SIMPLE MACHINES are used to perform a variety of tasks with considerable efficiency. In this example, a system of gears, pulleys, and levers function to produce accurate time measurements.
Objectives: After completing this module, you should be able to:

- Describe a simple machine in general terms and apply the concepts of efficiency, energy conservation, work, and power.
- Distinguish by definition and example between the concepts of the ideal and actual mechanical advantages.
- Describe and apply formulas for the mechanical advantage and efficiency of the following devices: levers, inclined planes, wedges, gears, pulley systems, wheel and axle, screw jacks, and the belt drive.
In a simple machine, input work is done by the application of a single force, and the machine does output work by means of a single force.

Conservation of energy demands that the work input be equal to the sum of the work output and the heat lost to friction.
A Simple Machine (Cont.)

Input work = output work + work against friction

**Efficiency** $e$ is defined as the ratio of work output to work input.

\[
e = \frac{\text{Work output}}{\text{Work input}} = \frac{F_{\text{out}} s_{\text{out}}}{F_{\text{in}} s_{\text{in}}}
\]

\[
W_{\text{in}} = F_{\text{in}} s_{\text{in}}
\]

\[
W_{\text{out}} = F_{\text{out}} s_{\text{out}}
\]
Example 1. The efficiency of a simple machine is 80% and a 400-N weight is lifted a vertical height of 2 m. If an input force of 20 N is required, what distance must be covered by the input force?

The advantage is a reduced input force, but it is at the expense of distance. The input force must move a greater distance.

\[
S_{in} = \frac{(400 \text{ N})(2 \text{ m})}{(0.80)(20 \text{ N})}
\]

\[
S_{in} = 5.0 \text{ m}
\]
Power and Efficiency

Since power is work per unit time, we may write

\[ P = \frac{\text{Work}}{t} \quad \text{or} \quad \text{Work} = Pt \]

\[ e = \frac{W_{\text{out}}}{W_{\text{in}}} = \frac{P_0}{P_i} \]

Efficiency is the ratio of the power output to the power input.

\[ e = \frac{\text{Power out}}{\text{Power in}} = \frac{P_0}{P_i} \]
Example 2. A 12-hp winch motor lifts a 900-lb load to a height of 8 ft. What is the output power in ft·lb/s if the winch is 95% efficient?

First we must find the power output, $P_o$:

\[ e = \frac{P_0}{P_i} \]

\[ P_0 = eP_i \]

\[ P_o = (0.95)(12 \text{ hp}) = 11.4 \text{ hp} \]

(1 hp = 550 ft/s): \[ P_o = (11.4 \text{ hp}) \left( \frac{550 \text{ ft} \cdot \text{lb}/s}{1 \text{ hp}} \right) = 6600 \text{ ft} \cdot \text{lb}/s \]

\[ P_o = 6270 \text{ ft} \cdot \text{lb}/s \]
Ex. 2 (cont.) A 12-hp winch motor lifts a 900 lb load to a height of 8 ft. How much time is required if the winch is 95% efficient?

We just found that

\[
P_o = 6270 \text{ W}
\]

\[
P_o = \frac{\text{Work out}}{t} = \frac{F_o s_o}{t}
\]

Now we solve for \( t \):

\[
t = \frac{F_o s_o}{P_o} = \frac{(900 \text{ lb})(8 \text{ ft})}{6270}
\]

Time required: \( t = 1.15 \text{ s} \)
Actual Mechanical Advantage

The actual mechanical advantage, $M_A$, is the ratio of $F_o$ to $F_i$.

$$M_A = \frac{\text{output force}}{\text{input force}} = \frac{F_o}{F_i}$$

For example, if an input force of 40 N lifts an 80 N weight, the actual mechanical advantage is:

$$M_A = \frac{80 \text{ N}}{40 \text{ N}} = 2.0$$
An Ideal Machine

Conservation of energy demands that:

\[ \text{Input work} = \text{output work} + \text{work against friction} \]

An ideal or perfect machine is 100% efficient and \((Work)_f = 0\), so that

\[ F_i s_i = F_o s_o \]

The ratio \(s/s_o\) is the ideal mechanical advantage.
Ideal Mechanical Advantage

The ideal mechanical advantage, \( M_I \), is the ratio of \( s_{in} \) to \( s_{out} \).

\[
M_A = \frac{\text{in distance}}{\text{out distance}} = \frac{s_i}{s_o}
\]

For example, if an input force moves a distance of 6 m while the output force moves 2 m, the ideal mechanical advantage is:

\[
M_I = \frac{6 \text{ m}}{2 \text{ m}} = 3.0
\]
Efficiency for an Ideal Engine

For 100% efficiency $M_A = M_I$. In other words, in the absence of friction, the machine IS an ideal machine and $e = 1$.

