a multiplicative inverse)
III.	$\left(\frac{1}{a}\right)\left(\frac{1}{\frac{1}{a}}\right) = 1$	Property of reciprocals (existence of a multiplicative inverse)
IV.	$\frac{1}{\frac{1}{a}} = a$	Property of reciprocals (uniqueness)