Chapter 3A. Measurement and Significant Figures

A PowerPoint Presentation by

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PARCS is an atomic-clock mission scheduled to fly on the International Space Station (ISS) in 2008. The mission, funded by NASA, involves a laser-cooled cesium atomic clock to improve the accuracy of timekeeping on earth.
Objectives: After completing this module, you should be able to:

- Name and give the SI units of the seven fundamental quantities.
- Write the base units for mass, length, and time in SI and USCU units.
- Convert one unit to another for the same quantity when given necessary definitions.
- Discuss and apply conventions for significant digits and precision of measurements.
A physical quantity is a quantifiable or assignable property ascribed to a particular phenomenon, body, or substance.
A unit is a particular physical quantity with which other quantities of the same kind are compared in order to express their value.

A meter is an established unit for measuring length.

Based on definition, we say the diameter is 0.12 m or 12 centimeters.
One **meter** is the length of path traveled by a light wave in a vacuum in a time interval of $1/299,792,458$ seconds.
The **kilogram** is the unit of **mass** - it is equal to the mass of the international prototype of the kilogram.

This standard is the only one that requires comparison to an artifact for its validity. A copy of the standard is kept by the International Bureau of Weights and Measures.
The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

Cesium Fountain Atomic Clock: The primary time and frequency standard for the USA (NIST)
# Seven Fundamental Units


<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Meter</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>Kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>Second</td>
<td>S</td>
</tr>
<tr>
<td>Electric Current</td>
<td>Ampere</td>
<td>A</td>
</tr>
<tr>
<td>Temperature</td>
<td>Kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela</td>
<td>cd</td>
</tr>
<tr>
<td>Amount of Substance</td>
<td>Mole</td>
<td>mol</td>
</tr>
</tbody>
</table>
Systems of Units

**SI System:** The international system of units established by the International Committee on Weights and Measures. Such units are based on strict definitions and are the only official units for physical quantities.

**US Customary Units (USCU):** Older units still in common use by the United States, but definitions must be based on SI units.
In *mechanics* we use only three fundamental quantities: mass, length, and time. An additional quantity, force, is derived from these three.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>SI unit</th>
<th>USCS unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kilogram (kg)</td>
<td>slug (slug)</td>
</tr>
<tr>
<td>Length</td>
<td>meter (m)</td>
<td>foot (ft)</td>
</tr>
<tr>
<td>Time</td>
<td>second (s)</td>
<td>second (s)</td>
</tr>
<tr>
<td>Force</td>
<td>newton (N)</td>
<td>pound (lb)</td>
</tr>
</tbody>
</table>
Procedure for Converting Units

1. Write down quantity to be converted.

2. Define each unit in terms of desired unit.

3. For each definition, form two conversion factors, one being the reciprocal of the other.

4. Multiply the quantity to be converted by those factors that will cancel all but the desired units.
**Example 1:** Convert 12 in. to centimeters given that 1 in. = 2.54 cm.

Step 1: Write down quantity to be converted.  
12 in.

Step 2. Define each unit in terms of desired unit.  
1 in. = 2.54 cm

Step 3. For each definition, form two conversion factors, one being the reciprocal of the other.

\[
\begin{align*}
\frac{1 \text{ in.}}{2.54 \text{ cm}} & \quad \text{and} \quad \frac{2.54 \text{ cm}}{1 \text{ in}}
\end{align*}
\]
Example 1 (Cont.): Convert 12 in. to centimeters given that 1 in. = 2.54 cm.

From Step 3. \[
\frac{1 \text{ in.}}{2.54 \text{ cm}} \quad \text{or} \quad \frac{2.54 \text{ cm}}{1 \text{ in.}}
\]

Step 4. Multiply by those factors that will cancel all but the desired units. Treat unit symbols algebraically.

Wrong Choice!

\[
12 \text{ in.} \left( \frac{1 \text{ in.}}{2.54 \text{ cm}} \right) = 4.72 \text{ in.}^2 \text{ cm}
\]

Correct Answer!

\[
12 \text{ in.} \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right) = 30.5 \text{ cm}
\]
**Example 2:** Convert $60 \text{ mi/h}$ to units of $\text{km/s}$ given $1 \text{ mi.} = 5280 \text{ ft}$ and $1 \text{ h} = 3600 \text{ s}$.

