Chapter 32A – AC Circuits

A PowerPoint Presentation by

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Objectives: After completing this module, you should be able to:

- Describe the sinusoidal variation in ac current and voltage, and calculate their effective values.
- Write and apply equations for calculating the inductive and capacitive reactances for inductors and capacitors in an ac circuit.
- Describe, with diagrams and equations, the phase relationships for circuits containing resistance, capacitance, and inductance.

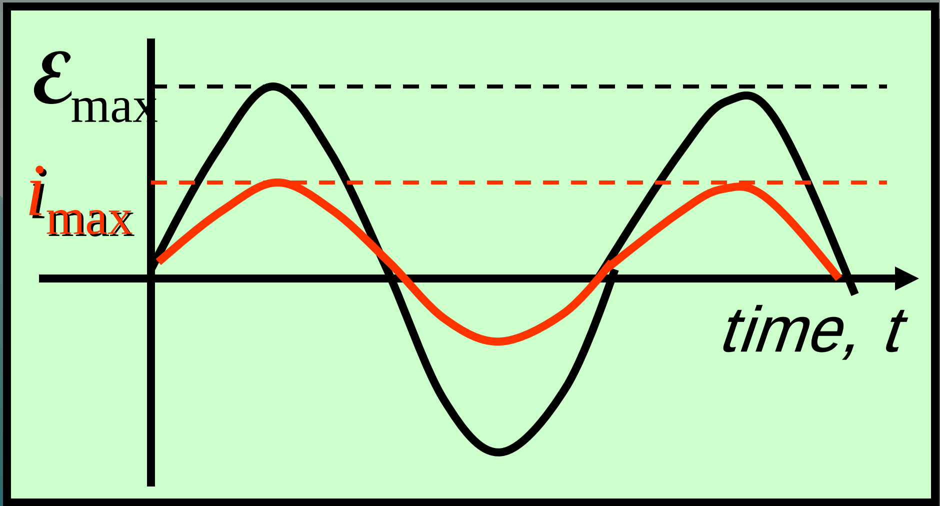
Objectives (Cont.)

- Write and apply equations for calculating the impedance, the phase angle, the effective current, the average power, and the resonant frequency for a series ac circuit.
- Describe the basic operation of a step-up and a step-down transformer.
- Write and apply the transformer equation and determine the efficiency of a transformer.

Alternating Currents

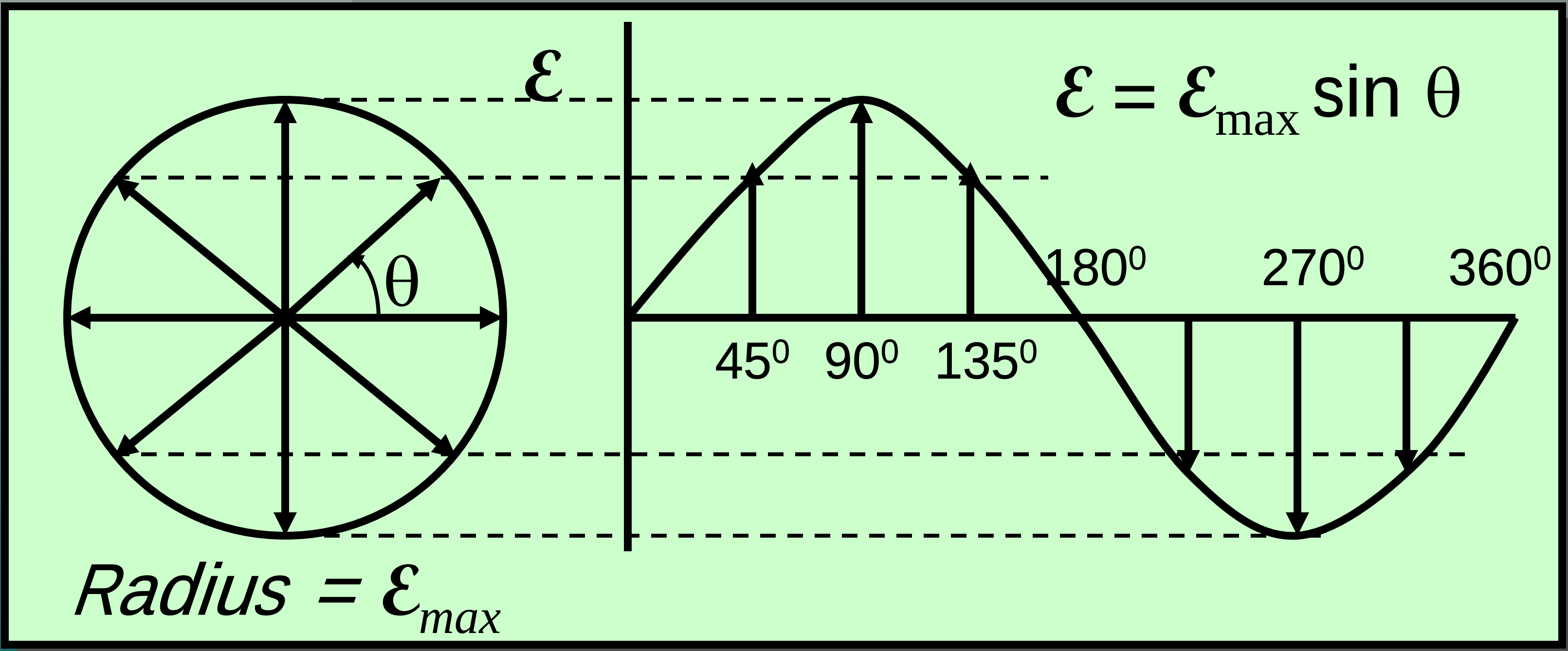
An alternating current such as that produced by a generator has no direction in the sense that direct current has. The magnitudes vary sinusoidally with time as given by:

AC-voltage and current
$\mathcal{E} = \mathcal{E}_{max} \sin \theta$ $i = i_{max} \sin \theta$



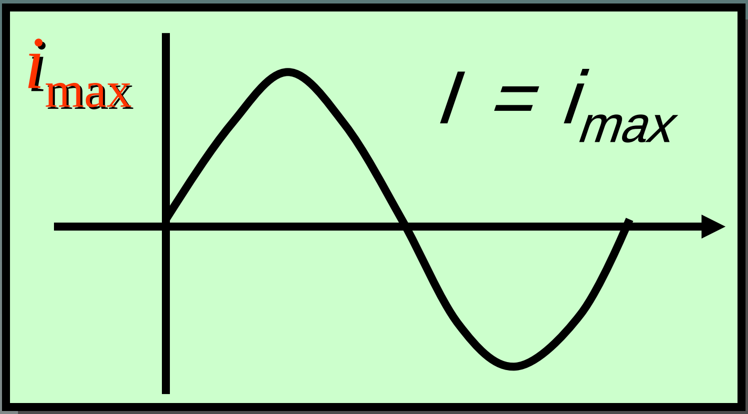
Rotating Vector Description

The coordinate of the emf at any instant is the value of $\mathcal{E}_{max} \sin \theta$. Observe for incremental angles in steps of 45^0. Same is true for *i*.



Effective AC Current

The average current in a cycle is zero—half + and half -.



But energy is expended, regardless of direction. So the "root-mean-square" value is useful.

$$I_{rms} = \sqrt{\frac{I^2}{2}} = \frac{I}{0.707}$$

The rms value I_{rms} is sometimes called the effective current I_{eff}:

The effective ac current:

$$i_{eff} = 0.707\ i_{max}$$

AC Definitions

One effective ampere is that ac current for which the power is the same as for one ampere of dc current.

Effective current: $i_{eff} = 0.707\ i_{max}$

One effective volt is that ac voltage that gives an effective ampere through a resistance of one ohm.

