Objectives: After completing this module, you should be able to:

- Discuss the early models of the atom leading to the Bohr theory of the atom.
- Demonstrate your understanding of emission and absorption spectra and predict the wavelengths or frequencies of the Balmer, Lyman, and Pashen spectral series.
- Calculate the energy emitted or absorbed by the hydrogen atom when the electron moves to a higher or lower energy level.
Properties of Atoms

- Atoms are stable and electrically neutral.
- Atoms have chemical properties which allow them to combine with other atoms.
- Atoms emit and absorb electromagnetic radiation with discrete energy and momentum.
- Early experiments showed that most of the mass of an atom was associated with positive charge.
- Atoms have angular momentum and magnetism.
J. J. Thompson’s plum pudding model consists of a sphere of positive charge with electrons embedded inside. This model would explain that most of the mass was positive charge and that the atom was electrically neutral. The size of the atom ($\approx 10^{-10} \text{ m}$) prevented direct confirmation.
Rutherford’s Experiment

The Thompson model was abandoned in 1911 when Rutherford bombarded a thin metal foil with a stream of positively charged alpha particles.

Most particles pass right through the foil, but a few are scattered in a backward direction.
The Nucleus of an Atom

If electrons were distributed uniformly, particles would pass straight through an atom. Rutherford proposed an atom that is open space with positive charge concentrated in a very dense nucleus.

Electrons must orbit at a distance in order not to be attracted into the nucleus of atom.
Consider the planetary model for electrons which move in a circle around the positive nucleus. The figure below is for the hydrogen atom.

**Coulomb’s law:**

\[ F_C = \frac{e^2}{4\pi\varepsilon_0 r^2} \]

**Centripetal \(F_C\):**

\[ F_C = \frac{mv^2}{r^2} \]

\[ \frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \]

\[ r = \frac{e^2}{4\pi\varepsilon_0 mv^2} \]
When an electron is accelerated by the central force, it must radiate energy.

The loss of energy should cause the velocity $v$ to decrease, sending the electron crashing into the nucleus.

This does NOT happen and the Rutherford atom fails.
Atomic Spectra

Earlier, we learned that objects continually emit and absorb electromagnetic radiation.

In an emission spectrum, light is separated into characteristic wavelengths.

In an absorption spectrum, a gas absorbs certain wavelengths, which identify the element.
Balmer worked out a mathematical formula, called the **Balmer series** for predicting the absorbed wavelengths from hydrogen gas.

\[ \frac{1}{\lambda} = R \left( \frac{1}{2^2} + \frac{1}{n^2} \right); \quad n = 3, 4, 5, \ldots \]

\[ R = 1.097 \times 10^7 \text{ m}^{-1} \]
**Example 1:** Use the Balmer equation to find the wavelength of the first line \((n = 3)\) in the Balmer series. How can you find the energy?

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} + \frac{1}{n^2} \right); \quad n = 3 \quad R = 1.097 \times 10^7 \text{ m}^{-1}
\]

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} + \frac{1}{3^2} \right) = R(0.361); \quad \lambda = \frac{1}{0.361R}
\]

\[
\lambda = \frac{1}{0.361(1.097 \times 10^7 \text{ m}^{-1})} \quad \lambda = 656 \text{ nm}
\]

The frequency and the energy are found from:

\[
c = f\lambda \quad \text{and} \quad E = hf
\]
The Bohr Atom

Atomic spectra indicate that atoms emit or absorb energy in discrete amounts. In 1913, Neils Bohr explained that classical theory did not apply to the Rutherford atom.

An electron can only have certain orbits and the atom must have definite energy levels which are analogous to standing waves.
Wave Analysis of Orbits

Stable orbits exist for integral multiples of de Broglie wavelengths.

\[ 2\pi r = n\lambda \quad n = 1,2,3, \ldots \]

Recalling that angular momentum is \( mvr \), we write:

\[ L = mvr = n \frac{h}{2\pi} ; \quad n = 1,2,3, \ldots \]
The Bohr Atom

An electron can have only those orbits in which its angular momentum is:

\[ L = n \frac{\hbar}{2\pi} \; ; \; n = 1, 2, 3, \ldots \]

Bohr’s postulate: When an electron changes from one orbit to another, it gains or loses energy equal to the difference in energy between initial and final levels.
Bohr’s Atom and Radiation

When an electron drops to a lower level, radiation is emitted; when radiation is absorbed, the electron moves to a higher level.

Energy: \[ hf = E_f - E_i \]

By combining the idea of energy levels with classical theory, Bohr was able to predict the radius of the hydrogen atom.
Radius of the Hydrogen Atom

Radius as function of energy level:

\[ L = mvr = n \frac{h}{2\pi} \; ; \; n = 1, 2, 3, \ldots \]

Bohr’s radius

\[ r = \frac{nh}{mv} \]

Classical radius

\[ r = \frac{e^2}{4\pi\varepsilon_0 mv^2} \]

By eliminating \( r \) from these equations, we find the velocity \( v \); elimination of \( v \) gives possible radii \( r_n \):

\[ v_n = \frac{e^2}{2\varepsilon_0 nh} \]

\[ r_n = \frac{n^2\varepsilon_0 h^2}{\pi me^2} \]
Example 2: Find the radius of the Hydrogen atom in its most stable state \((n = 1)\).

