Objectives: After completing this module, you should be able to:

• Determine the effective resistance for a number of resistors connected in series and in parallel.

• For simple and complex circuits, determine the voltage and current for each resistor.

• Apply Kirchoff’s laws to find currents and voltages in complex circuits.
Electrical circuits often contain one or more resistors grouped together and attached to an energy source, such as a battery.

The following symbols are often used:

- **Ground**: 
  ![Ground Symbol](Image)

- **Battery**: 
  ![Battery Symbol](Image)

- **Resistor**: 
  ![Resistor Symbol](Image)
Resistances in Series

Resistors are said to be connected in **series** when there is a **single path** for the current.

The current $I$ is the same for each resistor $R_1$, $R_2$ and $R_3$.

The energy gained through $E$ is lost through $R_1$, $R_2$ and $R_3$.

The same is true for voltages:

$$I = I_1 = I_2 = I_3$$

$$V_T = V_1 + V_2 + V_3$$

For series connections:
Equivalent Resistance: Series

The equivalent resistance $R_e$ of a number of resistors connected in series is equal to the sum of the individual resistances.

$V_T = V_1 + V_2 + V_3; \quad (V = IR)$

$I_T R_e = I_1 R_1 + I_2 R_2 + I_3 R_3$

But $\ldots I_T = I_1 = I_2 = I_3$

$R_e = R_1 + R_2 + R_3$
Example 1: Find the equivalent resistance $R_e$. What is the current $I$ in the circuit?

\[ R_e = R_1 + R_2 + R_3 \]
\[ R_e = 3\,\Omega + 2\,\Omega + 1\,\Omega = 6\,\Omega \]

Equivalent $R_e = 6\,\Omega$

The current is found from Ohm’s law: $V = IR_e$

\[ I = \frac{V}{R_e} = \frac{12\,V}{6\,\Omega} \]

$I = 2\,A$
Example 1 (Cont.): Show that the voltage drops across the three resistors totals the 12-V emf.

Current $I = 2 \text{ A}$ same in each R.

$$V_1 = IR_1; \quad V_2 = IR_2; \quad V_3 = IR_3$$

$$V_1 = (2 \text{ A})(1 \Omega) = 2 \text{ V}$$
$$V_1 = (2 \text{ A})(2 \Omega) = 4 \text{ V}$$
$$V_1 = (2 \text{ A})(3 \Omega) = 6 \text{ V}$$

$$V_1 + V_2 + V_3 = V_T$$
$$2 \text{ V} + 4 \text{ V} + 6 \text{ V} = 12 \text{ V}$$

Check!
Sources of EMF in Series

The output direction from a source of emf is from $+ \text{ side}$:

Thus, from $a$ to $b$ the potential increases by $\mathcal{E}$; From $b$ to $a$, the potential decreases by $\mathcal{E}$.

**Example:** Find $\Delta V$ for path $AB$ and then for path $BA$.

$AB$: $\Delta V = +9 \text{ V} - 3 \text{ V} = +6 \text{ V}$

$BA$: $\Delta V = +3 \text{ V} - 9 \text{ V} = -6 \text{ V}$
A Single Complete Circuit

Consider the simple **series circuit** drawn below:

Path ABCD: Energy and V increase through the 15-V source and decrease through the 3-V source.

\[ \Sigma \mathcal{E} = 15 \text{ V} - 3 \text{ V} = 12 \text{ V} \]

The net gain in potential is lost through the two resistors: these voltage drops are \( IR_2 \) and \( IR_4 \), so that the sum is zero for the entire loop.
Finding I in a Simple Circuit.

