Section 1-1

Sets and Symbols

Objectives for Section 1-1

- to specify the union and the intersection of sets by roster
- to list the subsets of a given set
- to count the number of subsets of a given set

Sets and Symbols

- { }: read as "the set whose members are" means a collection or set
- $A \subset B$: read as "A is a subset of B" means that every member of A is a member of B
- ϕ: read as "the null set or the empty set" means the set that contains no elements. The empty set is a subset of every set.
- $A = B$: read as "A is equal to B" means that A and B contain exactly the same elements
- $A \cap B$: read as "A intersect B" means the set of elements belonging to both A and B
- $A \cup B$: read as "A union B" means the set of elements in at least one of the given sets

Section 1-2

Open Sentences and Graphs

Objectives for Section 1-2

- to solve an open sentence over a given domain
- to graph a set of numbers on a number line
- to solve an open sentence over the set of positive integers and graph the solution set

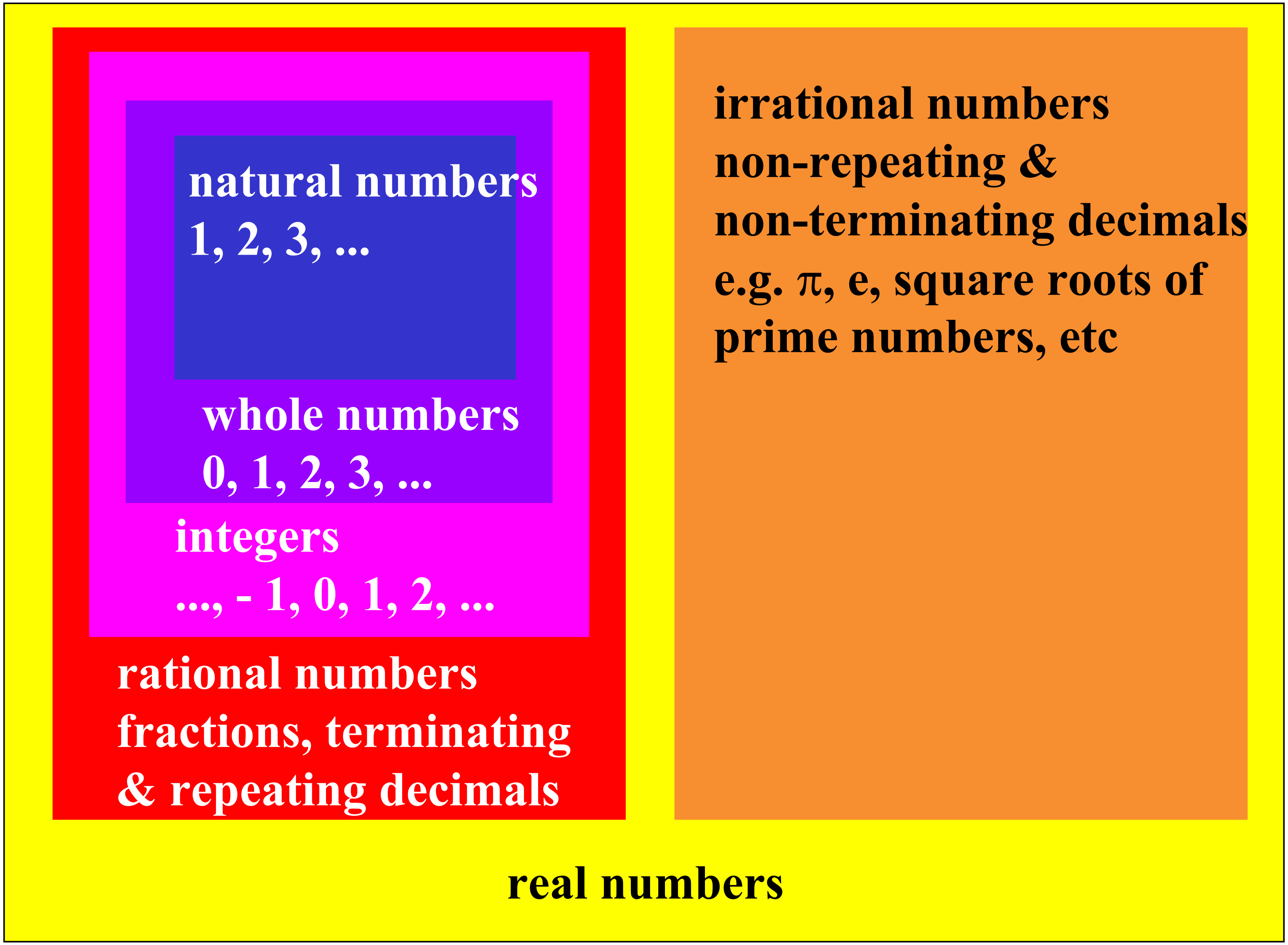
Open Sentences and Graphs

- expression: is a number, variable, or a sum, difference, product or quotient that contains one or more variables
- variable: a symbol that represents the members of a specified set
- domain: the values of a variable
- constant: a number or variable with only one value
- mathematical sentence: expresses a relationship between two mathematical expressions
- open sentence: a mathematical sentence with one or more variables
- solution (roots): the values of the variable(s) that make an open sentence true
- solution set: the set of all solutions that make an open sentence true