Effective voltage: $V_{eff} = 0.707\ V_{max}$

Example 1: For a particular device, the house ac voltage is 120-V and the ac current is 10 A. What are their maximum values?

$$i_{eff} = 0.707\ i_{max}$$

$$V_{eff} = 0.707\ V_{max}$$

$$i_{max} = \frac{i_{eff}}{0.707} = \frac{10\ \text{A}}{0.707}$$

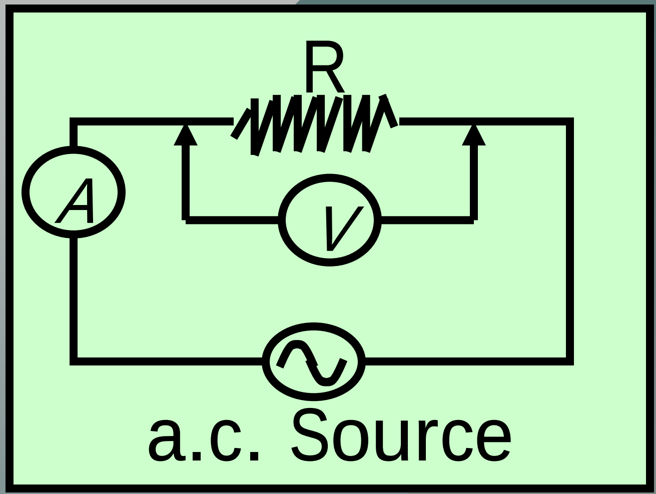
$$V_{max} = \frac{V_{eff}}{0.707} = \frac{120\text{V}}{0.707}$$

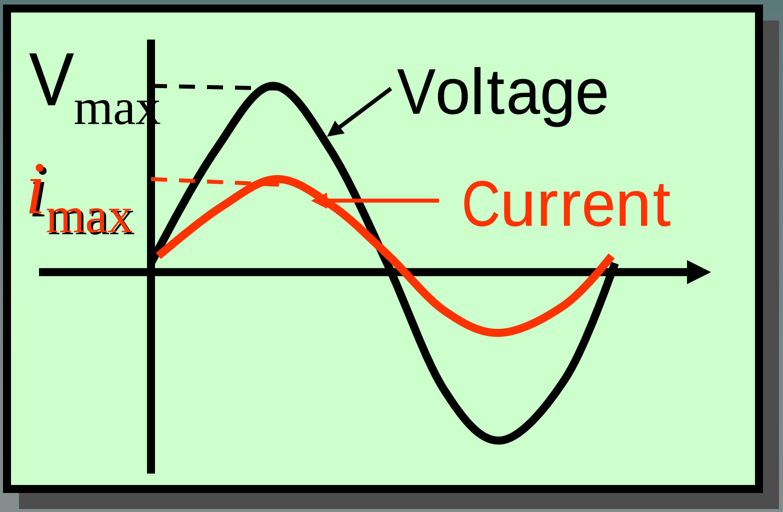
$$i_{max} = 14.14\ \text{A}$$

$$V_{max} = 170\ \text{V}$$

The ac voltage actually varies from +170 V to -170 V and the current from 14.1 A to –14.1 A.

Pure Resistance in AC Circuits

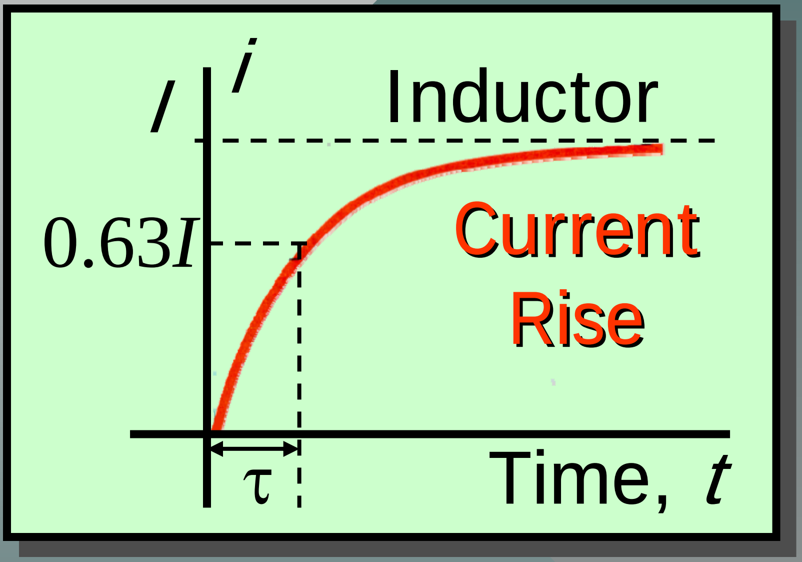


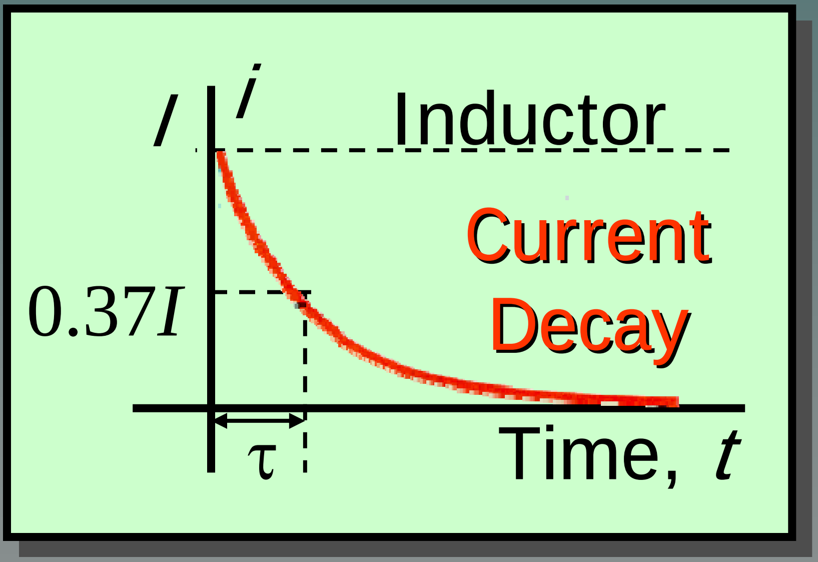


Voltage and current are in phase, and Ohm's law applies for effective currents and voltages.

Ohm's law: $V_{eff} = i_{eff}R$

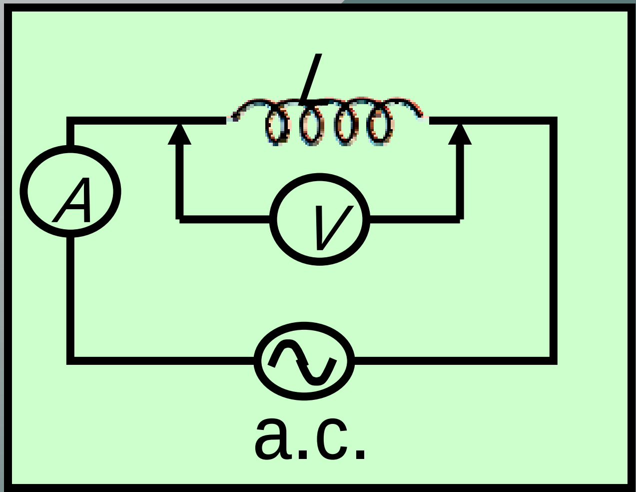
AC and Inductors

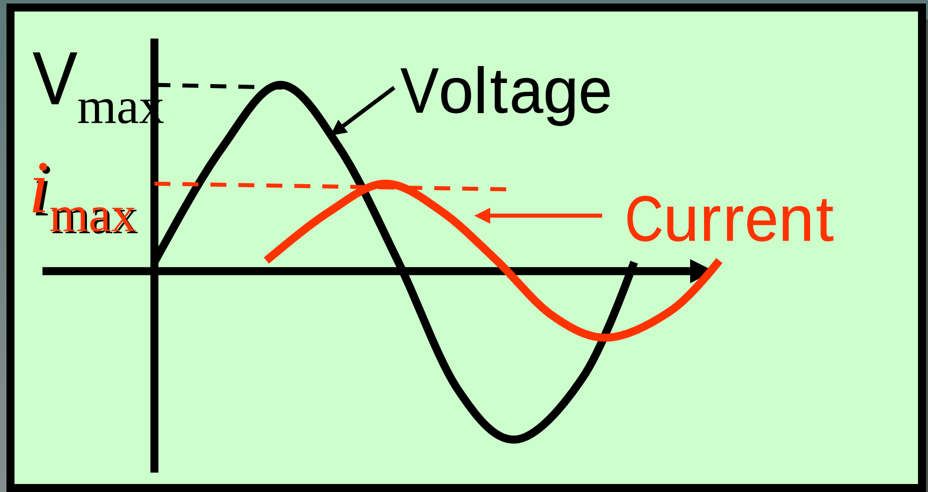




The voltage *V* peaks first, causing rapid rise in *i* current which then peaks as the emf goes to zero. Voltage leads (peaks before) the current by 90^0. Voltage and current are out of phase.

