Chapter 30 - Magnetic Fields and Torque

A PowerPoint Presentation by

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Objectives: After completing this module, you should be able to:

- Determine the magnitude and direction of the force on a current-carrying wire in a \(B\)-field.
- Calculate the magnetic torque on a coil or solenoid of area \(A\), turns \(N\), and current \(I\) in a given \(B\)-field.
- Calculate the magnetic field induced at the center of a loop or coil or at the interior of a solenoid.
Recall that the magnetic field $B$ in teslas (T) was defined in terms of the force on a moving charge:

$$F = qvB \sin \theta$$

Magnetic Field Intensity $B$:

$$B = \frac{F}{qv \sin \theta}$$

1 T = $\frac{1 \text{ N}}{\text{C(m/s)}} = \frac{1 \text{ N}}{\text{A} \cdot \text{m}}$
Since a current $I$ is charge $q$ moving through a wire, the magnetic force can be given in terms of current.

Since $v = L/t$, and $I = q/t$, we can rearrange to find:

$$F = q\left(\frac{L}{t}\right)B = \left(\frac{q}{t}\right)LB$$

The force $F$ on a conductor of length $L$ and current $I$ in perpendicular $B$-field:

$$F = IBL$$

Right-hand rule: force $F$ is upward.
Force Depends on Current Angle

Just as for a moving charge, the force on a wire varies with direction.

\[ F = IBL \sin \theta \]

**Example 1.** A wire of length 6 cm makes an angle of 20° with a 3 mT magnetic field. What current is needed to cause an upward force of \( 1.5 \times 10^{-4} \) N?

\[
I = \frac{F}{BL \sin \theta} = \frac{1.5 \times 10^{-4} \text{N}}{(3 \times 10^{-3} \text{T})(0.06 \text{ m}) \sin 20^\circ}
\]

\[ I = 2.44 \text{ A} \]
Consider a loop of area $A = ab$ carrying a current $I$ in a constant $B$ field as shown below.

The right-hand rule shows that the side forces cancel each other and the forces $F_1$ and $F_2$ cause a torque.

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Torque on Current Loop

Recall that torque is product of force and moment arm.

The moment arms for $F_1$ and $F_2$ are:

\[ \frac{a}{2} \sin \theta \]

\[ F_1 = F_2 = IBb \]

\[ \tau_1 = (IBb)(\frac{a}{2} \sin \theta) \]

\[ \tau_2 = (IBb)(\frac{a}{2} \sin \theta) \]

\[ \tau = 2(IBb)(\frac{a}{2} \sin \theta) = IB(ab) \sin \theta \]

In general, for a loop of $N$ turns carrying a current $I$, we have:

\[ \tau = NIBA \sin \theta \]
Example 2: A 200-turn coil of wire has a radius of 20 cm and the normal to the area makes an angle of 30° with a 3 mT B-field. What is the torque on the loop if the current is 3 A?

\[ \tau = NIBA \sin \theta \]

\[ A = \pi R^2 = \pi (0.2 \text{ m})^2 \]

\[ A = 0.126 \text{ m}^2; \quad N = 200 \text{ turns} \]

\[ B = 3 \text{ mT}; \quad \theta = 30^\circ; \quad I = 3 \text{ A} \]

\[ \tau = NIBA \sin \theta = (200)(3 \text{ A})(0.003 \text{ T})(0.126 \text{ m}^2) \sin 30^\circ \]

Resultant torque on loop:

\[ \tau = 0.113 \text{ N\cdot m} \]
When a current $I$ passes through a long straight wire, the magnetic field $B$ is circular as is shown by the pattern of iron filings below and has the indicated direction.

The right-hand thumb rule: Grasp wire with right hand; point thumb in direction of $I$. Fingers wrap wire in direction of the circular B-field.
Calculating B-field for Long Wire

The magnitude of the magnetic field \( B \) at a distance \( r \) from a wire is proportional to current \( I \).

Magnitude of \( B \)-field for current \( I \) at distance \( r \):

\[
B = \frac{\mu_0 I}{2\pi r}
\]

The proportionality constant \( \mu_0 \) is called the permeability of free space:

*Permeability: \( \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \)*
**Example 3:** A long straight wire carries a current of 4 A to the right of page. Find the magnitude and direction of the B-field at a distance of 5 cm above the wire.

\[
B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4 \text{ A})}{2\pi(0.05 \text{ m})}
\]

\[
B = 1.60 \times 10^{-5} \text{ T or 16 } \mu\text{T}
\]

**Right-hand thumb rule:** Fingers point out of paper in direction of B-field.
Example 4: Two parallel wires are separated by 6 cm. Wire 1 carries a current of 4 A and wire 2 carries a current of 6 A in the same direction. What is the resultant $B$-field at the midpoint between the wires?

\[ B = \frac{\mu_0 I}{2\pi r} \]

$B_1$ is positive

$B_2$ is negative

Resultant is vector sum: $B_R = \Sigma B$
Example 4 (Cont.): Find resultant B at midpoint.

\[ B_1 = \frac{(4\pi x 10^{-7} \, \text{Tm} / \text{A})(4 \, \text{A})}{2\pi(0.03 \, \text{m})} = +26.7 \, \mu\text{T} \]

\[ B_2 = \frac{(4\pi x 10^{-7} \, \text{Tm} / \text{A})(6 \, \text{A})}{2\pi(0.03 \, \text{m})} = -40.0 \, \mu\text{T} \]

Resultant is vector sum: \[ B_R = \Sigma B \]

\[ B_R = 26.7 \, \mu\text{T} - 40 \, \mu\text{T} = -13.3 \, \mu\text{T} \]

\[ B_R \text{ is into paper: } B = -13.3 \, \mu\text{T} \]
Force Between Parallel Wires

Recall wire with $I_1$ creates $B_1$ at P:

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Out of paper!