\[
r_n = \frac{n^2 \varepsilon_0 h^2}{\pi m e^2}
\]

\[
m = 9.1 \times 10^{-31} \text{ kg}
\]
\[
e = 1.6 \times 10^{-19} \text{ C}
\]

\[
r = \frac{(1)^2 (8.85 \times 10^{-12} \text{ Nm}^2/\text{C}^2)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{\pi (9.1 \times 10^{-31} \text{ kg})(1.6 \times 10^{-19} \text{ C})^2}
\]

\[
r = 5.31 \times 10^{-11} \text{ m}
\]

\[
r = 53.1 \text{ pm}
\]
Total Energy of an Atom

The total energy at level $n$ is the sum of the kinetic and potential energies at that level.

$$ E = K + U; \quad K = \frac{1}{2} m v^2; \quad U = \frac{e^2}{4\pi\varepsilon_0 r} $$

But we recall that:

$$ v_n = \frac{e^2}{2\varepsilon_0 n\hbar} \quad r_n = \frac{n^2 \varepsilon_0 \hbar^2}{\pi me^2} $$

Substitution for $v$ and $r$ gives expression for total energy.

Total energy of Hydrogen atom for level $n$:

$$ E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 \hbar^2} $$
Energy for a Particular State

It will be useful to simplify the energy formula for a particular state by substitution of constants.

\[
m = 9.1 \times 10^{-31} \text{ kg}
\]
\[
e = 1.6 \times 10^{-19} \text{ C}
\]
\[
\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2
\]
\[
h = 6.63 \times 10^{-34} \text{ J s}
\]

\[
E_n = -\frac{me^4}{8\varepsilon_0 n^2 h^2} = -\frac{(9.1 \times 10^{-31} \text{ kg})(1.6 \times 10^{-19} \text{ C})^4}{8(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)^2 n^2 (6.63 \times 10^{-34} \text{ Js})^2}
\]

\[
E_n = -\frac{2.17 \times 10^{-18} \text{ J}}{n^2}
\]

Or

\[
E_n = \frac{-13.6 \text{ eV}}{n^2}
\]
Balmer Revisited

Total energy of Hydrogen atom for level $n$.

$$E_n = -\frac{me^4}{8\varepsilon_0 n^2 h^2}$$

Negative because outside energy to raise $n$ level.

When an electron moves from an initial state $n_i$ to a final state $n_f$, energy involved is:

$$E = \frac{1}{\lambda} = \frac{1}{E} = \frac{me^4}{\lambda} E_f \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

Balmer’s Equation:

$$\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_0^2} \right); \quad R = 1.097 \times 10^7 \text{ m}^{-1}$$
We can now visualize the hydrogen atom with an electron at many possible energy levels.

The energy of the atom increases on absorption ($n_f > n_i$) and decreases on emission ($n_f < n_i$). The change in energy of the atom can be given in terms of initial $n_i$ and final $n_f$ levels:

$$E = -13.6 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{n_0^2} \right)$$
Spectral Series for an Atom

The **Lyman series** is for transitions to \( n = 1 \) level.

The **Balmer series** is for transitions to \( n = 2 \) level.

The **Pashen series** is for transitions to \( n = 3 \) level.

The **Brackett series** is for transitions to \( n = 4 \) level.

\[
E = -13.6 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{n_0^2} \right)
\]
Example 3: What is the energy of an emitted photon if an electron drops from the \( n = 3 \) level to the \( n = 1 \) level for the hydrogen atom?

\[
E = -13.6 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{n_0^2} \right)
\]

\[
\Delta E = -12.1 \text{ eV}
\]

The energy of the atom decreases by 12.1 eV as a photon of that energy is emitted.

You should show that 13.6 eV is required to move an electron from \( n = 1 \) to \( n = \infty \).
Modern Theory of the Atom

The model of an electron as a point particle moving in a circular orbit has undergone significant change.

- The quantum model now presents the location of an electron as a probability distribution - a cloud around the nucleus.
- Additional quantum numbers have been added to describe such things as shape, orientation, and magnetic spin.
- Pauli’s exclusion principle showed that no two electrons in an atom can exist in the exact same state.
The Bohr atom for Beryllium suggests a planetary model which is not strictly correct.

The n = 2 level of the Hydrogen atom is shown here as a probability distribution.
Bohr’s model of the atom assumed the electron to follow a circular orbit around a positive nucleus.

Radius of Hydrogen Atom

\[ r = \frac{e^2}{4\pi\varepsilon_0 mv^2} \]
In an emission spectrum, characteristic wavelengths appear on a screen. For an absorption spectrum, certain wavelengths are omitted due to absorption.
### Summary (Cont.)

**Spectrum for** $n_f = 2$ (*Balmer*)

<table>
<thead>
<tr>
<th>$n$</th>
<th>Emission spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

434 nm

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>653 nm</th>
<th>486 nm</th>
<th>410 nm</th>
</tr>
</thead>
</table>

The general equation for a change from one level to another:

**Balmer’s Equation:**  
$$\frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_0^2} \right); \quad R = 1.097 \times 10^7 \text{ m}^{-1}$$
Bohr’s model sees the hydrogen atom with an electron at many possible energy levels.

The energy of the atom increases on absorption ($n_f > n_i$) and decreases on emission ($n_f < n_i$).

Energy of $n$th level:

$$E = \frac{-13.6 \text{ eV}}{n^2}$$

The change in energy of the atom can be given in terms of initial $n_i$ and final $n_f$ levels:

$$E = -13.6 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$
CONCLUSION: Chapter 38C
Atomic Physics