Section 5-1

Quotients of Monomials

Quotients of Monomials

- When you have the product of fractions you multiply numerator-to-numerator and denominator-to-denominator

$$\left(\frac{3}{5}\right)\left(\frac{4}{7}\right) = \frac{(3)(4)}{(5)(7)} = \frac{12}{35}$$

- When reducing a fraction you are factoring the numerator and the denominator so that any number that appears in both will reduce to one and cancel out.

$$\frac{20}{25} = \frac{(5)(4)}{(5)(5)} = \left(\frac{5}{5}\right)\left(\frac{4}{5}\right) = 1\left(\frac{4}{5}\right) = \frac{4}{5}$$

Quotients of Monomials

- When multiplying like bases add the exponents
- When dividing like bases subtract the exponents.
  - Negative exponents indicate that you are to take the same positive power of the reciprocal.

$$\frac{x^3}{x^8} = x^{3-8} = x^{-5} = \frac{1}{x^5}$$

$$\frac{x^3}{x^8} = \frac{x \bullet x \bullet x}{x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x} = \left(\frac{x}{x}\right)\left(\frac{x}{x}\right)\left(\frac{x}{x}\right)\left(\frac{1}{x \bullet x \bullet x \bullet x \bullet x}\right) = \frac{1}{x^5}$$

- When taking the power of a product raise each factor in the product to the power.
- When taking the power of a fraction raise everything in both the numerator and the denominator to that power.
- When raising a power to a power multiply the exponents.

Examples for 1-30

$$\frac{u^2}{v}\left(\frac{3v}{u^2}\right)^2$$

$$\frac{u^2}{v}\left(\frac{3^2v^2}{(u^2)^2}\right)$$

$$\frac{u^2}{v}\left(\frac{9v^2}{u^4}\right)$$

$$\frac{9u^2v^2}{u^4v}$$

$$9u^{2-4}v^{2-1}$$

$$9u^{-2}v$$

$$\frac{9v}{u^2}$$

$$\frac{a^{2m}b^{2m+1}}{\left(a^2b^2\right)^m}$$

$$\frac{a^{2m}b^{2m+1}}{\left(a^{2m}b^{2m}\right)}$$

$$\left(\frac{a^{2m}}{a^{2m}}\right)\left(\frac{b^{2m+1}}{b^{2m}}\right)$$

$$1\left(\frac{b^{2m+1}}{b^{2m}}\right)$$

$$b^{2m+1-2m}$$

$$\mathbf{b}$$

Section 5-2

Zero and Negative Exponents

Zero and Negative Exponents

- Any number other than zero with an exponent of zero equals 1
  - If $x \in \Re$ and $x \neq 0$, then $x^0 = 1$.
  - 0^0 is undefined
- A negative exponent means to place the value in the denominator of a fraction or to take the same power of its reciprocal.
  - If n is a positive integer and $x \neq 0$, then $x^{-n} = \frac{1}{x^n}$

Example for 1-8

$(2^{-2} \bullet 3^{-1} \bullet 5^{0})^{-1}$

power of product $(2^{2} \bullet 3^{1} \bullet 5^{0})$

no multiplication or division of like bases

no negative exponents remaining

convert the power of zero to a one $(2^{2} \bullet 3^{1} \bullet 1)$

simplify (4)(3)(1) = 12

Example for 9-12

$$\frac{7}{10{,}000}$$

Rewrite 10,000 as 10^4

Rewrite the denominator as 10^{-4} so that the fraction (quotient)

becomes the product $(7)(10^{-4})$

Example for 13-20

596×10^{-2}

converting the negative exponent creates the fraction: $\frac{596}{10^2} = \frac{596}{100}$

performing the division creates the decimal **5.96**

Example for 21-42

$$4x^3y^{-6} + (x^{-1}y^2)^{-3}$$

Perform power of a product and power of a power to get

$$4x^3y^{-6} + (x^3y^{-6})$$

Combine like terms $5x^3y^{-6}$

Convert the negative exponent into the fraction

$$\frac{5x^3}{y^6}$$

Example for 43-46

$(x+y)^{-1}$ & $x^{-1}+y^{-1}$

Let $x = 3$ and $y = 4$

$(3+4)^{-1} = 7^{-1} = \frac{1}{7}$ and $3^{-1}+4^{-1} = \frac{1}{3}+\frac{1}{4} = \frac{4}{12}+\frac{3}{12} = \frac{7}{12}$

the two expressions are not equal so $x = 3$ & $y = 4$ is a counterexample.