A Pure Inductor in AC Circuit



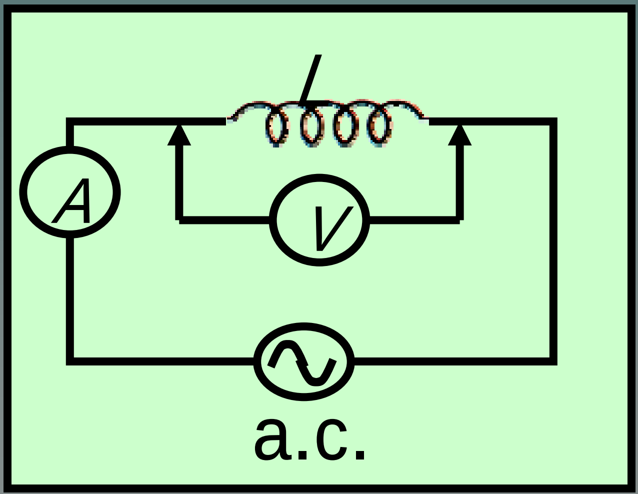


The voltage peaks 90^0 before the current peaks. One builds as the other falls and vice versa.

The reactance may be defined as the nonresistive opposition to the flow of ac current.

Inductive Reactance

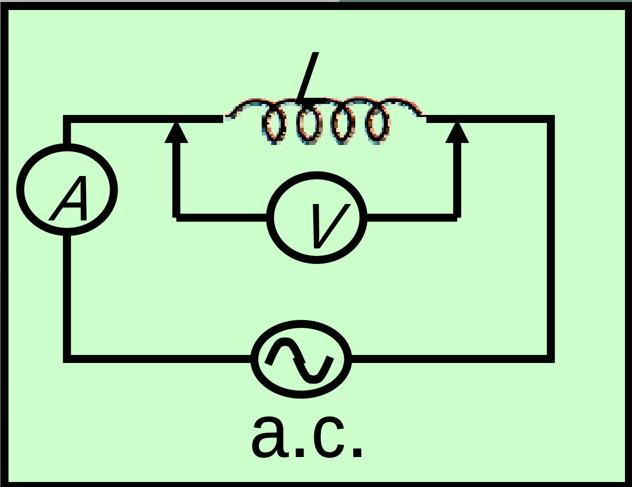
The back emf induced by a changing current provides opposition to current, called inductive reactance X_L.



Such losses are temporary, however, since the current changes direction, periodically re-supplying energy so that no net power is lost in one cycle.

Inductive reactance X_L is a function of both the inductance and the frequency of the ac current.

Calculating Inductive Reactance



Inductive Reactance:

$X_L = 2\pi f L$ Unit is the Ω

Ohm's law: $V_L = iX_L$

The voltage reading *V* in the above circuit at the instant the ac current is *i* can be found from the inductance in H and the frequency in Hz.

$$V_L = i(2\pi fL)$$

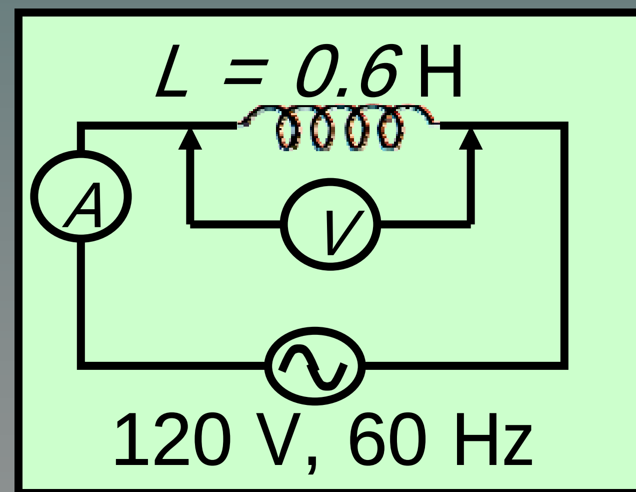
Ohm's law: $V_L = i_{eff} X_L$

Example 2: A coil having an inductance of 0.6 H is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?

Reactance: $X_L = 2\pi fL$

$X_L = 2\pi(60\text{ Hz})(0.6\text{ H})$

$X_L = 226\ \Omega$

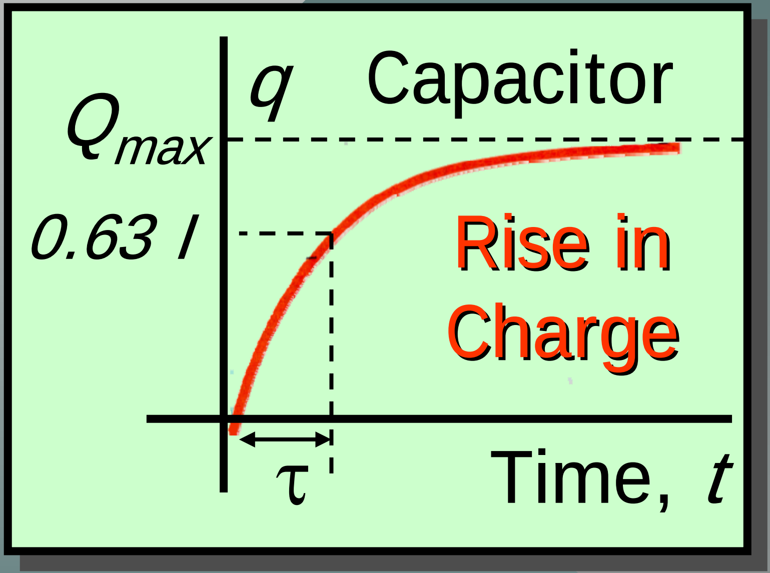


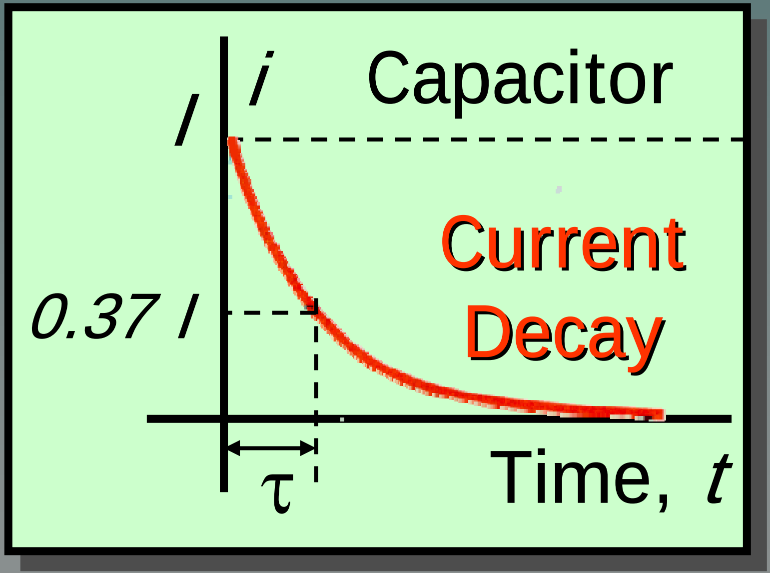
$$i_{eff} = \frac{V_{eff}}{X_L} = \frac{120\text{V}}{226\ \Omega}$$