**Example 2:** Find the current $I$ in the circuit below:

\[
\begin{align*}
\Sigma \mathcal{E} &= 18 \text{ V} - 3 \text{ V} = 15 \text{ V} \\
\Sigma R &= 3 \Omega + 2 \Omega = 5 \Omega
\end{align*}
\]

Applying Ohm’s law:

\[
I = \frac{\Sigma \mathcal{E}}{\Sigma R} = \frac{15 \text{ V}}{5 \Omega} = 3 \text{ A}
\]

In general for a single loop circuit:

\[
I = \frac{\Sigma \mathcal{E}}{\Sigma R}
\]
Summary: Single Loop Circuits:

**Resistance Rule:** \( R_e = \Sigma R \)

**Current:** \( I = \frac{\Sigma \mathcal{E}}{\Sigma R} \)

**Voltage Rule:** \( \Sigma \mathcal{E} = \Sigma IR \)
A complex circuit is one containing more than a single loop and different current paths.

At junctions m and n:

\[ I_1 = I_2 + I_3 \text{  or  } I_2 + I_3 = I_1 \]

Junction Rule:

\[ \sum I \text{ (enter)} = \sum I \text{ (leaving)} \]
Parallel Connections

Resistors are said to be connected in parallel when there is more than one path for current.

**Parallel Connection:**

- For Parallel Resistors:
  \[ V_2 = V_4 = V_6 = V_T \]
  \[ I_2 + I_4 + I_6 = I_T \]

**Series Connection:**

- For Series Resistors:
  \[ I_2 = I_4 = I_6 = I_T \]
  \[ V_2 + V_4 + V_6 = V_T \]
Equivalent Resistance: Parallel

\[ V_T = V_1 = V_2 = V_3 \]
\[ I_T = I_1 + I_2 + I_3 \]

Ohm's law: \[ I = \frac{V}{R} \]

\[ \frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \]

The equivalent resistance for Parallel resistors:

\[ \frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

\[ \frac{1}{R_e} = \sum_{i=1}^{N} \frac{1}{R_i} \]
Example 3. Find the equivalent resistance \( R_e \) for the three resistors below.

\[
\frac{1}{R_e} = \sum_{i=1}^{N} \frac{1}{R_i}
\]

\[
\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

\[
\frac{1}{R_e} = \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{6\,\Omega} = 0.500 + 0.250 + 0.167
\]

\[
\frac{1}{R_e} = 0.917; \quad R_e = \frac{1}{0.917} = 1.09\,\Omega
\]

\( R_e = 1.09\,\Omega \)

For parallel resistors, \( R_e \) is less than the least \( R_i \).
Example 3 (Cont.): Assume a 12-V emf is connected to the circuit as shown. What is the total current leaving the source of emf?

Ohm’s Law: \( I = \frac{V}{R} \)

\[ I_e = \frac{V_T}{R_e} = \frac{12 \text{ V}}{1.09 \Omega} \]

Total current: \( I_T = 11.0 \text{ A} \)
Example 3 (Cont.): Show that the current leaving the source $I_T$ is the sum of the currents through the resistors $R_1$, $R_2$, and $R_3$.

$I_T = 11 \text{ A}; \quad R_e = 1.09 \ \Omega$

$V_1 = V_2 = V_3 = 12 \text{ V}$

$I_T = I_1 + I_2 + I_3$

$I_1 = \frac{12 \ \text{V}}{2 \ \Omega} = 6 \ \text{A}$

$I_2 = \frac{12 \ \text{V}}{4 \ \Omega} = 3 \ \text{A}$

$I_3 = \frac{12 \ \text{V}}{6 \ \Omega} = 2 \ \text{A}$

$6 \ \text{A} + 3 \ \text{A} + 2 \ \text{A} = 11 \ \text{A}$

Check!
Short Cut: Two Parallel Resistors

The equivalent resistance $R_e$ for two parallel resistors is the product divided by the sum.

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2};$$

$$R_e = \frac{R_1 R_2}{R_1 + R_2}$$

**Example:**

$$R_e = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega}$$

$$R_e = 2\,\Omega$$
In complex circuits resistors are often connected in both series and parallel.

In such cases, it’s best to use rules for series and parallel resistances to reduce the circuit to a simple circuit containing one source of emf and one equivalent resistance.
Example 4. Find the equivalent resistance for the circuit drawn below (assume $V_T = 12$ V).

\[ R_{3,6} = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = 2\,\Omega \]

\[ R_e = 4\,\Omega + 2\,\Omega \]

\[ R_e = 6\,\Omega \]
Example 3 (Cont.) Find the total current $I_T$.

