Chapter 22A – Sound Waves

A PowerPoint Presentation by

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Objectives: After completion of this module, you should be able to:

- Define *sound* and solve problems relating to its velocity in solids, liquids, and gases.
- Use boundary conditions to apply concepts relating to *frequencies* in *open* and *closed* pipes.
Definition of Sound

**Sound** is a longitudinal mechanical wave that travels through an elastic medium.

Many things vibrate in air, producing a sound wave.

Source of sound: a tuning fork.
Is there sound in the forest when a tree falls?

Sound is a physical disturbance in an elastic medium.

Based on our definition, there IS sound in the forest, whether a human is there to hear it or not!

The elastic medium (air) is required!
Sound Requires a Medium

The sound of a ringing bell diminishes as air leaves the jar. No sound exists without air molecules.
Graphing a Sound Wave.

The sinusoidal variation of pressure with distance is a useful way to represent a sound wave graphically. Note the wavelengths $\lambda$ defined by the figure.
Factors That Determine the Speed of Sound.

Longitudinal mechanical waves (sound) have a wave speed dependent on elasticity factors and density factors. Consider the following examples:

A denser medium has greater inertia resulting in lower wave speeds.

A medium that is more elastic springs back quicker and results in faster speeds.
Speeds for different media

\[ v = \sqrt{\frac{Y}{\rho}} \]

Metal rod

Young’s modulus, \( Y \)

Metal density, \( \rho \)

\[ v = \sqrt{\frac{B + \frac{4}{3}S}{\rho}} \]

Extended Solid

Bulk modulus, \( B \)

Shear modulus, \( S \)

Density, \( \rho \)

\[ v = \sqrt{\frac{B}{\rho}} \]

Fluid

Bulk modulus, \( B \)

Fluid density, \( \rho \)
Example 1: Find the speed of sound in a steel rod.

\[ \rho = 7800 \text{ kg/m}^3 \]
\[ Y = 2.07 \times 10^{11} \text{ Pa} \]

\[ v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.07 \times 10^{11} \text{ Pa}}{7800 \text{ kg/m}^3}} \]

\[ v = 5150 \text{ m/s} \]
Speed of Sound in Air

For the speed of sound in air, we find that:

\[ B = \gamma P \quad \text{and} \quad \frac{P}{\rho} = \frac{RT}{M} \]

\[ \gamma = 1.4 \text{ for air} \]
\[ R = 8.34 \text{ J/kg mol} \]
\[ M = 29 \text{ kg/mol} \]

\[ v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} \]
\[ v = \sqrt{\frac{\gamma RT}{M}} \]

Note: Sound velocity increases with temperature T.
Example 2: What is the speed of sound in air when the temperature is 20°C?

Given: \( \gamma = 1.4; \quad R = 8.314 \text{ J/mol K}; \quad M = 29 \text{ g/mol} \)

\[
T = 20^0 + 273^0 = 293 \text{ K} \quad M = 29 \times 10^{-3} \text{ kg/mol}
\]

\[
\nu = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.314 \text{ J/mol K})(293 \text{ K})}{29 \times 10^{-3} \text{kg/mol}}}
\]

\[
\nu = 343 \text{ m/s}
\]
Dependence on Temperature

Note: $v$ depends on Absolute T:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Now $v$ at 273 K is 331 m/s. $\gamma$, $R$, $M$ do not change, so a simpler formula might be:

$$v = 331 \text{ m/s} + \left(0.6 \frac{\text{m/s}}{\text{C}^0}\right)t_c$$

Alternatively, there is the approximation using °C:
Example 3: What is the velocity of sound in air on a day when the temperature is equal to 27°C?

Solution 1: \( v = 331 \text{ m/s} \sqrt{\frac{T}{273 \text{ K}}} \)

\[
T = 27^0 + 273^0 = 300 \text{ K}; \quad v = 331 \text{ m/s} \sqrt{\frac{300 \text{ K}}{273 \text{ K}}}
\]

\[ v = 347 \text{ m/s} \]

Solution 2: \( v = 331 \text{ m/s} + (0.6)(27^0 \text{C}); \quad v = 347 \text{ m/s} \)
Musical Instruments

Sound waves in air are produced by the vibrations of a violin string. Characteristic frequencies are based on the length, mass, and tension of the wire.
Vibrating Air Columns

Just as for a vibrating string, there are characteristic wavelengths and frequencies for longitudinal sound waves. Boundary conditions apply for pipes:

The open end of a pipe must be a displacement antinode A.