Example for 47-50

Divide each term of the polynomial on the left side by the monomial on the right side.

$$x^{-1} - 4x^{-2} + 2x^{-3} = x^{-3}\,(?)$$

$$\frac{x^{-1}}{x^{-3}} - \frac{4x^{-2}}{x^{-3}} + \frac{2x^{-3}}{x^{-3}} = (?)$$

$$x^{-1-(-3)} - 4x^{-2-(-3)} + 2x^{-3-(-3)} = (?)$$

$$x^{2} - 4x + 2x^{0} = (?)$$

$$x^{2} - 4x + 2 = (?)$$

Section 5-3

Scientific Notation and Significant Digits

Scientific Notation

- scientific notation is used to express very large or very small numbers without writing all of the zeroes
- significant digit: is any nonzero digit or any zero that has a purpose other than placing the decimal point.
- when multiplying or dividing scientific numbers the estimate should have the same number of digits as the least accurate factor.
- rounding error: when rounding by the rule of five the difference between a number and its approximation is at most half the unit of the last digit retained.
- precision: is given by the unit used in making the measurement
- maximum possible error: is half the unit of precision
- accuracy: is the relative error, usually expressed as the ratio of the maximum possible error in the measurement to the measurement itself

Example for 1-12

7500

Move the decimal between the 7 & 5

the number was not a decimal so the exponent will be positive and we moved it three places so it will be positive 3 7.5×10^3

Example for 13-20

$$5 \times 10^3$$

The exponent is a positive 3 so you move the decimal 3 units right so that the number is 5000.

Example for 21-24

$(2 \times 10^3)^2$ ______ 5×10^5

The left side equals 4×10^6

The exponents are different and the one on the left side is larger; therefore, $(2 \times 10^3)^2 > 5 \times 10^5$

Example for 25-31

$$\frac{73.1\times(0.493)^2}{0.620\times(32.6)^2}=\frac{(7.31\times10^1)(4.93\times10^{-1})^2}{(6.20\times10^{-1})(3.26\times10^1)^2}$$

$$\frac{(7.31\times10^1)(4.93\times10^{-1})^2}{(6.20\times10^{-1})(3.26\times10^1)^2}=\frac{(7\times10^1)(5\times10^{-1})^2}{(6\times10^{-1})(3\times10^1)^2}$$

$$\frac{(7\times10^1)(5\times10^{-1})^2}{(6\times10^{-1})(3\times10^1)^2}=\frac{(7\times10^1)(25\times10^{-2})}{(6\times10^{-1})(9\times10^2)}$$

$$\frac{(7\times10^1)(25\times10^{-2})}{(6\times10^{-1})(9\times10^2)}=\frac{(7)(25)(10^{-1})}{(6)(9)(10^1)}=\frac{(7)(25)}{(6)(9)}\times10^{-2}$$

$$\frac{(7)(25)}{(6)(9)}\times10^{-2}=3.2\times10^{-2}=3\times10^{-2}=\ .03$$

Section 5-4

Rational and Algebraic Expressions

Rational Algebraic Expressions

- rational algebraic expression is a polynomial fraction
- simplifying rational expressions: remember you are using the same process to do these problems as you use to reduce numerical fractions
  - factor the top and the bottom into prime factors
  - cancel any factors which appear in both the numerator and the denominator.
  - negative exponents indicate that the value should be on the other side of the fraction bar.

    If it is in the numerator, then put it in the denominator.

    If it is in the denominator then put it in the numerator.

Example for 1-20, 29-38

$$\frac{x^2 - 5x + 6}{x^2 - 7x + 12}$$

$$\frac{x^2 - 5x + 6}{x^2 - 7x + 12} = \frac{(x-2)(x-3)}{(x-4)(x-3)} = \frac{x-2}{x-4}$$

Example for 21-28

$$\frac{t^2 - 9}{t^2 - 9t} = \frac{(t+3)(t-3)}{t(t-9)}$$

domain {t: t ≠ 0 or t ≠ 9}

solution {3, - 3}

Section 5-5

Products and Quotients of Rational Expressions

Products & Quotients of Rational Expressions

- Remember that division of a fraction is the same as multiplying by the reciprocal.