$R_e = 6 \Omega$

$I = \frac{V_T}{R_e} = \frac{12 \text{ V}}{6 \Omega}$

$I_T = 2.00 \text{ A}$
Example 3 (Cont.) Find the currents and the voltages across each resistor.

\[ I_4 = I_T = 2 \text{ A} \]

\[ V_4 = (2 \text{ A})(4 \text{ } \Omega) = 8 \text{ V} \]

The remainder of the voltage: \((12 \text{ V} - 8 \text{ V} = 4 \text{ V})\) drops across EACH of the parallel resistors.

\[ V_3 = V_6 = 4 \text{ V} \]

This can also be found from \(V_{3,6} = I_{3,6}R_{3,6} = (2 \text{ A})(2 \text{ } \Omega)\)

(Continued . . .)
Example 3 (Cont.) Find the currents and voltages across each resistor.

\[ V_4 = 8 \, \text{V} \]
\[ V_6 = V_3 = 4 \, \text{V} \]

\[ I_3 = \frac{V_3}{R_3} = \frac{4 \, \text{V}}{3 \, \Omega} \]
\[ I_3 = 1.33 \, \text{A} \]

\[ I_6 = \frac{V_6}{R_6} = \frac{4 \, \text{V}}{6 \, \Omega} \]
\[ I_6 = 0.667 \, \text{A} \]

\[ I_4 = 2 \, \text{A} \]

Note that the junction rule is satisfied:

\[ \Sigma I \, (\text{enter}) = \Sigma I \, (\text{leaving}) \]
\[ I_T = I_4 = I_3 + I_6 \]
Kirchoff’s Laws for DC Circuits

**Kirchoff’s first law:** The sum of the currents entering a junction is equal to the sum of the currents leaving that junction.

\[ \sum I_{\text{enter}} = \sum I_{\text{leaving}} \]

**Kirchoff’s second law:** The sum of the emf’s around any closed loop must equal the sum of the IR drops around that same loop.

\[ \sum \mathcal{E} = \sum IR \]
Sign Conventions for Emf’s

- When applying Kirchoff’s laws you must assume a consistent, positive tracing direction.

- When applying the voltage rule, emf’s are positive if normal output direction of the emf is with the assumed tracing direction.

- If tracing from A to B, this emf is considered positive.

- If tracing from B to A, this emf is considered negative.
Signs of IR Drops in Circuits

- When applying the voltage rule, IR drops are positive if the assumed current direction is with the assumed tracing direction.

- If tracing from A to B, this IR drop is positive.

- If tracing from B to A, this IR drop is negative.
Kirchoff’s Laws: Loop 1

1. Assume possible consistent flow of currents.
2. Indicate positive output directions for emf’s.
3. Indicate consistent tracing direction. (clockwise)

\[ I_2 = I_1 + I_3 \]

**Junction Rule:**

\[ \Sigma E = \Sigma IR \]

\[ E_1 + E_2 = I_1 R_1 + I_2 R_2 \]
Kirchoff’s Laws: Loop II

4. Voltage rule for Loop II:
Assume counterclockwise positive tracing direction.

Voltage Rule: \( \sum \mathcal{E} = \sum IR \)

\( \mathcal{E}_2 + \mathcal{E}_3 = I_2R_2 + I_3R_3 \)

Would the same equation apply if traced clockwise?

Yes!

\( -\mathcal{E}_2 - \mathcal{E}_3 = -I_2R_2 - I_3R_3 \)
Kirchoff’s laws: Loop III

5. Voltage rule for Loop III:
Assume counterclockwise positive tracing direction.

Voltage Rule: $\sum \mathcal{E} = \sum I R$

$\mathcal{E}_3 - \mathcal{E}_1 = -I_1 R_1 + I_3 R_3$

Would the same equation apply if traced clockwise?

Yes!