$i_{eff} = 0.531$ A

Show that the peak current is $I_{max} = 0.750$ A

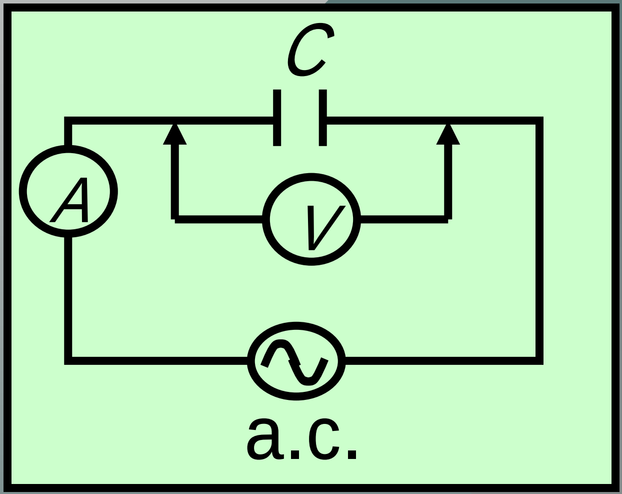
AC and Capacitance

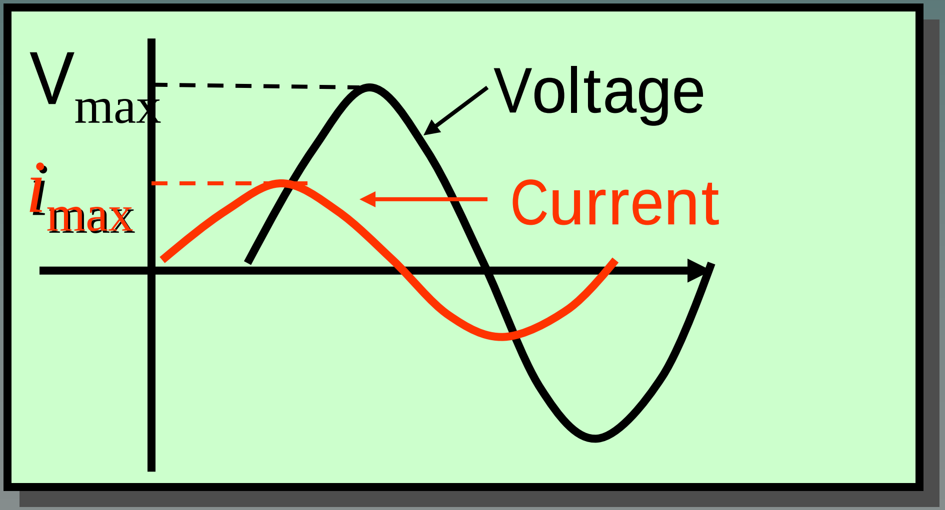




The voltage V peaks ¼ of a cycle after the current *i* reaches its maximum. The voltage lags the current. Current *i* and V out of phase.

A Pure Capacitor in AC Circuit



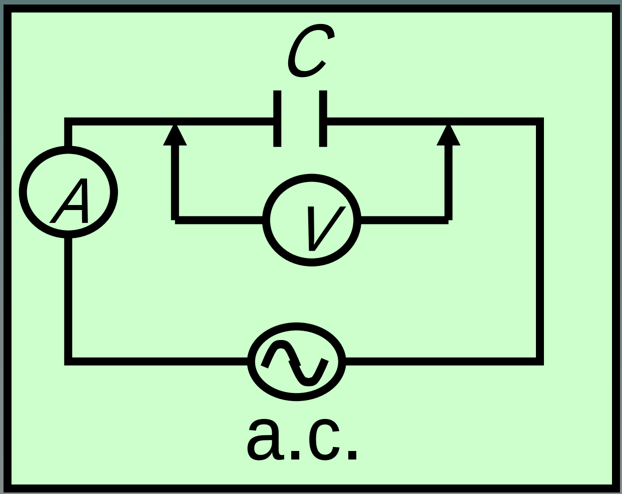


The voltage peaks 90^0 after the current peaks. One builds as the other falls and vice versa.

The diminishing current *i* builds charge on C which increases the back emf of V_C.

Capacitive Reactance

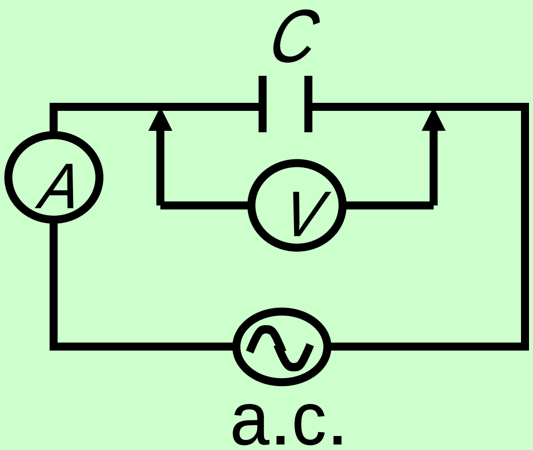
Energy gains and losses are also temporary for capacitors due to the constantly changing ac current.



No net power is lost in a complete cycle, even though the capacitor does provide nonresistive opposition (reactance) to the flow of ac current.

Capacitive reactance X_C is affected by both the capacitance and the frequency of the ac current.

Calculating Inductive Reactance



<u>Capacitive Reactance:</u>

$$X_C = \frac{1}{2\pi fC} \quad \text{Unit is the } \Omega$$

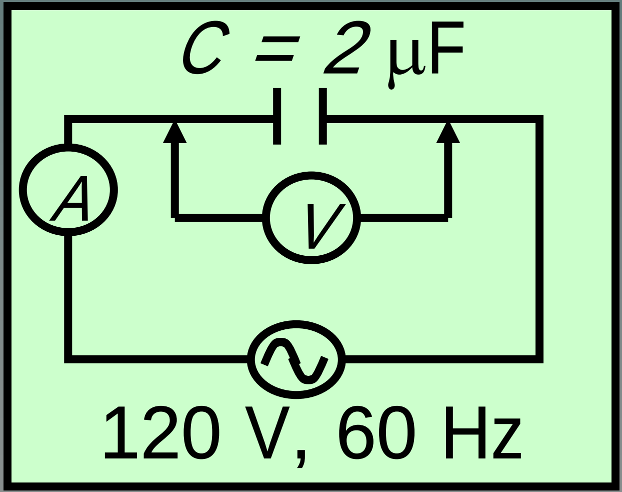
Ohm's law: $V_C = iX_C$

The voltage reading *V* in the above circuit at the instant the ac current is *i* can be found from the inductance in F and the frequency in Hz.

$$V_L = \frac{i}{2\pi fL}$$

Ohm's law: $V_C = i_{eff}X_C$

Example 3: A 2-μF capacitor is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?



Reactance: $X_C = \frac{1}{2\pi fC}$

$$X_C = \frac{1}{2\pi(60\,\text{Hz})(2 \text{ x } 10^{-6}\text{F})}$$

X_C = 1330 Ω

$$i_{eff} = \frac{V_{eff}}{X_C} = \frac{120\text{V}}{1330\ \Omega}$$

i_{eff} = 90.5 mA

Show that the peak current is i_{max} = 128 mA

Memory Aid for AC Elements

An old, but very effective, way to remember the phase differences for inductors and capacitors is :

"ℰ L I" the "*i* C ℰ" Man

Emf ℰ is before current *i* in inductors L;
Emf ℰ is after current *i* in capacitors C.

Frequency and AC Circuits

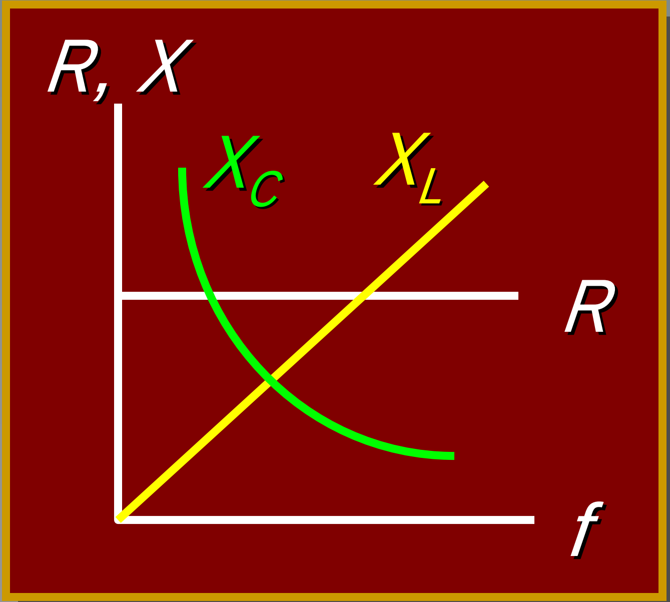
Resistance *R* is constant and not affected by *f*.

Inductive reactance X_L varies directly with frequency as expected since $\mathcal{E} \propto \Delta i/\Delta t$.