$$\frac{3}{5} \div \frac{6}{10} = \frac{3}{5} \bullet \frac{10}{6} = 1 \text{ or } \frac{\frac{3}{5}}{\frac{6}{10}} = \frac{3}{5} \bullet \frac{10}{6} = 1$$

Example for 1-24

$$\frac{3x^2+xy-2y^2}{3x^2-xy-2y^2} \div \frac{3x^2+7xy-6y^2}{3x^2-2xy-y^2} \div \frac{3x+y}{3x+2y}$$

Turn division into multiplication by reciprocals.

$$\frac{3x^2+xy-2y^2}{3x^2-xy-2y^2} \bullet \frac{3x^2-2xy-y^2}{3x^2+7xy-6y^2} \bullet \frac{3x+2y}{3x+y}$$

Factor

$$\frac{(3x-2y)(x+y)}{(3x+2y)(x-y)} \bullet \frac{(3x+y)(x-y)}{(3x-2y)(x+3y)} \bullet \frac{3x+2y}{3x+y}$$

Cancel

$$\frac{\cancel{(3x-2y)}(x+y)}{\cancel{(3x+2y)}\cancel{(x-y)}} \bullet \frac{\cancel{(3x+y)}\cancel{(x-y)}}{\cancel{(3x-2y)}(x+3y)} \bullet \frac{\cancel{3x+2y}}{\cancel{3x+y}}$$

Answer: $\frac{x+y}{x+3y}$

Section 5-6

Sums and Differences of Rational Expressions

Sums & Differences of Rational Expressions

- Follow the same procedure for adding and subtracting polynomial fractions as you would for adding and subtracting numerical fractions.
- Rules for adding and Subtracting Fractions
  - Find the least common denominator (LCM of denominators) of all the fractions.
  - Multiply the numerator and the denominator by the factor of the LCM not present in the denominator already.
  - Add or subtract the fractions by combining similar terms in the numerators and placing this over the LCD.
  - Reduce the sum or difference by factoring and canceling, if possible.

Example for 1-38

$$\frac{1}{2pq^4} + \frac{2}{p^3q^2}$$

The LCM of $2pq^4$ & p^3q^3 is $2p^3q^4$. The first term must be multiplied by p^2 on top and bottom and the second term must be multiplied by $2q^2$ on the top and bottom.

$$\left(\frac{p^2}{p^2}\right)\left(\frac{1}{2pq^4}\right) + \left(\frac{2q^2}{2q^2}\right)\left(\frac{2}{p^3q^2}\right)$$

Multiplying creates the new sum.

$$\left(\frac{p^2}{2p^3q^4}\right) + \left(\frac{4q^2}{2p^3q^4}\right)$$

Adding creates the fraction:

$$\frac{p^2 + 4q^2}{2p^3q^4}$$

Example for 1-38

$$\frac{1}{s^2 + 2s + 1} - \frac{1}{s^2 - 1}$$

After factoring the denominators, find the LCM of the denominators which in this case is (s + 1)(s + 1)(s - 1).

$$\frac{1}{(s+1)(s+1)} - \frac{1}{(s+1)(s-1)}$$

Multiply the numerator & denominator of the first term by (s - 1) and the second term by (s + 1).

$$\left[\frac{(s-1)}{(s-1)}\right]\left[\frac{1}{(s+1)(s+1)}\right] - \left[\frac{(s+1)}{(s+1)}\right]\left[\frac{1}{(s+1)(s-1)}\right]$$

Problem Continued

The new difference is: $$\left[\frac{s-1}{(s+1)(s+1)(s-1)}\right]-\left[\frac{s+1}{(s+1)(s+1)(s-1)}\right]$$

Combining these terms you get: $$\frac{s-1-(s+1)}{(s+1)(s+1)(s-1)}$$

Distributing the negative: $$\frac{s-1-s-1}{(s+1)(s+1)(s-1)}$$

Simplifying the numerator you get: $$\frac{-2}{(s+1)(s+1)(s-1)}$$

Section 5-7

Complex Fractions

Complex Fractions

- complex fractions have fractions within fractions.
- to simplify complex fractions:
  - Method 1

    simplify the numerator

    simplify the denominator

    multiply the numerator by the reciprocal of the denominator
  - Method 2

    multiply both the numerator and the denominator by the LCD of all the fractions in the problem

Example for 1-26, Method 1.