Now suppose another wire with current $I_2$ in same direction is parallel to first wire. Wire 2 experiences force $F_2$ due to $B_1$.

From right-hand rule, what is direction of $F_2$?

Force $F_2$ is Downward

$B_2$, $d$, $I_2$, $I_1$
Parallel Wires (Cont.)

Now start with wire 2. \( I_2 \) creates \( B_2 \) at P:

\[ B_2 = \frac{\mu_0 I_2}{2\pi d} \]

INTO paper!

Now the wire with current \( I_1 \) in same direction is parallel to first wire. Wire 1 experiences force \( F_1 \) due to \( B_2 \).

From right-hand rule, what is direction of \( F_1 \)?

Force \( F_1 \) is Upward
Parallel Wires (Cont.)

We have seen that two parallel wires with currents in the same direction are attracted to each other.

Use right-hand force rule to show that oppositely directed currents repel each other.
Calculating Force on Wires

The field from current in wire 2 is given by:

\[ B_2 = \frac{\mu_0 I_2}{2\pi d} \]

The force \( F_1 \) on wire 1 is:

\[ F_1 = I_1 B_2 L \]

The force per unit length for two wires separated by \( d \) is:

\[ F = \frac{\mu_0 I_1 I_2}{L} \]

The same equation results when considering \( F_2 \) due to \( B_1 \).
Example 5: Two wires 5 cm apart carry currents. The upper wire has 4 A north and the lower wire has 6 A south. What is the mutual force per unit length on the wires?

\[ F = \frac{\mu_0 I_1 I_2}{2\pi d} \]

I_1 = 6 A; I_2 = 4 A; \ d = 0.05 \text{ m}

Right-hand rule applied to either wire shows repulsion.

\[ F = \frac{(4\pi \times 10^{-7} \ T \cdot m/A)(6 \ A)(4 \ A)}{2\pi(0.05 \text{ m})} \]

\[ F = 9.60 \times 10^{-5} \text{ N/m} \]
Magnetic Field in a Current Loop

Right-hand rule shows $B$ field directed out of center.

Single loop: $B = \frac{\mu_0 I}{2R}$

Coil of $N$ loops: $B = \frac{\mu_0 NI}{2R}$
**The Solenoid**

A solenoid consists of many turns $N$ of a wire in shape of a helix. The magnetic $\mathbf{B}$-field is similar to that of a bar magnet. The core can be air or any material.

If the core is air:  $\mu = \mu_0 = 4\pi \times 10^{-7} \text{Tm/A}$

The relative permeability $\mu_r$ uses this value as a comparison.

The relative permeability for a medium ($\mu_r$):

$$\mu_r = \frac{\mu}{\mu_0} \quad \text{or} \quad \mu = \mu_r \mu_0$$
For a solenoid of length $L$, with $N$ turns and current $I$, the $B$-field is given by:

$$B = \frac{\mu NI}{L}$$

Such a $B$-field is called the magnetic induction since it arises or is produced by the current. It applies to the interior of the solenoid, and its direction is given by the right-hand thumb rule applied to any current coil.
Example 6: A solenoid of length 20 cm and 100 turns carries a current of 4 A. The relative permeability of the core is 12,000. What is the magnetic induction of the coil?

\[ I = 4 \text{ A}; \quad N = 100 \text{ turns} \]
\[ L = 0.20 \text{ m}; \quad \mu = \mu_r \mu_0 \]
\[ \mu = (12000)(4\pi \times 10^{-7} \frac{T\cdot m}{A}) \]
\[ \mu = 0.0151 \frac{T\cdot m}{A} \]
\[ B = \frac{(0.0151 \frac{T\cdot m}{A})(100)(4 \text{ A})}{0.200 \text{ m}} \]

\[ B = 30.2 \text{ T} \]

A ferromagnetic core can significantly increase the B-field!
Summary of Formulas

The force $F$ on a wire carrying current $I$ in a given B-field.

$$F = IBL \sin \theta$$

The torque on a loop or coil of $N$ turns and current $I$ in a B-field at known angle $\theta$.

$$\tau = NIBA \sin \theta$$
A circular magnetic field $B$ is induced by a current in a wire. The direction is given by the right-hand thumb rule.

The magnitude depends on the current $I$ and the distance $r$ from the wire.

$$B = \frac{\mu_0 I}{2\pi r}$$

Permeability: $\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$
Summary (Continued)

The force per unit length for two wires separated by \( d \) is:

\[
\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}
\]

Single loop:

\[
B = \frac{\mu_0 I}{2R}
\]

Coil of \( N \) loops:

\[
B = \frac{\mu_0 NI}{2R}
\]

For a solenoid of length \( L \), with \( N \) turns and current \( I \), the \( B \)-field is given by:

\[
B = \frac{\mu NI}{L}
\]
CONCLUSION: Chapter 30
Torque and Magnetic Fields