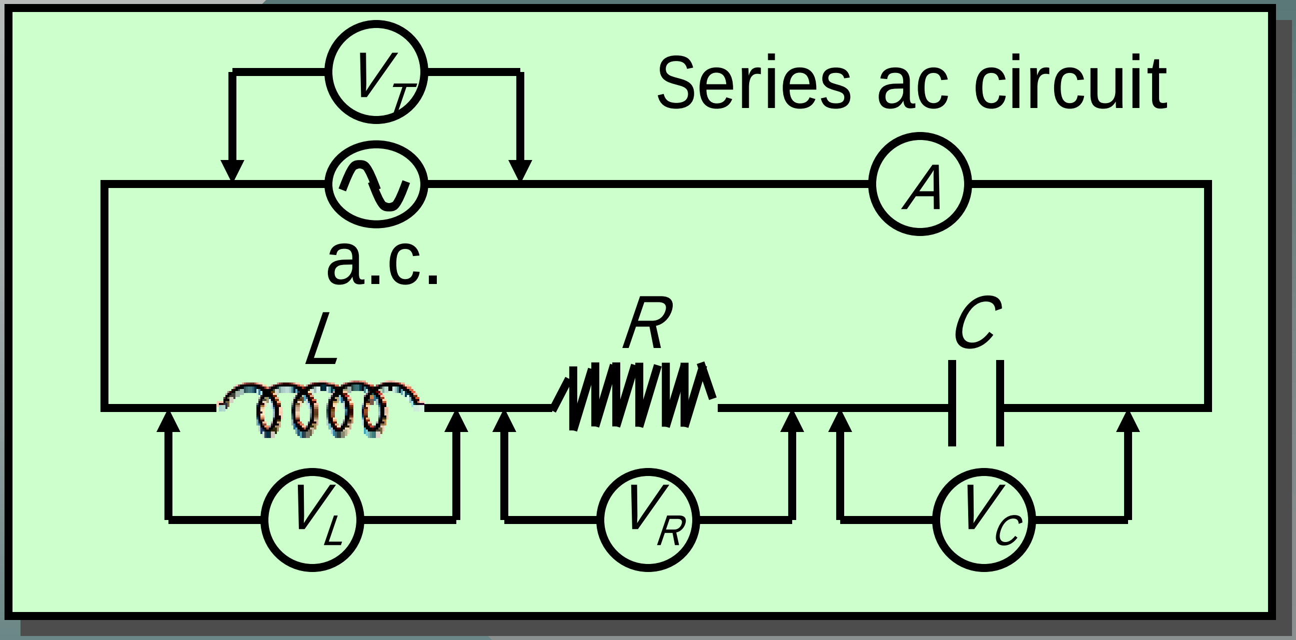
$$X_L = 2\pi fL$$

$$X_C = \frac{1}{2\pi fC}$$

Capacitive reactance X_C varies inversely with *f* since rapid ac allows little time for charge to build up on capacitors.



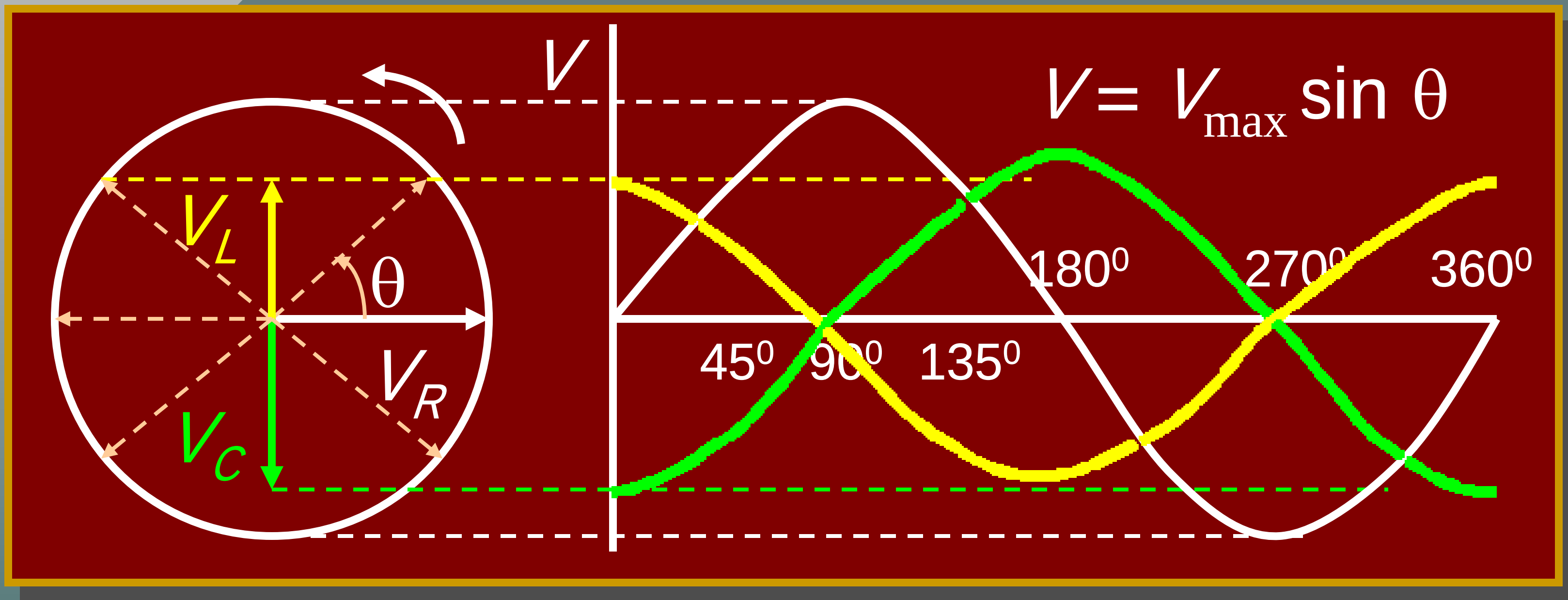
Series LRC Circuits



Consider an inductor *L*, a capacitor *C*, and a resistor *R* all connected in series with an ac source. The instantaneous current and voltages can be measured with meters.

Phase in a Series AC Circuit

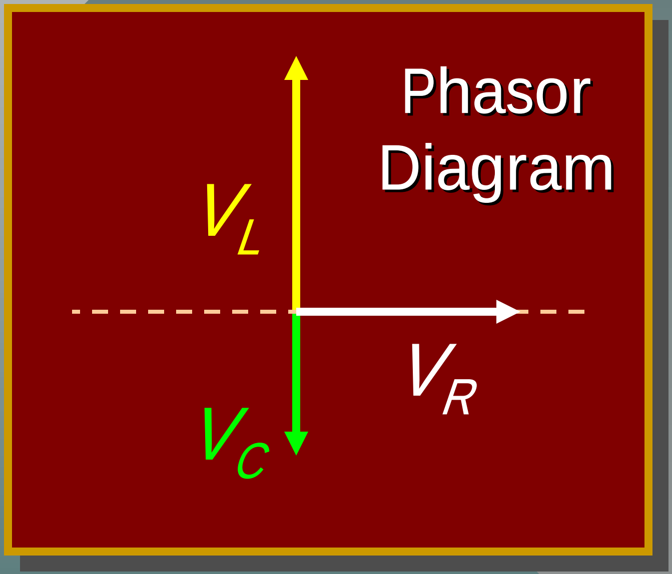
The voltage leads current in an inductor and lags current in a capacitor. In phase for resistance *R*.

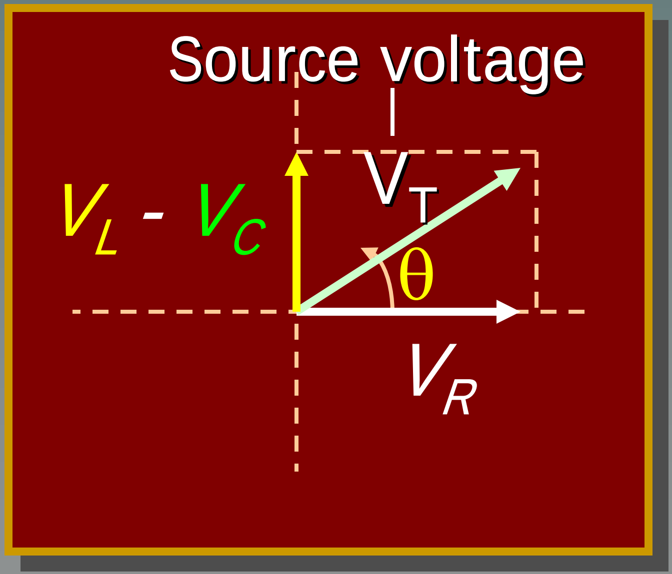


Rotating phasor diagram generates voltage waves for each element *R*, *L*, and *C* showing phase relations. Current *i* is always in phase with V_R.