$$\frac{\frac{1}{a+1}+\frac{1}{a-1}}{\frac{1}{a+1}-\frac{1}{a-1}}$$

Multiplying the numerator by the LCD and simplifying you get:

$$\left(\frac{a-1}{a-1}\right)\frac{1}{a+1}+\left(\frac{a+1}{a+1}\right)\frac{1}{a-1}=\frac{a-1+a+1}{(a+1)(a-1)}=\frac{2a}{(a+1)(a-1)}$$

Multiplying the denominator by the LCD and simplifying you get:

$$\left(\frac{a-1}{a-1}\right)\frac{1}{a+1}-\left(\frac{a+1}{a+1}\right)\frac{1}{a-1}=\frac{a-1-(a+1)}{(a+1)(a-1)}=\frac{-2}{(a+1)(a-1)}$$

Problem: Method #1 Continued

The fraction with the numerator and denominator simplified is:

$$\frac{\dfrac{2a}{(a+1)(a-1)}}{\dfrac{-2}{(a+1)(a-1)}}$$

Multiplying by the reciprocal of the denominator you get:

$$\frac{2a}{(a+1)(a-1)} \bullet \frac{(a+1)(a-1)}{-2} = \frac{2a(a+1)(a-1)}{-2(a+1)(a-1)}$$

After canceling like terms you are left with: - a

Example 1-26, Method 2

$$\frac{1+\frac{1}{x-1}}{1+\frac{1}{x^2-1}}$$

Multiplying the numerator and denominator by the LCD, x^2 - 1, of all of the fractions you get:

$$\frac{1+\frac{1}{x-1}}{1+\frac{1}{x^2-1}} = \frac{(x+1)(x-1)\left(1+\frac{1}{x-1}\right)}{(x+1)(x-1)\left(1+\frac{1}{(x+1)(x-1)}\right)} = \frac{(x+1)(x-1)+(x+1)}{(x+1)(x-1)+1}$$

Simplifying the numerator and denominator you get:

$$\frac{(x+1)(x-1)+(x+1)}{(x+1)(x-1)+1} = \frac{x^2-1+x+1}{x^2-1+1} = \frac{x^2+x}{x^2} = \frac{x(x+1)}{x^2}$$

The answer after reducing this fraction is: $\frac{x+1}{x}$

Section 5-8

Fractional Coefficients

Solving rational open sentences.

I. Find the LCD of all of the fractions.

II. Multiply every term in the problem by the LCD.

III. Reduce each term in the problem and if you’ve used the correct LCD then no fractions should remain.

IV. Solve the remaining problem according to the appropriate methods you’ve learned throughout the year.

Example for 1-24

$$\frac{x(x+1)}{5} - \frac{x+1}{6} = \frac{1}{3}$$

The LCD is 30. Multiplying every term by 30 you get:

$$(30)\left(\frac{x(x+1)}{5}\right) - (30)\left(\frac{x+1}{6}\right) = (30)\left(\frac{1}{3}\right)$$

Which simplifies to:

$$6x(x+1) - 5(x+1) = 10$$

Distributing you get: $6x^2 + 6x - 5x - 5 = 10$

Collecting like terms you get: $6x^2 + x - 15 = 0$

Factoring you get: $(3x+5)(2x-3) = 0$

Solution is: $x = \left\{-\frac{5}{3}, \frac{3}{2}\right\}$

Section 5-9

Fractional Equations

Solving rational open sentences.

I. Identify the LCD.

II. Identify the domain so that you know which values to exclude from the solution set.

III. Multiply every term in the open sentence by the LCD and reduce so that no fractions remain.

IV. Solve the resulting problem in the appropriate fashion.

V. Check all of the answers in the solution set to identify any extraneous roots (values excluded in the domain).

Example for 1-28

$$\frac{5}{u^2+u-6} = 2 - \frac{u-3}{u-2}$$

u = - 4 or 2 but 2 is extraneous
so - 4 is the answer

$$\frac{5}{(u+3)(u-2)} = 2 - \frac{u-3}{u-2}$$

LCD = (u - 2)(u + 3)
Domain = {u: u $\neq$ 2 or - 3}

$$5 = 2(u+3)(u-2) - (u+3)(u-3)$$

$5 = 2(u^2 + u - 6) - (u^2 - 9)$
$5 = 2u^2 + 2u - 12 - u^2 + 9$
$5 = u^2 + 2u - 3$
$0 = u^2 + 2u - 8$
$0 = (u + 4)(u - 2)$