$\mathcal{E}_3 - \mathcal{E}_1 = I_1 R_1 - I_3 R_3$
Four Independent Equations

6. Thus, we now have four independent equations from Kirchoff’s laws:

\[ I_2 = I_1 + I_3 \]

\[ \mathcal{E}_1 + \mathcal{E}_2 = I_1R_1 + I_2R_2 \]

\[ \mathcal{E}_2 + \mathcal{E}_3 = I_2R_2 + I_3R_3 \]

\[ \mathcal{E}_3 - \mathcal{E}_1 = -I_1R_1 + I_3R_3 \]
Example 4. Use Kirchoff’s laws to find the currents in the circuit drawn to the right.

**Junction Rule:** \( I_2 + I_3 = I_1 \)

Consider Loop I tracing clockwise to obtain:

**Voltage Rule:** \( \sum E = \sum I R \)

12 V = (5 \( \Omega \))\( I_1 \) + (10 \( \Omega \))\( I_2 \)

Recalling that \( V/\Omega = A \), gives

\[ 5I_1 + 10I_2 = 12 \text{ A} \]
Example 5 (Cont.) Finding the currents.

Consider Loop II tracing clockwise to obtain:

Voltage Rule: \( \sum V = \sum I R \)

6 V = (20 \Omega)/3 - (10 \Omega)/2

Simplifying: Divide by 2 and \( V/\Omega = A \), gives

\[ \frac{10}{3} - \frac{5}{2} = 3 \text{ A} \]
Example 5 (Cont.) Three independent equations can be solved for $I_1$, $I_2$, and $I_3$.

(1) $I_2 + I_3 = I_1$

(2) $5I_1 + 10I_2 = 12 \text{ A}$

(3) $10I_3 - 5I_2 = 3 \text{ A}$

Substitute Eq. (1) for $I_1$ in (2):

$5(I_2 + I_3) + 10I_3 = 12 \text{ A}$

Simplifying gives:

$5I_2 + 15I_3 = 12 \text{ A}$
Example 5 (Cont.) Three independent equations can be solved.

(1) \[ I_2 + I_3 = I_1 \]

(2) \[ 5I_1 + 10I_2 = 12 A \]

(3) \[ 10I_3 - 5I_2 = 3 A \]

(4) \[ 15I_3 + 5I_2 = 12 A \]

Eliminate \( I_2 \) by adding equations above right:

\[
\begin{align*}
10I_3 - 5I_2 &= 3 A \\
15I_3 + 5I_2 &= 12 A \\
\hline
25I_3 &= 15 A \\
\end{align*}
\]

Putting \( I_3 = 0.600 \) A in (3) gives:

\[ 10(0.6 A) - 5I_2 = 3 A \]

\[ I_2 = 0.600 \text{ A} \]

Then from (1):

\[ I_1 = 1.20 \text{ A} \]
Summary of Formulas:

Rules for a simple, single loop circuit containing a source of emf and resistors.

**Resistance Rule:** \( R_e = \Sigma R \)

**Current:** \( I = \frac{\Sigma \mathcal{E}}{\Sigma R} \)

**Voltage Rule:** \( \Sigma \mathcal{E} = \Sigma I R \)
Summary (Cont.)

For resistors connected in series:

\[ R_e = R_1 + R_2 + R_3 \]

For series connections:

\[ I = I_1 = I_2 = I_3 \]
\[ V_T = V_1 + V_2 + V_3 \]
Resistors connected in parallel:

For parallel connections:

\[ V = V_1 = V_2 = V_3 \]

\[ I_T = I_1 + I_2 + I_3 \]

\[ \frac{1}{R_e} = \sum_{i=1}^{N} \frac{1}{R_i} \]

\[ R_e = \frac{R_1 R_2}{R_1 + R_2} \]
Summary Kirchoff’s Laws

**Kirchoff’s first law:** The sum of the currents entering a junction is equal to the sum of the currents leaving that junction.

\[ \sum I \text{ (enter)} = \sum I \text{ (leaving)} \]

**Kirchoff’s second law:** The sum of the emf’s around any closed loop must equal the sum of the IR drops around that same loop.

\[ \sum \mathcal{E} = \sum IR \]