The closed end of a pipe must be a displacement node N.
Velocity and Wave Frequency.

The period T is the time to move a distance of one wavelength. Therefore, the wave speed is:

\[ v = \frac{\lambda}{T} \text{ but } T = \frac{1}{f} \text{ so } v = f \lambda \]

The frequency \( f \) is in s\(^{-1}\) or hertz (Hz).

The velocity of any wave is the product of the frequency and the wavelength:

\[ v = f \lambda \]
\[ f = \frac{v}{\lambda} \]
Possible Waves for Open Pipe

Fundamental, $n = 1$

1st Overtone, $n = 2$

2nd Overtone, $n = 3$

3rd Overtone, $n = 4$

All harmonics are possible for open pipes:

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, 4 \ldots$$
Characteristic Frequencies for an Open Pipe.

Fundamental, $n = 1$

1st Overtone, $n = 2$

2nd Overtone, $n = 3$

3rd Overtone, $n = 4$

All harmonics are possible for open pipes:

\[ f_n = \frac{nv}{2L} \quad n = 1, 2, 3, 4 \ldots \]
Possible Waves for Closed Pipe.

Fundamental, \( n = 1 \)

1st Overtone, \( n = 3 \)

2nd Overtone, \( n = 5 \)

3rd Overtone, \( n = 7 \)

Only the odd harmonics are allowed:

\[
\lambda_n = \frac{4L}{n} \quad n = 1, 3, 5, 7 \ldots
\]
Possible Waves for Closed Pipe.

Fundamental, $n = 1$

$1$st Overtone, $n = 3$

$2$nd Overtone, $n = 5$

$3$rd Overtone, $n = 7$

Only the odd harmonics are allowed:

$$f_n = \frac{nv}{4L} \quad n = 1, 3, 5, 7 \ldots$$
Example 4. What length of closed pipe is needed to resonate with a fundamental frequency of 256 Hz? What is the second overtone? Assume that the velocity of sound is 340 m/s.

\[ f_n = \frac{nv}{4L} \quad n = 1, 3, 5, 7 \ldots \]

\[ f_1 = \frac{(1)v}{4L}; \quad L = \frac{v}{4f_1} = \frac{340 \text{ m/s}}{4(256 \text{ Hz})} \]

\[ L = 33.2 \text{ cm} \]

The second overtone occurs when \( n = 5 \):

\[ f_5 = 5f_1 = 5(256 \text{ Hz}) \]

\[ 2\text{nd Ovt.} = 1280 \text{ Hz} \]
### Summary of Formulas for Speed of Sound

<table>
<thead>
<tr>
<th>Medium</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid rod</td>
<td>[ v = \sqrt{\frac{Y}{\rho}} ]</td>
</tr>
<tr>
<td>Extended Solid</td>
<td>[ v = \sqrt{\frac{B + \frac{4}{3} S}{\rho}} ]</td>
</tr>
<tr>
<td>Liquid</td>
<td>[ v = \sqrt{\frac{B}{\rho}} ]</td>
</tr>
</tbody>
</table>

Sound for any gas:

\[ v = \sqrt{\frac{\gamma RT}{M}} \]

Approximation:

\[ v = 331 \text{ m/s} + \left( 0.6 \frac{\text{m/s}}{C^0} \right) t_c \]
Summary of Formulas (Cont.)

For any wave:

\[ v = f \lambda \]
\[ f = \frac{v}{\lambda} \]

Characteristic frequencies for open and closed pipes:

**OPEN PIPE**

\[ f_n = \frac{nv}{2L} \quad n = 1, 2, 3, 4 \ldots \]

**CLOSED PIPE**

\[ f_n = \frac{nv}{4L} \quad n = 1, 3, 5, 7 \ldots \]
CONCLUSION: Chapter 22
Sound Waves