The **Golden Gate Bridge** provides an excellent example of balanced forces and torques. Engineers must design such structures so that rotational and translational equilibrium is maintained.
Objectives: After completing this module, you should be able to:

- State and describe with examples your understanding of the first and second conditions for equilibrium.
- Write and apply the first and second conditions for equilibrium to the solution of physical problems similar to those in this module.
Translational Equilibrium

The linear speed is **not** changing with time. There is no resultant force and therefore zero acceleration. **Translational equilibrium exists.**
Rotational Equilibrium

Wheel at rest  

\[ \Sigma \tau = 0; \text{ No change in rotation.} \]

Constant rotation

The angular speed is not changing with time. There is no resultant torque and, therefore, zero change in rotational velocity. Rotational equilibrium exists.
Equilibrium

• An object is said to be in equilibrium if and only if there is no resultant force and no resultant torque.

First Condition: \[ \sum F_x = 0; \quad \sum F_y = 0 \]

Second Condition: \[ \sum \tau = 0 \]
Does Equilibrium Exist?

Is the system at left in equilibrium both translationally and rotationally? **YES!**

Observation shows that no part of the system is changing its state of motion.

A sky diver moments after the jump? **No**

A sky diver who reaches terminal speed? **Yes**

A fixed pulley rotating at constant speed? **Yes**
Statics or Total Equilibrium

Statics is the physics that treats objects at rest or objects in constant motion.

In this module, we will review the first condition for equilibrium (treated in Part 5A of these modules); then we will extend our treatment by working with the second condition for equilibrium. Both conditions must be satisfied for true equilibrium.
If all forces act at the same point, then there is no torque to consider and one need only apply the first condition for equilibrium:

\[ \sum F_x = 0; \quad \sum F_y = 0 \]

- Construct free-body diagram.
- Sum forces and set to zero:
- Solve for unknowns.
Review: Free-body Diagrams

- Read problem; draw and label sketch.
- Construct force diagram for each object, vectors at origin of x,y axes.
- Dot in rectangles and label x and y components opposite and adjacent to angles.
- Label all components; choose positive direction.
**Example 1.** Find the tension in ropes A and B.

- Read problem; draw sketch; construct a free-body diagram, indicating components.
- Choose x-axis horizontal and choose right direction as positive (+). There is no motion.
Example 1 (Continued). Find \( A \) and \( B \).

Free-body Diagram:

Note: The components \( B_x \) and \( B_y \) can be found from right triangle trigonometry:

\[
B_x = B \cos 60^0; \quad B_y = B \sin 60^0
\]
Example 1 (Cont.). Find tension in ropes A and B.

- Apply the first condition for equilibrium.

\[ \sum F_x = 0; \quad \sum F_y = 0; \]
**Example 2.** Find tension in ropes A and B.

Recall: \( \sum F_x = 0 \) and \( \sum F_y = 0 \)

\[ \sum F_x = B_x - A_x = 0 \]

\[ \sum F_y = B_y + A_y - 500 \text{ N} = 0 \]

\( W = 500 \text{ N} \)
Example 2 (Cont.) Simplify by rotating axes:

\[ \sum F_x = B - W_x = 0 \]

\[ B = W_x = (500 \text{ N}) \cos 35^\circ \]

\[ B = 410 \text{ N} \]

\[ \sum F_y = A - W_y = 0 \]

\[ A = W_x = (500 \text{ N}) \sin 35^\circ \]

\[ A = 287 \text{ N} \]

Recall that \( W = 500 \text{ N} \)
Total Equilibrium

In general, there are six degrees of freedom (right, left, up, down, ccw, and cw):

\[ \sum F_x = 0 \quad \text{Right} = \text{left} \]
\[ \sum F_x = 0 \quad \text{Up} = \text{down} \]
\[ \sum \tau = 0 \]
\[ \sum \tau_{(ccw)} = \sum \tau_{(ccw)} \]

ccw (+)  

cw (-)
General Procedure:

- Draw free-body diagram and label.
- Choose axis of rotation at point where least information is given.
- Extend line of action for forces, find moment arms, and sum torques about chosen axis:
  \[ \sum \tau = \tau_1 + \tau_2 + \tau_3 + \ldots = 0 \]
- Sum forces and set to zero: \( \sum F_x = 0; \quad \sum F_y = 0 \)
- Solve for unknowns.
**Example 3:** Find the forces exerted by supports A and B. Neglect the weight of the 10-m boom.

**Draw free-body diagram**

**Rotational Equilibrium:**

Choose axis at point of unknown force.

At A for example.
Example 3 (Cont.)

Note: When applying

\[ \Sigma \tau_{(ccw)} = \Sigma \tau_{(cw)} \]

we need only the absolute (positive) magnitudes of each torque.

\[ |\tau (+)| = |\tau (-)| \]

Essentially, we are saying that the torques are balanced about a chosen axis.
Example 3: (Cont.)

Rotational Equilibrium:
\[ \Sigma \tau = \tau_1 + \tau_2 + \tau_3 + \tau_4 = 0 \]

or

\[ \Sigma \tau(\text{ccw}) = \Sigma \tau(\text{cw}) \]

With respect to Axis A:

**CCW Torques:** Forces \( B \) and 40 N.

**CW Torques:** 80 N force.

*Force A is ignored: Neither ccw nor cw*
Example 3 (Cont.)