Open Sentences and Graphs

- graph: a visual representation of a mathematical relationship
- number line: a visual representation of the set of real numbers

The Subsets of the Set of Real Numbers



Section 1-3

Axioms for the Real Numbers

Objectives for Section 1-3

- to complete an open sentence so that it is true for all values of the set of real numbers
- to identify the property that justifies a statement
- to use the commutative and associative properties to simplify expressions
- to identify the properties of a new set and its defined operations

Axioms for Real Numbers

- Closure for Addition: when two real numbers are added the sum is a real number.
- Associative for Addition: the way terms are grouped in addition does not effect the value of the sum. $(a + b) + c = a + (b + c)$.
- Commutative for Addition: the order of the terms in addition does not effect the value of the sum. $a + b = b + a$.
- Identity for Addition: for every real number a there exists a unique real number 0 such that $a + 0 = a$.
- Additive Inverse:for every real number a there exists a real number - a such that $a + (- a) = 0$

Axioms for Real Numbers

- Closure for Multiplication: when two real numbers are multiplied the product is a real number.
- Associative for Multiplication: the way the factors are grouped in multiplication does not effect the product. (ab)c = a(bc)
- Commutative for Multiplication: the order of the factors in multiplication does not effect the value of the product. ab = ba.
- Identity for Multiplication: for every real number a there exists a unique real number 1 such that (a)(1) = a
- Multiplicative Inverse: for every real number a there exists a real number $\frac{1}{a}$ such that $\left(\frac{a}{1}\right)\left(\frac{1}{a}\right) = 1$

Axioms for Real Numbers

- Substitution: the process of replacing an expression with a known logical equivalent for that expression.
- Distribution: real numbers can be multiplied into the terms of a sum or real numbers can be divided out of the terms of a sum. $a(b + c) = ab + ac$.
- Reflexive: a number equals itself. $a = a$
- Symmetric: you may exchange the expressions on either side of the equals sign. If $a = b$, then $b = a$.
- Transitive: value can be transferred from a first term through a second an into a third. If $a = b$ and $b = c$, then $a = c$.

Section 1-4

Theorems and Proof: Addition

Objectives for Section 1-4

- to complete and expression so that it makes a true statement
- to simplify variable expressions using the properties of real numbers
- to solve simple equations over the set of real numbers
- to state the property or axiom that justifies a statement
- to prove theorems concerning real numbers

Theorems and Proof: Addition

- For all real numbers b and c, $(b + c) + (- c) = b$
- For all real numbers a, b and c, if $a + c = b + c$, then $a = b$
- For all real numbers a, b and c, if $c + a = c + b$, then $a = b$
- For all real numbers a, b and c, if $a + c = b + c$ or $c + a = c + b$, then $a = b$
- For all real numbers a and b, $- (a + b) = (- a) + (- b)$
- For all real numbers a, $- (- a) = a$

Section 1-5

Properties of Products

Objectives for Section 1-5

- to simplify variable expressions using the properties of real numbers
- to determine the sign of an expression without simplifying
- to state the property or axiom that justifies a statement
- to prove theorems concerning real numbers

Properties of Products

- For all real numbers b and all nonzero real numbers c,
  $(bc)\frac{1}{c} = b$
- For all real numbers a and b and all nonzero real numbers c, if ac = bc or ca = cb, then a = b.
- For all nonzero real numbers a and b, $\frac{1}{ab} = \frac{1}{a} \bullet \frac{1}{b}$
- For all real numbers a, $a \bullet 0 = 0$ and $0 \bullet a = 0$.
- For all real numbers a, a(- 1) = - a and (- 1)a = - a.
- For all real numbers a and b, (- a)b = - ab, a(- b) = - ab, (- a)(- b) = ab.
- The product of an even number of nonzero negative numbers is positive; the product of an odd number of nonzero negative numbers is negative.

Section 1-6

Properties of Differences

Objectives for Section 1-6

- to simplify variable expressions using the properties of real numbers
- to evaluate expressions for given values of a variable
- to state the property or axiom that justifies a statement
- to prove theorems concerning real numbers

Properties of Differences

- For all real numbers a and b, $a - b = a + (-b)$

Section 1-7

Properties of Quotients

Objectives for Section 1-7

- to simplify variable expressions using the properties of real numbers
- to evaluate an equation for a given value of a variable and determine if the statement is true or false
- to state the property or axiom that justifies a statement
- to prove theorems concerning the real numbers

Properties of Quotients

- For all real numbers a and all nonzero real numbers b,

$$\frac{a}{b} = a \bullet \frac{1}{b}$$

Simplifying Expressions

- Order of Operations:
  - Simplify the expressions within each grouping symbol working outward from the innermost grouping. If the grouping symbol has a mathematical operation associated with it such as absolute value or roots, this operation cannot be performed until the expression within the symbol has been simplified to one term.
  - Perform any types of multiplication which include powers, multiplication of factors or division in order from left to right.
  - Perform any types of addition including addition of negatives and subtraction in order from left to right.