Phasors and Voltage

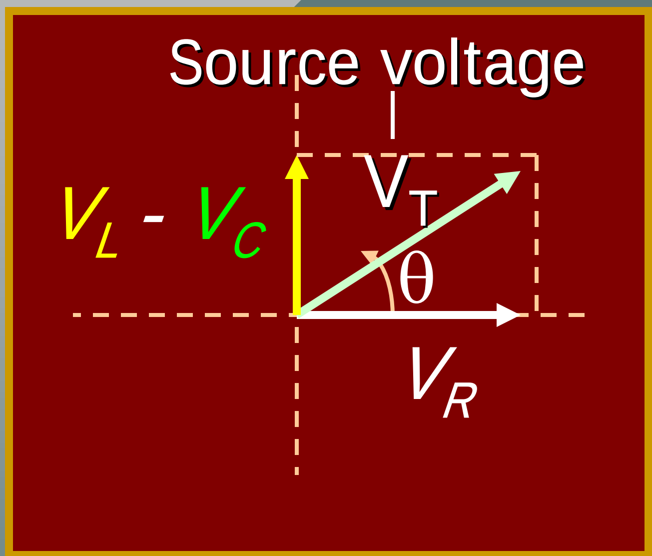
At time t = 0, suppose we read V_L, V_R and V_C for an ac series circuit. What is the source voltage V_T?





We handle phase differences by finding the vector sum of these readings. $V_T = \Sigma \mathbf{V_i}$. The angle θ is the phase angle for the ac circuit.

Calculating Total Source Voltage



Treating as vectors, we find:

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

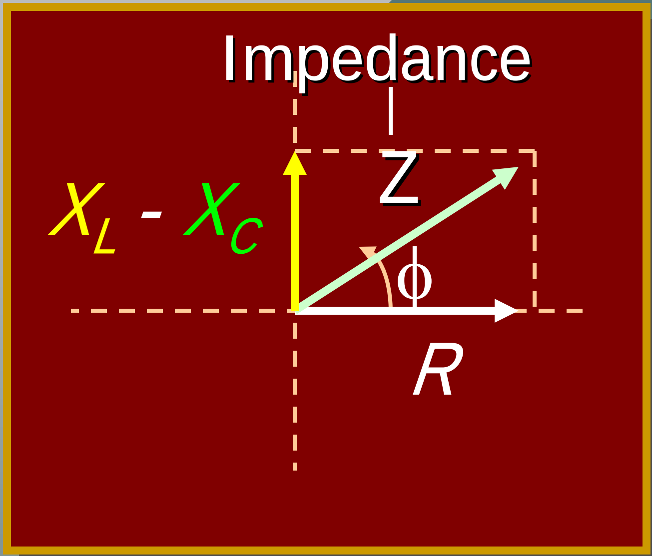
$$\tan\phi = \frac{V_L - V_C}{V_R}$$

Now recall that: $V_R = iR$; $V_L = iX_L$; and $V_C = iV_C$

Substitution into the above voltage equation gives:

$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance in an AC Circuit



$$V_T = i\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance Z is defined:

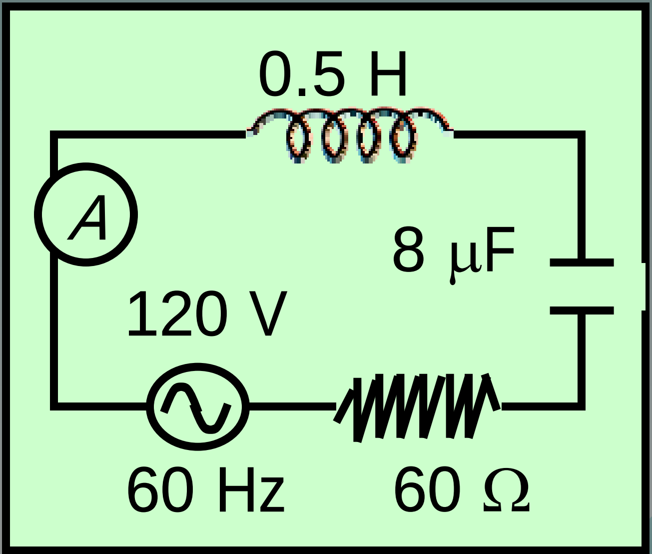
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Ohm's law for ac current and impedance:

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

The impedance is the combined opposition to ac current consisting of both resistance and reactance.

Example 3: A 60-Ω resistor, a 0.5 H inductor, and an 8-μF capacitor are connected in series with a 120-V, 60 Hz ac source. Calculate the impedance for this circuit.



$$X_L = 2\pi fL \quad \text{and} \quad X_C = \frac{1}{2\pi fC}$$

$$X_L = 2\pi(60\text{Hz})(0.6\text{ H}) = 226\,\Omega$$

$$X_C = \frac{1}{2\pi(60\text{Hz})(8 \text{ x } 10^{-6}\text{F})} = 332\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(60\,\Omega)^2 + (226\,\Omega - 332\,\Omega)^2}$$

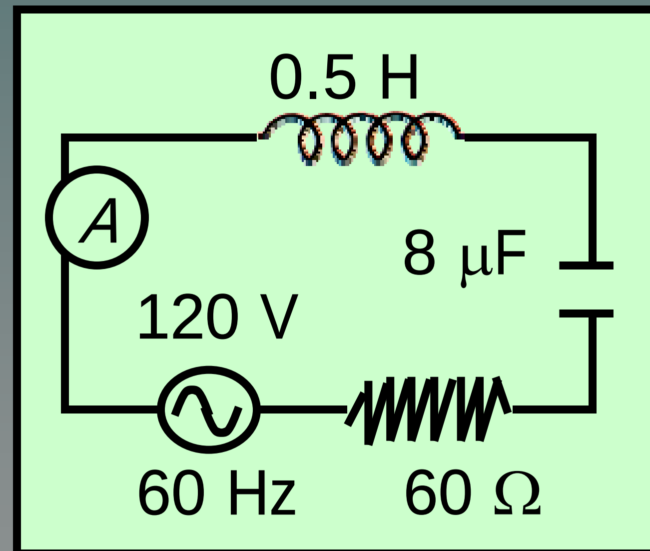
Thus, the impedance is: $Z = 122\ \Omega$

Example 4: Find the effective current and the phase angle for the previous example.

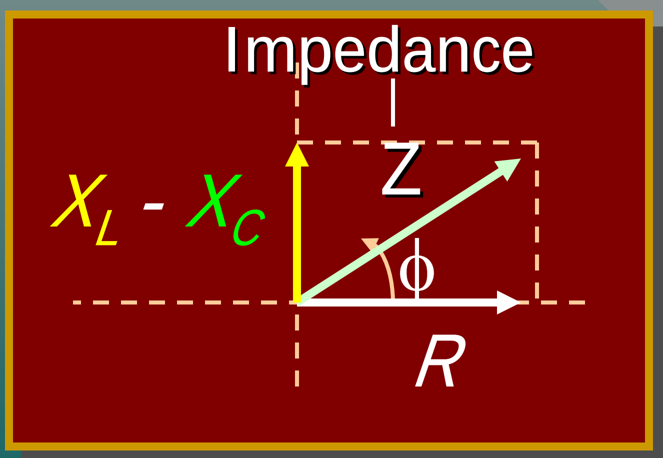
X_L = 226 Ω; X_C = 332 Ω; *R* = 60 Ω; *Z* = 122 Ω

$$i_{eff} = \frac{V_T}{Z} = \frac{120\text{ V}}{122\ \Omega}$$

i_{eff} = 0.985 A



Next we find the phase angle:



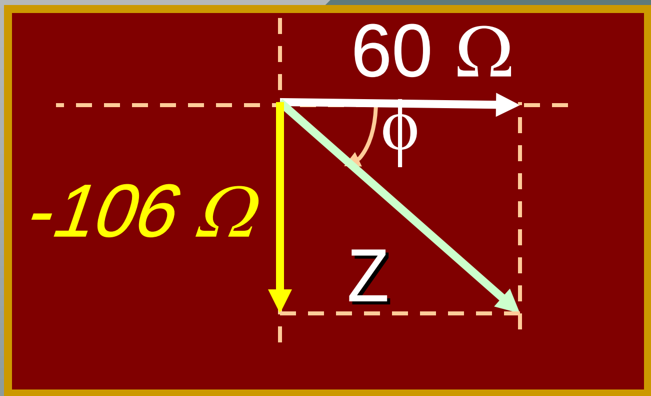
X_L – X_C = 226 – 332 = -106 Ω

R = 60 Ω

$$\tan\phi = \frac{X_L - X_C}{R}$$

Continued . . .