First: $\Sigma \tau (ccw)$

$\tau_1 = B \ (10 \ m)$

$\tau_2 = (40 \ N) \ (2 \ m)$

$= 80 \ N \cdot m$

Next: $\Sigma \tau (cw)$

$\tau_3 = (80 \ N) \ (7 \ m)$

$= 560 \ N \cdot m$

$\Sigma \tau (ccw) = \Sigma \tau (cw)$

$B(10 \ m) + 80 \ N \cdot m = 560 \ N \cdot m$

$B = 48.0 \ N$
Example 3 (Cont.)

Translational Equilibrium

\[ \sum F_x = 0; \quad \sum F_y = 0 \]

\[ \Sigma F(up) = \Sigma F(down) \]

\[ A + B = 40 \text{ N} + 80 \text{ N} \]

\[ A + B = 120 \text{ N} \]

Recall that \( B = 48.0 \text{ N} \)

\[ A + 48 \text{ N} = 120 \text{ N} \]

\[ A = 72.0 \text{ N} \]
Example 3 (Cont.)

Check answer by summing torques about right end to verify $A = 72.0 \text{ N}$

$\Sigma \tau(\text{ccw}) = \Sigma \tau(\text{cw})$

$(40 \text{ N})(12 \text{ m}) + (80 \text{ N})(3 \text{ m}) = A (10 \text{ m})$

$480 \text{ N} \cdot \text{m} + 240 \text{ N} \cdot \text{m} = A (10 \text{ m})$

$A = 72.0 \text{ N}$
Reminder on Signs:

Absolute values apply for:

\[ \sum F^{\text{up}} = \sum F^{\text{down}} \]

We used absolute (+) values for both UP and DOWN terms.

Instead of: \[ \sum F_y = A + B - 40 \text{ N} - 80 \text{ N} = 0 \]

We wrote: \[ A + B = 40 \text{ N} + 90 \text{ N} \]
Example 4: Find the tension in the rope and the force by the wall on the boom. The 10-m boom weighing 200 N. Rope is 2 m from right end.

For purposes of summing torques, we consider entire weight to act at center of board.
Example 4 (Cont.)

Choose axis of rotation at wall (least information)

\[ \Sigma \tau (ccw): \quad Tr = T (8 \text{ m}) \sin 30^\circ = (4 \text{ m}) T \]

\[ \Sigma \tau (cw): \quad (200 \text{ N})(5 \text{ m}) + (800 \text{ N})(10 \text{ m}) = 9000 \text{ Nm} \]

\[ (4 \text{ m})T = 9000 \text{ N} \cdot \text{m} \]

\[ T = 2250 \text{ N} \]
**Example 4 (Cont.)**

\[ \sum F_{\text{up}} = \sum F_{\text{down}}: \]

\[ T_y + F_y = 200 \text{ N} + 800 \text{ N} \]

\[ F_y = 200 \text{ N} + 800 \text{ N} - T_y; \quad F_y = 1000 \text{ N} - T \sin 30^0 \]

\[ F_y = 1000 \text{ N} - (2250 \text{ N}) \sin 30^0 \]

\[ F_y = -125 \text{ N} \]

\[ \sum F_{\text{right}} = \sum F_{\text{left}}: \]

\[ F_x = T_y = (2250 \text{ N}) \cos 30^0 \]

\[ F_x = 1950 \text{ N} \quad \text{or} \]

\[ F = 1954 \text{ N, 356.3}^0 \]
The center of gravity of an object is the point at which all the weight of an object might be considered as acting for purposes of treating forces and torques that affect the object.

The single support force has line of action that passes through the c. g. in any orientation.
Examples of Center of Gravity

Note: C. of G. is not always inside material.
**Example 5:** Find the center of gravity of the apparatus shown below. Neglect the weight of the connecting rods.

*C. of G.* is point at which a single upward force $F$ will balance the system.

Choose axis at left, then sum torques:

$$
\Sigma \tau(\text{ccw}) = \Sigma \tau(\text{cw})
$$

$$
Fx = (10 \text{ N})(4 \text{ m}) + (5 \text{ N})(10 \text{ m})
$$

$$
Fx = 90.0 \text{ Nm}
$$

$$
\Sigma F(\text{up}) = \Sigma F(\text{down}):
F = 30 \text{ N} + 10 \text{ N} + 5 \text{ N}
$$

$$
(45 \text{ N}) \times = 90 \text{ N}
$$

$$
\boxed{x = 2.00 \text{ m}}
$$
Summary

Conditions for Equilibrium:

An object is said to be in equilibrium if and only if there is no resultant force and no resultant torque.

\[ \Sigma F_x = 0 \]
\[ \Sigma F_y = 0 \]
\[ \Sigma \tau = 0 \]
Summary: Procedure

- Draw free-body diagram and label.
- Choose axis of rotation at point where least information is given.
- Extend line of action for forces, find moment arms, and sum torques about chosen axis:
  \[ \Sigma \tau = \tau_1 + \tau_2 + \tau_3 + \ldots = 0 \]
- Sum forces and set to zero: \[ \Sigma F_x = 0; \quad \Sigma F_y = 0 \]
- Solve for unknowns.
CONCLUSION: Chapter 5B
Rotational Equilibrium