Chapter 11A – Angular Motion

A PowerPoint Presentation by

Paul E. Tippens, Professor of Physics

Southern Polytechnic State University

© 2007
WIND TURBINES such as these can generate significant energy in a way that is environmentally friendly and renewable. The concepts of rotational acceleration, angular velocity, angular displacement, rotational inertia, and other topics discussed in this chapter are useful in describing the operation of wind turbines.
Objectives: After completing this module, you should be able to:

- Define and apply concepts of angular displacement, velocity, and acceleration.
- Draw analogies relating rotational-motion parameters ($\theta$, $\omega$, $\alpha$) to linear ($x$, $v$, $a$) and solve rotational problems.
- Write and apply relationships between linear and angular parameters.
Objectives: (Continued)

- Define moment of inertia and apply it for several regular objects in rotation.
- Apply the following concepts to rotation:
  1. Rotational work, energy, and power
  2. Rotational kinetic energy and momentum
  3. Conservation of angular momentum
Rotational Displacement, $\theta$

Consider a disk that rotates from A to B:

Angular displacement $\theta$:
Measured in revolutions, degrees, or radians.

$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$

The best measure for rotation of rigid bodies is the radian.
Definition of the Radian

One radian is the angle $\theta$ subtended at the center of a circle by an arc length $s$ equal to the radius $R$ of the circle.

$$\theta = \frac{s}{R}$$

1 rad $= \frac{R}{R} = 57.3^0$
Example 1: A rope is wrapped many times around a drum of radius 50 cm. How many revolutions of the drum are required to raise a bucket to a height of 20 m?

\[ \theta = \frac{s}{R} = \frac{20 \text{ m}}{0.50 \text{ m}} \quad \theta = 40 \text{ rad} \]

Now, 1 rev = 2\pi rad

\[ \theta = (40 \text{ rad}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \]

\[ \theta = 6.37 \text{ rev} \]

\[ h = 20 \text{ m} \]
**Example 2:** A bicycle tire has a radius of 25 cm. If the wheel makes 400 rev, how far will the bike have traveled?

\[ \theta = (400 \text{ rev}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \]

\[ \theta = 2513 \text{ rad} \]

\[ s = \theta R = 2513 \text{ rad} \times 0.25 \text{ m} \]

\[ s = 628 \text{ m} \]
Angular Velocity

**Angular velocity**, $\omega$, is the rate of change in angular displacement. (radians per second.)

$$\omega = \frac{\Delta \theta}{\Delta t} \quad \text{Angular velocity in rad/s.}$$

Angular velocity can also be given as the frequency of revolution, $f$ (rev/s or rpm):

$$\omega = 2\pi f \quad \text{Angular frequency } f \text{ (rev/s).}$$
**Example 3:** A rope is wrapped many times around a drum of radius 20 cm. What is the angular velocity of the drum if it lifts the bucket to 10 m in 5 s?

\[
\theta = \frac{s}{R} = \frac{10 \text{ m}}{0.20 \text{ m}} \quad \theta = 50 \text{ rad}
\]

\[
\omega = \frac{\Delta \theta}{\Delta t} = \frac{50 \text{ rad}}{5 \text{ s}} \quad \omega = 10.0 \text{ rad/s}
\]
Example 4: In the previous example, what is the frequency of revolution for the drum? Recall that $\omega = 10.0 \text{ rad/s}$.

\[ \omega = 2\pi f \quad \text{or} \quad f = \frac{\omega}{2\pi} \]

\[ f = \frac{10.0 \text{ rad/s}}{2\pi \text{ rad/rev}} = 1.59 \text{ rev/s} \]

Or, since $60 \text{ s} = 1 \text{ min}$:

\[ f = 1.59 \frac{\text{rev}}{\text{s}} \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 95.5 \frac{\text{rev}}{\text{min}} \]

\[ f = 95.5 \text{ rpm} \]
Angular Acceleration

**Angular acceleration** is the rate of change in angular velocity. (Radians per sec per sec.)

\[ \alpha = \frac{\Delta \omega}{\Delta t} \]

Angular acceleration (rad/s²)

The angular acceleration can also be found from the change in frequency, as follows:

\[ \alpha = \frac{2\pi(\Delta f)}{t} \]

Since \( \omega = 2\pi f \)
Example 5: The block is lifted from rest until the angular velocity of the drum is 16 rad/s after a time of 4 s. What is the average angular acceleration?

\[ \alpha = \frac{\omega_f - \omega_o}{t} \quad \text{or} \quad \alpha = \frac{\omega_f}{t} \]

\[ \alpha = \frac{16 \text{ rad/s}}{4 \text{ s}} = 4.00 \text{ rad/s}^2 \]

\[ \alpha = 4.00 \text{ rad/s}^2 \]
Angular and Linear Speed

From the definition of angular displacement:

\[ s = \theta \cdot R \quad \text{Linear vs. angular displacement} \]

\[ v = \frac{\Delta s}{\Delta t} = \left( \frac{\Delta \theta \cdot R}{\Delta t} \right) = \left( \frac{\Delta \theta}{\Delta t} \right) R \]

\[ v = \omega \cdot R \]

Linear speed = angular speed \times radius
Angular and Linear Acceleration:

From the velocity relationship we have:

\[ v = \omega R \quad \text{Linear vs. angular velocity} \]

\[ v = \frac{\Delta v}{\Delta t} = \frac{\Delta v \cdot R}{\Delta t} = \left( \frac{\Delta v}{\Delta t} \right) R \]

\[ a = \alpha R \]