**IDEAL EXAMPLE:**

\[
M_A = \frac{F_o}{F_i} = \frac{80 \text{ N}}{20 \text{ N}} = 4
\]

\[
M_I = \frac{s_i}{s_o} = \frac{8 \text{ m}}{2 \text{ m}} = 4
\]

\[
e = \frac{M_A}{M_I} = 1.0
\]
Efficiency for an Actual Engine

The actual efficiency is always less than the ideal efficiency because friction always exits. The efficiency is still equal to the ratio \( \frac{M_A}{M_i} \).

\[
e = \frac{M_A}{M_i}
\]

In our previous example, the ideal mechanical advantage was equal to 4. If the engine was only 50% efficient, the actual mechanical advantage would be 0.5(4) or 2. Then 160 N (instead of 80 N) would be needed to lift the 400-N weight.
A **lever** shown here consists of input and output forces at different distances from a fulcrum.

The input torque $F_i r_i$ is equal to the output torque $F_o r_o$.

$$F_i r_i = F_o r_o$$

The actual mechanical advantage is, therefore:

$$M_A = \frac{F_o}{F_i} = \frac{r_i}{r_o}$$
The Lever

Friction is negligible so that $W_{out} = W_{in}$:

$$F_i s_i = F_s o \text{ or } \frac{F_o}{F_i} = \frac{s_i}{s_o}$$

Note from figure that angles are the same and arc length $s$ is proportional to $r$. Thus, the ideal mechanical advantage is the same as actual.

The **ideal** $M_I$ is:

$$M_I = \frac{F_o}{F_i} = \frac{r_i}{r_o} \text{ and } M_I = M_A$$
Example 3. A 1-m metal lever is used to lift a 800-N rock. What force is required at the left end if the fulcrum is placed 20 cm from the rock?

1. Draw and label sketch:

2. List given info:

   \[ F_o = 700 \text{ N}; \quad r_2 = 20 \text{ cm} \]
   
   \[ r_1 = 100 \text{ cm} - 20 \text{ cm} = 80 \text{ cm} \]

3. To find \( F_i \) we recall the definition of \( M_i \):

   \[ M_i = \frac{r_i}{r_o} \quad \text{and} \quad M_i = \frac{80 \text{ cm}}{20 \text{ cm}} = 4; \quad \text{For lever:} \quad M_A = M_i \]

   Thus,

   \[ M_A = \frac{F_o}{F_i} = 4 \quad \text{and} \quad F_i = \frac{800 \text{ N}}{4} = 200 \text{ N} \]
Other Examples of Levers
Wheel and Axel:

Application of Lever Principle:

With no friction \( M_I = M_A \) and

\[
M_A = \frac{F_o}{F_i} = \frac{r_i}{r_o}
\]

For example, if \( R = 30 \text{ cm} \) and \( r = 10 \text{ cm} \), an input force of only 100 N will lift a 300-N weight!

If the smaller radius is 1/3 of the larger radius, your output force is 3 times the input force.
Single Fixed Pulleys

Single fixed pulleys serve only to change the direction of the input force. See examples:

\[ F_{in} = F_{out} \]
A free-body diagram shows an actual mechanical advantage of $M_A = 2$ for a single moveable pulley. Note that the rope moves a distance of 2 m while the weight is lifted only 1 m. 

$$M_I = \frac{S_{in}}{S_{out}} = 2$$
The lifter must pull 4 m of rope in order to lift the weight 1 m.
A **belt drive** is a device used to transmit torque from one place to another. The actual mechanical advantage is the ratio of the torques.

The actual mechanical advantage is given by the ratio of the output torque to the input torque:

\[
M_A = \frac{\text{output torque}}{\text{input torque}} = \frac{\tau_o}{\tau_i}
\]

Since torque is defined as \( Fr \), the ideal advantage is:

\[
M_I = M_A = \frac{F_o r_o}{F_i r_i}
\]

Belt Drive: \( M_I = \frac{r_o}{r_i} = \frac{D_o}{D_i} \)
Angular Speed Ratio

The mechanical advantage of a belt drive can also be expressed in terms of the diameters $D$ or in terms of the angular speeds $\omega$.

\[
M_I = \frac{D_o}{D_i} = \frac{\omega_i}{\omega_o}
\]

Belt Drive:

Note that the smaller pulley diameter always has the greater rotational speed.

Speed ratio: $\frac{\omega_i}{\omega_o}$
Example 4. A 200 N·m torque is applied to an input pulley 12 cm in diameter. (a) What should be the diameter of the output pulley to give an ideal mechanical advantage of 4? (b) What is the belt tension?

To find $D_o$, we use the fact that

$$M_I = \frac{D_o}{D_i} = 4; \quad D_o = 4D_i$$

$$D_o = 4(12 \text{ cm}) = 48 \text{ cm}$$

Now, $\tau_i = F_i r_i$ and $r_i = D_i/2$. Belt tension is $F_i$ and $r_i$ is equal to $1/2D_i = 0.06 \text{ m}$. 

$$\tau_i = F_i r_i = 200 \text{ N} \cdot \text{m}$$

$$F_i = \frac{200 \text{ N} \cdot \text{m}}{0.06 \text{ m}} = 3330 \text{ N}$$
Gears

Mechanical advantage of gears is similar to that for belt drive:

\[ M_I = \frac{D_o}{D_i} = \frac{N_o}{N_i} \]

In this case, \( D_o \) is the diameter of the driving gear and \( D_i \) is diameter of the driven gear. \( N \) is the number of teeth.