Example 4 (Cont.): Find the phase angle ϕ for the previous example.



$X_L - X_C = 226 - 332 = -106\ \Omega$

$R = 60\ \Omega$

$$\tan\phi = \frac{X_L - X_C}{R}$$

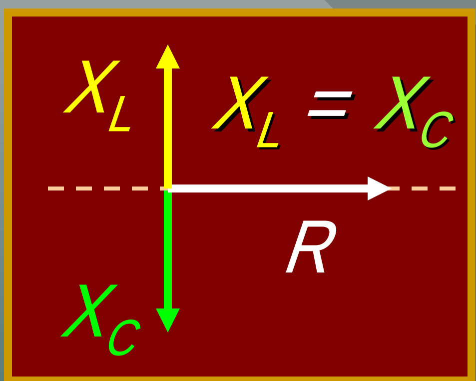
$$\tan\phi = \frac{-106\,\Omega}{60\,\Omega}$$

$$\phi = -60.5^0$$

The negative phase angle means that the ac voltage lags the current by 60.5^0. This is known as a capacitive circuit.

Resonant Frequency

Because inductance causes the voltage to lead the current and capacitance causes it to lag the current, they tend to cancel each other out.



Resonance (Maximum Power) occurs when $X_L = X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

Resonant f_r

$X_L = X_C$ ⇨

$$2\pi fL = \frac{1}{2\pi fC}$$

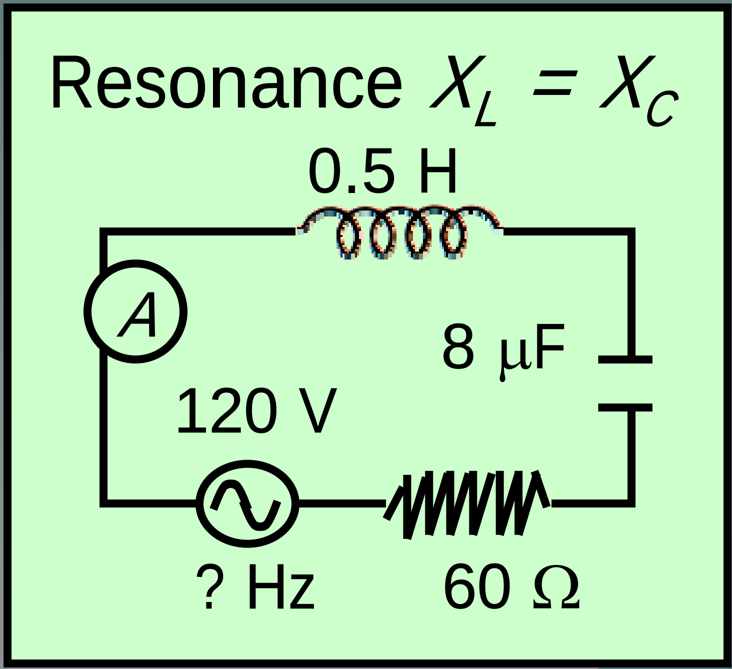
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Example 5: Find the resonant frequency for the previous circuit example: L = .5 H, C = 8 μF

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{(0.5\text{H})(8 \text{ x } 10^{-6}\text{F}}}$$

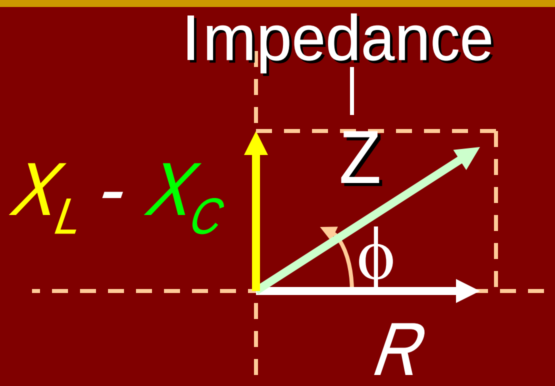
Resonant f_r = 79.6 Hz



At resonant frequency, there is zero reactance (only resistance) and the circuit has a phase angle of zero.

Power in an AC Circuit

No power is consumed by inductance or capacitance. Thus power is a function of the component of the impedance along resistance:



P lost in *R* only

In terms of ac voltage:

$$P = iV \cos \phi$$

In terms of the resistance R:

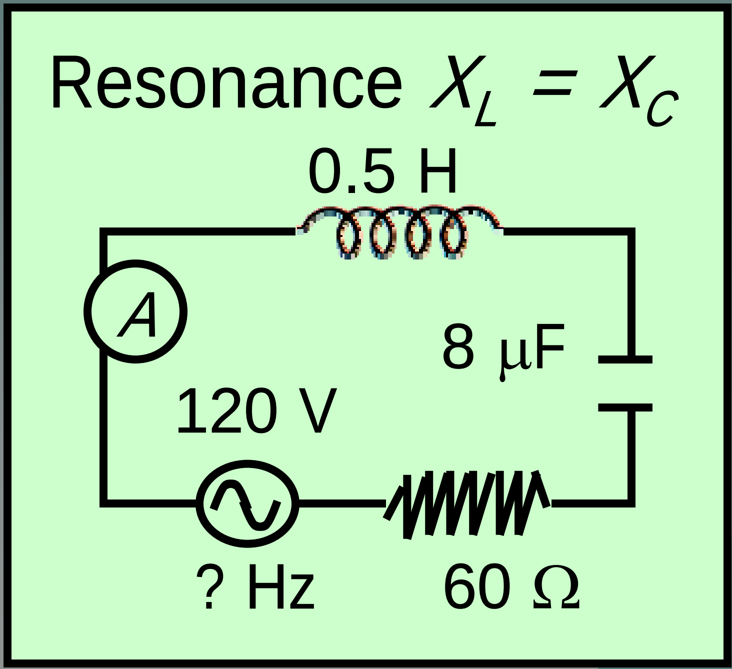
$$P = i^2 R$$

The fraction Cos ϕ is known as the power factor.

Example 6: What is the average power loss for the previous example: V = 120 V, ϕ = -60.5^0, i = 90.5 A, and R = 60Ω .

$P = i^2R$ = $(0.0905\ \text{A})^2(60\ \Omega)$

Average P = 0.491 W



The power factor is: Cos 60.5^0

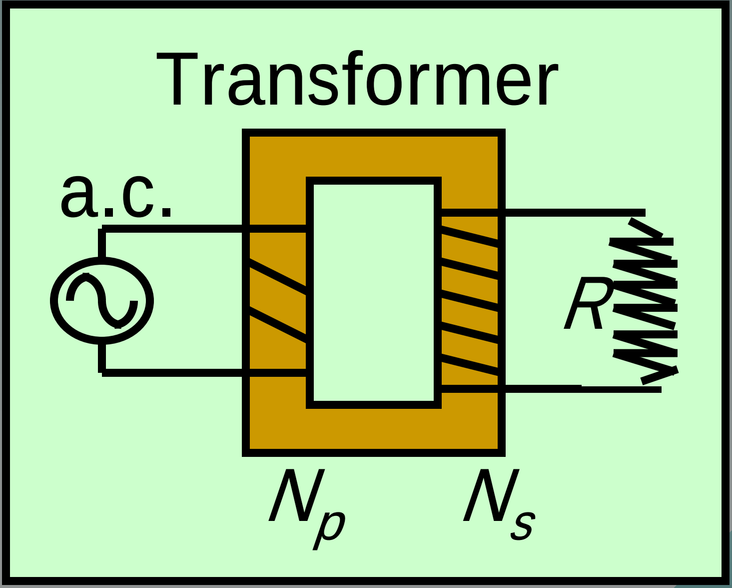
Cos ϕ = 0.492 or 49.2%

The higher the power factor, the more efficient is the circuit in its use of ac power.