Linear accel. = angular accel. x radius
Examples:

Consider flat rotating disk:

\[ \omega_o = 0; \quad \omega_f = 20 \text{ rad/s} \]
\[ t = 4 \text{ s} \]

What is final linear speed at points A and B?

\[ v_{Af} = \omega_{Af} R_1 = (20 \text{ rad/s})(0.2 \text{ m}); \quad v_{Af} = 4 \text{ m/s} \]

\[ v_{Af} = \omega_{Bf} R_1 = (20 \text{ rad/s})(0.4 \text{ m}); \quad v_{Bf} = 8 \text{ m/s} \]
**Acceleration Example**

Consider flat rotating disk:

\[ \omega_0 = 0; \quad \omega_f = 20 \text{ rad/s} \]
\[ t = 4 \text{ s} \]

**What is the average angular and linear acceleration at B?**

\[ \alpha = \frac{\omega_f - \omega_0}{t} = \frac{20 \text{ rad/s}}{4 \text{ s}} \]
\[ \alpha = 5.00 \text{ rad/s}^2 \]

\[ a = \alpha R = (5 \text{ rad/s}^2)(0.4 \text{ m}) \]
\[ a = 2.00 \text{ m/s}^2 \]
Angular vs. Linear Parameters

Recall the definition of linear acceleration $a$ from kinematics.

$$a = \frac{v_f - v_0}{t}$$

But, $a = \alpha R$ and $v = \omega R$, so that we may write:

$$a = \frac{v_f - v_0}{t} \quad \text{becomes} \quad \alpha R = \frac{R\omega_f - R\omega_0}{t}$$

Angular acceleration is the time rate of change in angular velocity.

$$\alpha = \frac{\omega_f - \omega_0}{t}$$
A Comparison: Linear vs. Angular

\[ s = \bar{v}t = \left( \frac{v_0 + v_f}{2} \right) t \]
\[ \theta = \bar{\omega}t = \left( \frac{\omega_0 + \omega_f}{2} \right) t \]

\[ v_f = v_o + at \]
\[ \omega_f = \omega_o + \alpha t \]

\[ s = v_0 t + \frac{1}{2} at^2 \]
\[ \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ s = v_f t - \frac{1}{2} at^2 \]
\[ \theta = \omega_f t - \frac{1}{2} \alpha t^2 \]

\[ 2as = v_f^2 - v_0^2 \]
\[ 2\alpha\theta = \omega_f^2 - \omega_0^2 \]
**Linear Example:** A car traveling initially at 20 m/s comes to a stop in a distance of 100 m. What was the acceleration?

**Select Equation:**

\[ 2as = v_f^2 - v_0^2 \]

\[
\begin{align*}
a &= \frac{0 - v_o^2}{2s} = \frac{-(20 \text{ m/s})^2}{2(100 \text{ m})} \\
a &= -2.00 \text{ m/s}^2
\end{align*}
\]
Angular analogy: A disk \( (R = 50 \text{ cm}) \), rotating at 600 rev/min comes to a stop after making 50 rev. What is the acceleration?

Select Equation:

\[
2\alpha \theta = \omega_f^2 - \omega_0^2
\]

\( \omega_o = 600 \text{ rpm} \)
\( \omega_f = 0 \text{ rpm} \)
\( \theta = 50 \text{ rev} \)

\[
600 \text{ rev/min} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 62.8 \text{ rad/s}
\]

\[
50 \text{ rev} = 314 \text{ rad}
\]

\[
\alpha = \frac{0 - \omega_0^2}{2\theta} = \frac{-(62.8 \text{ rad/s})^2}{2(314 \text{ rad})}
\]

\[\alpha = -6.29 \text{ m/s}^2\]
Problem Solving Strategy:

- Draw and label sketch of problem.
- Indicate + direction of rotation.
- List givens and state what is to be found.
  
  Given: _____, _____, _____ ($\theta, \omega_0, \omega_f, \alpha, t$)
  
  Find: _____, _____

- Select equation containing one and not the other of the unknown quantities, and solve for the unknown.
Example 6: A drum is rotating clockwise initially at **100 rpm** and undergoes a constant counterclockwise acceleration of **3 rad/s^2** for 2 s. What is the angular displacement?

*Given:* \( \omega_o = -100 \text{ rpm}; \ t = 2 \text{ s} \)
\( \alpha = +2 \text{ rad/s}^2 \)

\[
100 \text{ rev/min} \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 10.5 \text{ rad/s}
\]

\[
\theta = \omega_o t + \frac{1}{2} \alpha t^2 = (-10.5)(2) + \frac{1}{2} (3)(2)^2
\]

\[
\theta = -20.9 \text{ rad} + 6 \text{ rad} \quad \theta = -14.9 \text{ rad}
\]

*Net displacement is clockwise (-)*
Summary of Formulas for Rotation

\[ s = \bar{v}t = \left( \frac{v_0 + v_f}{2} \right) t \]

\[ \theta = \bar{\omega}t = \left( \frac{\omega_0 + \omega_f}{2} \right) t \]

\[ v_f = v_o + at \]

\[ \omega_f = \omega_o + \alpha t \]

\[ s = v_0 t + \frac{1}{2} at^2 \]

\[ \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ s = v_f t - \frac{1}{2} at^2 \]

\[ \theta = \omega_f t - \frac{1}{2} \alpha t^2 \]

\[ 2as = v_f^2 - v_0^2 \]

\[ 2\alpha \theta = \omega_f^2 - \omega_0^2 \]
CONCLUSION: Chapter 11A
Angular Motion