If 200 teeth are in the input (driving) gear, and 100 teeth in the output (driven) gear, the mechanical advantage is \( \frac{1}{2} \).
Example 5. The driving gear on a bicycle has 40 teeth and the wheel gear has only 20 teeth. What is the mechanical advantage? If the driving gear makes 60 rev/min, what is the rotational speed of the rear wheel?

\[ M_I = \frac{N_o}{N_i} = \frac{22}{44} ; \quad M_I = 0.5 \]

Remember that the angular speed ratio is opposite to the gear ratio.

\[ M_I = \frac{N_o}{N_i} = \frac{\omega_i}{\omega_o} ; \quad \frac{\omega_i}{\omega_o} = \frac{1}{2} \]

\[ \omega_o = 2\omega_i = 2(60 \text{ rpm}) \]

\begin{align*}
N_o &= 20 \\
N_i &= 40 \\
\omega_o &= 120 \text{ rpm}
\end{align*}
The Inclined Plane

Ideal Mechanical Advantage

\[ M_I = \frac{slope}{height} = \frac{s_i}{s_o} \]

Actual Advantage:

\[ M_A = \frac{W}{F_i} \]

Because of friction, the actual mechanical advantage \( M_A \) of an inclined plane is usually much less than the ideal mechanical advantage \( M_I \).
Example 6. An inclined plane has a slope of 8 m and a height of 2 m. What is the ideal mechanical advantage and what is the necessary input force needed to push a 400-N weight up the incline? The efficiency is 60 percent.

\[
M_I = \frac{s_i}{s_o} = \frac{8 \text{ m}}{2 \text{ m}}; \quad M_I = 4
\]

\[
e = \frac{M_A}{M_I}; \quad M_A = eM_I = (0.60)(4)
\]

\[
M_A = 2.4 = \frac{F_o}{F_i} \quad F_i = \frac{F_o}{2.4} = \frac{400 \text{ N}}{2.4} \quad F_i = 167 \text{ N}
\]
The Screw Jack

An application of the inclined plane:

Input distance: \( s_i = 2\pi R \)

Output distance: \( s_o = \rho \)

\[ M_I = \frac{2\pi R}{\rho} \]

Due to friction, the screw jack is an **inefficient** machine with an actual mechanical advantage significantly **less** than the ideal advantage.
Efficiency $e$ is defined as the ratio of work output to work input.

\[ e = \frac{\text{Work output}}{\text{Work input}} \]

Efficiency is the ratio of the power output to the power input.

\[ e = \frac{\text{Power out}}{\text{Power in}} = \frac{P_0}{P_i} \]
Summary

The actual mechanical advantage, $M_A$, is the ratio of $F_o$ to $F_i$.

$$M_A = \frac{\text{output force}}{\text{input force}} = \frac{F_o}{F_i}$$

The ideal mechanical advantage, $M_I$, is the ratio of $s_{in}$ to $s_{out}$.

$$M_A = \frac{\text{in distance}}{\text{out distance}} = \frac{s_i}{s_o}$$

Efficiency

$$e = \frac{P_{out}}{P_{in}}$$
Summary (Cont.)

The actual mechanical advantage for a lever:

\[ M_A = \frac{F_o}{F_i} = \frac{r_i}{r_o} \]

Application of lever principle:

With no friction \( M_f = M_A \)

For Wheel and axel:

\[ M_A = \frac{F_o}{F_i} = \frac{r_i}{r_o} \]
Belt Drive: $M_I = \frac{D_o}{D_i} = \frac{\omega_i}{\omega_o}$

$M_A = \frac{\text{output torque}}{\text{input torque}} = \frac{\tau_o}{\tau_i}$

Belt Drive: $M_I = \frac{r_o}{r_i} = \frac{D_o}{D_i}$
Summary

Gears: \( M_I = \frac{D_o}{D_i} = \frac{N_o}{N_i} \)

The Inclined Plane

Ideal Mechanical Advantage

\[ M_I = \frac{slope}{height} = \frac{s_i}{s_o} \]

Actual Advantage: \( M_A = \frac{W}{F_i} \)
Summary (Cont.)

An application of the inclined plane:

Input distance: \( s_i = 2\pi R \)

Output distance: \( s_o = p \)

\[
M_l = \frac{2\pi R}{p}
\]

Screw Jack

\[
M_l = \frac{s_i}{s_o} = \frac{2\pi R}{p}
\]
CONCLUSION: Chapter 12
Simple Machines