**Step 1:** Write down quantity to be converted.

$$60 \frac{\text{mi}}{\text{h}}$$

**Note:** Write units so that numerators and denominators of fractions are clear.

**Step 2.** Define each unit in terms of desired units.

$$1 \text{ mi.} = 5280 \text{ ft}$$

$$1 \text{ h} = 3600 \text{ s}$$
Ex. 2 (Cont): Convert 60 mi/h to units of km/s given that 1 mi. = 5280 ft and 1 h = 3600 s.

Step 3. For each definition, form 2 conversion factors, one being the reciprocal of the other.

\[
\begin{align*}
1 \text{ mi} &= 5280 \text{ ft} \\
1 \text{ h} &= 3600 \text{ s}
\end{align*}
\]

\[
\begin{align*}
\frac{1 \text{ mi}}{5280 \text{ ft}} & \quad \text{or} \quad \frac{5280 \text{ ft}}{1 \text{ mi}} \\
\frac{1 \text{ h}}{3600 \text{ s}} & \quad \text{or} \quad \frac{3600 \text{ s}}{1 \text{ h}}
\end{align*}
\]

Step 3, shown here for clarity, can really be done mentally and need not be written down.
Ex. 2 (Cont): Convert 60 mi/h to units of ft/s given that 1 mi. = 5280 ft and 1 h = 3600 s.

Step 4. Choose Factors to cancel non-desired units.

\[
60 \frac{\text{mi}}{\text{h}} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 88.0 \text{ m/s}
\]

Treating unit conversions algebraically helps to see if a definition is to be used as a multiplier or as a divider.
Uncertainty of Measurement

All measurements are assumed to be approximate with the last digit estimated.

The length in “cm” here is written as: 1.43 cm

The last digit “3” is estimated as 0.3 of the interval between 3 and 4.
Estimated Measurements (Cont.)

Length = 1.43 cm

The last digit is estimated, but is significant. It tells us the actual length is between 1.40 cm and 1.50. It would not be possible to estimate yet another digit, such as 1.436.

This measurement of length can be given in three significant digits—the last is estimated.
Significant Digits and Numbers

When writing numbers, zeros used ONLY to help in locating the decimal point are NOT significant—others are. See examples.

<table>
<thead>
<tr>
<th>Value</th>
<th>Significant Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0062 cm</td>
<td>2</td>
</tr>
<tr>
<td>4.0500 cm</td>
<td>5</td>
</tr>
<tr>
<td>0.1061 cm</td>
<td>4</td>
</tr>
<tr>
<td>50.0 cm</td>
<td>3</td>
</tr>
<tr>
<td>50,600 cm</td>
<td>3</td>
</tr>
</tbody>
</table>
Rule 1. When approximate numbers are multiplied or divided, the number of significant digits in the final answer is the same as the number of significant digits in the least accurate of the factors.

Example: \[ P = \frac{45 \text{ N}}{(3.22 \text{ m})(2.005 \text{ m})} = 6.97015 \text{ N/m}^2 \]

Least significant factor (45) has only two (2) digits so only two are justified in the answer.

The appropriate way to write the answer is: \[ P = 7.0 \text{ N/m}^2 \]
Rule 2. When approximate numbers are added or subtracted, the number of significant digits should equal the smallest number of decimal places of any term in the sum or difference.

Ex: $9.65\text{ cm} + 8.4\text{ cm} - 2.89\text{ cm} = 15.16\text{ cm}$

Note that the least precise measure is 8.4 cm. Thus, answer must be to nearest tenth of cm even though it requires 3 significant digits.

The appropriate way to write the answer is: $15.2\text{ cm}$
Example 3. Find the area of a metal plate that is 95.7 cm by 32 cm.

\[ A = LW = (8.71 \text{ cm})(3.2 \text{ cm}) = 27.872 \text{ cm}^2 \]

Only 2 digits justified:

\[ A = 28 \text{ cm}^2 \]

Example 4. Find the perimeter of the plate that is 95.7 cm long and 32 cm wide.

\[ p = 8.71 \text{ cm} + 3.2 \text{ cm} + 8.71 \text{ cm} + 3.2 \text{ cm} \]

Ans. to tenth of cm:

\[ p = 23.8 \text{ cm} \]
Rounding Numbers

Remember that significant figures apply to your reported result. Rounding off your numbers in the process can lead to errors.

Rule: Always retain at least one more significant figure in your calculations than the number you are entitled to report in the result.