The Transformer

A transformer is a device that uses induction and ac current to step voltages up or down.

An ac source of emf $\mathcal{E}_p$ is connected to primary coil with N_p turns. Secondary has N_s turns and emf of $\mathcal{E}_s$.

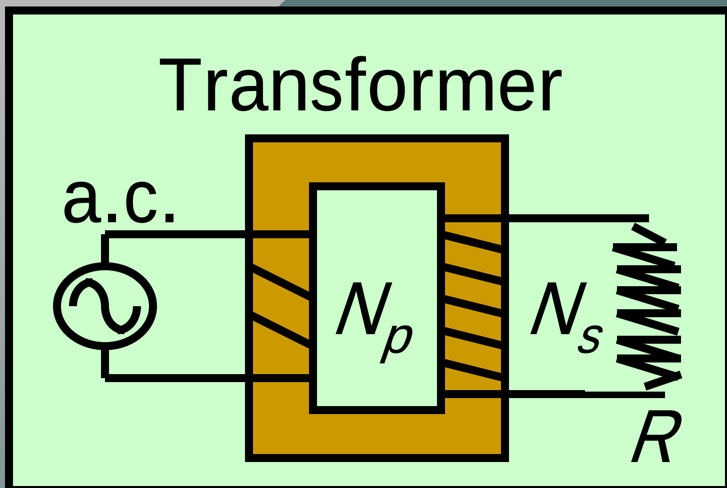


Induced emf's are:

$$\mathcal{E}_P = -N_P \frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E}_S = -N_S \frac{\Delta\Phi}{\Delta t}$$

Transformers (Continued):



$$\mathcal{E}_P = -N_P \frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E}_S = -N_S \frac{\Delta\Phi}{\Delta t}$$

Recognizing that $\Delta\phi/\Delta t$ is the same in each coil, we divide first relation by second and obtain:

The transformer equation:

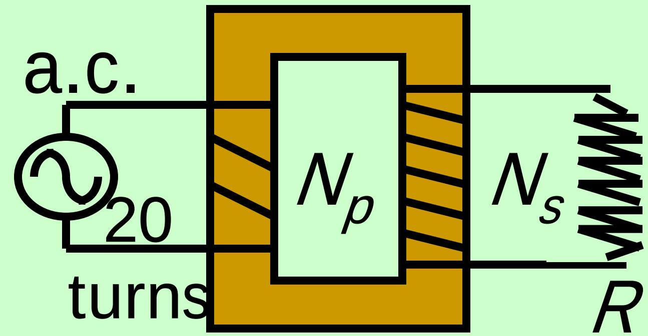
$$\frac{\mathcal{E}_P}{\mathcal{E}_S} = \frac{N_P}{N_S}$$

Example 7: A generator produces 10 A at 600 V. The primary coil in a transformer has 20 turns. How many secondary turns are needed to step up the voltage to 2400 V?

Applying the transformer equation:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

I = 10 A; V_p = 600 V



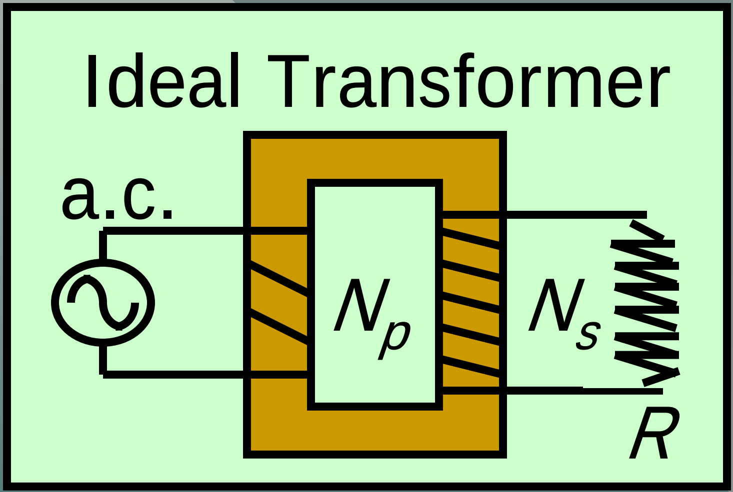
$$N_S = \frac{N_P V_S}{V_P} = \frac{(20)(2400\ \text{V})}{600\ \text{V}}$$

N_S = 80 turns

This is a step-up transformer; reversing coils will make it a step-down transformer.

Transformer Efficiency

There is no power gain in stepping up the voltage since voltage is increased by reducing current. In an ideal transformer with no internal losses:



An ideal transformer:

$$\mathcal{E}_P i_P = \mathcal{E}_S i_S \quad \text{or} \quad \frac{i_P}{i_s} = \frac{\mathcal{E}_S}{\mathcal{E}_P}$$

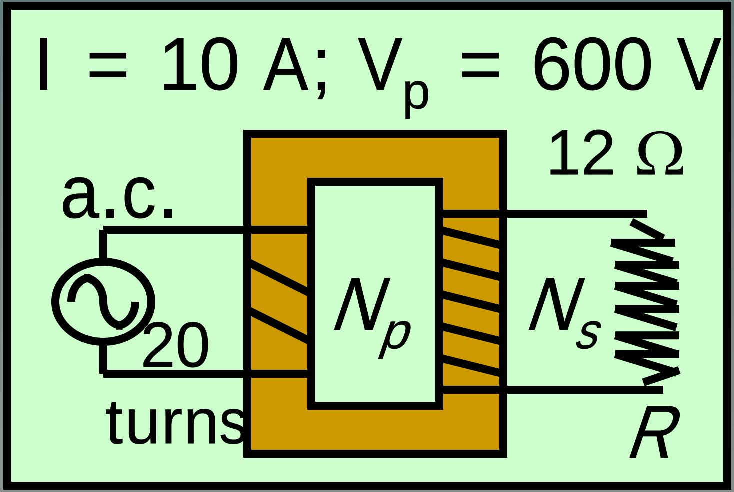
The above equation assumes no internal energy losses due to heat or flux changes. Actual efficiencies are usually between 90 and 100%.

Example 7: The transformer in Ex. 6 is connected to a power line whose resistance is 12 Ω. How much of the power is lost in the transmission line?

$$V_S = 2400\ V$$

$$\mathcal{E}_P i_P = \mathcal{E}_S i_S \qquad i_S = \frac{\mathcal{E}_P i_P}{\mathcal{E}_S}$$

$$i_S = \frac{(600\text{V})(10\text{ A})}{2400\text{ V}} = 2.50\text{ A}$$



$$P_{lost} = i^2R = (2.50\text{ A})^2(12\ \Omega) \qquad P_{lost} = 75.0\text{ W}$$

$$P_{in} = (600\text{ V})(10\text{ A}) = 6000\text{ W}$$

%Power Lost = (75 W/6000 W)(100%) = 1.25%

Summary

Effective current: $i_{eff} = 0.707\ i_{max}$

Effective voltage: $V_{eff} = 0.707\ V_{max}$

<u>Inductive Reactance:</u>

$$X_L = 2\pi f L \quad \text{Unit is the } \Omega$$

Ohm's law: $V_L = iX_L$

<u>Capacitive Reactance:</u>

$$X_C = \frac{1}{2\pi f C} \quad \text{Unit is the } \Omega$$

Ohm's law: $V_C = iX_C$

Summary (Cont.)

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$V_T = iZ \quad \text{or} \quad i = \frac{V_T}{Z}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Summary (Cont.)

Power in AC Circuits:

In terms of ac voltage:

$$P = iV \cos \phi$$

In terms of the resistance R:

$$P = i^2 R$$

Transformers:

$$\frac{\mathcal{E}_P}{\mathcal{E}_S} = \frac{N_P}{N_S} \qquad \mathcal{E}_P i_P = \mathcal{E}_S i_S$$

CONCLUSION: Chapter 32A AC Circuits