With calculators, it is usually easier to just keep all digits until you report the result.
Rules for Rounding Numbers

Rule 1. If the remainder beyond the last digit to be reported is less than 5, drop the last digit.

Rule 2. If the remainder is greater than 5, increase the final digit by 1.

Rule 3. To prevent rounding bias, if the remainder is exactly 5, then round the last digit to the closest even number.
Examples

Rule 1. If the remainder *beyond the last digit* to be reported is less than 5, drop the last digit.

Round the following to 3 significant figures:

<table>
<thead>
<tr>
<th>Number</th>
<th>Becomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.99499</td>
<td>becomes 4.99</td>
</tr>
<tr>
<td>0.09403</td>
<td>becomes 0.0940</td>
</tr>
<tr>
<td>95,632</td>
<td>becomes 95,600</td>
</tr>
<tr>
<td>0.02032</td>
<td>becomes 0.0203</td>
</tr>
</tbody>
</table>
Examples

Rule 2. If the remainder is greater than 5, increase the final digit by 1.

Round the following to 3 significant figures:

- 2.3452 becomes 2.35
- 0.08757 becomes 0.0876
- 23,650.01 becomes 23,700
- 4.99502 becomes 5.00
Rule 3. To prevent rounding bias, if the remainder is exactly 5, then round the last digit to the closest even number.

Round the following to 3 significant figures:

- $3.77500$ becomes $3.78$
- $0.024450$ becomes $0.0244$
- $96,6500$ becomes $96,600$
- $5.09500$ becomes $5.10$
Classroom work and laboratory work should be treated differently.

In class, the uncertainties in quantities are not usually known. Round to 3 significant figures in most cases.

In lab, we know the limitations of the measurements. We must not keep digits that are not justified.
Classroom Example: A car traveling initially at 46 m/s undergoes constant acceleration of 2 m/s\(^2\) for a time of 4.3 s. Find total displacement, given formula.

\[ x = v_0 t + \frac{1}{2} at^2 \]

\[ = (46 \text{ m/s})(4.3 \text{ s}) + \frac{1}{2} (2 \text{ m/s}^2)(4.3 \text{ s})^2 \]

\[ = 197.8 \text{ m} + 18.48 \text{ m} = 216.29 \text{ m} \]

For class work, we assume all given info is accurate to 3 significant figures.

\[ X = 217 \text{ m} \]
**Laboratory Example:** The length of a sheet of metal is measured as 233.3 mm and the width is 9.3 mm. Find area.

Note that the precision of each measure is to the nearest tenth of a millimeter. However, the length has four significant digits and the width has only three.

How many significant digits are in the product of length and width (area)?

Two (9.3 has least significant digits).
Lab Example (Cont.): The length of a sheet of metal is measured as 233.3 mm and the width is 9.3 mm. Find area.

Area = LW = (233.3 mm)(9.3 mm)

Area = 2169.69 mm²

But we are entitled to only two significant digits. Therefore, the answer becomes:

Area = 2200 mm²
Lab Example (Cont.): Find perimeter of sheet of metal measured $L = 233.3$ mm and $W = 9.3$ mm. (Addition Rule)

\[ p = 233.3 \text{ mm} + 9.3 \text{ mm} + 233.3 \text{ mm} + 9.3 \text{ mm} \]

\[ p = 485.2 \text{ mm} \]

Note: The result has more significant digits than the width factor in this case.
Scientific notation provides a short-hand method for expressing very small and very large numbers.

Examples:

- 93,000,000 mi = 9.30 x 10^7 mi
- 0.00457 m = 4.57 x 10^{-3} m
- \( v = \frac{876 \text{ m}}{0.00370 \text{ s}} = 8.76 \times 10^2 \text{ m/s} \)
- \( v = 3.24 \times 10^5 \text{ m/s} \)
Scientific Notation and Significant Figures

With **Scientific notation** one can easily keep track of significant digits by using only those digits that are necessary in the **mantissa** and letting the **power of ten** locate the decimal.

**Example.** Express the number 0.0006798 m, accurate to three significant digits.

Mantissa $\times 10^{-4}$ m $\rightarrow$ 6.80 $\times 10^{-4}$ m

The “0” is significant—the last digit in doubt.
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Classroom work and lab work should be treated differently unless told otherwise.

In the classroom, we assume all given info is accurate to 3 significant figures.

In lab, the number of significant figures will depend on limitations of the instruments.
Conclusion of Measurement
Significant Digits Module