Chapter 4A. Translational Equilibrium

A PowerPoint Presentation by

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A MOUNTAIN CLIMBER exerts action forces on crevices and ledges, which produce reaction forces on the climber, allowing him to scale the cliffs.

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Objectives: After completing this module, you should be able to:

- State and describe with examples Newton’s three laws of motion.
- State and describe with examples your understanding of the first condition for equilibrium.
- Draw free-body diagrams for objects in translational equilibrium.
- Write and apply the first condition for equilibrium to the solution of problems similar to those in this module.
Newton’s First Law:
An object at rest or an object in motion at constant speed will remain at rest or at constant speed in the absence of a resultant force.

A glass is placed on a board and the board is jerked quickly to the right. The glass tends to remain at rest while the board is removed.
Newton’s First Law: An object at rest or an object in motion at constant speed will remain at rest or at constant speed in the absence of a resultant force.

Assume glass and board move together at constant speed. If the board stops suddenly, the glass tends to maintain its constant speed.
Understanding the First Law:

Discuss what the driver experiences when a car accelerates from rest and then applies the brakes.

(a) The driver is forced to move forward. An object at rest tends to remain at rest.

(b) Driver must resist the forward motion as brakes are applied. A moving object tends to remain in motion.
Newton’s Second Law

Newton’s second law of motion will be discussed quantitatively in a later chapter, after we have covered acceleration.

Acceleration is the rate at which the speed of an object changes. An object with an acceleration of 2 m/s², for example, is an object whose speed increases by 2 m/s every second it travels.
Newton’s Second Law:

- **Second Law:** Whenever a resultant force acts on an object, it produces an acceleration - an acceleration that is directly proportional to the force and inversely proportional to the mass.

\[ a \propto \frac{F}{m} \]
Acceleration and Force With Zero Friction Forces

Pushing the cart with twice the force produces twice the acceleration. Three times the force triples the acceleration.
Pushing two carts with same force $F$ produces one-half the acceleration. The acceleration varies *inversely* with the amount of material (the mass).
Newton’s Third Law

• To every action force there must be an equal and opposite reaction force.

Action and reaction forces act on different objects.
Newton’s Third Law

Two More Examples:

Action and Reaction Forces Act on Different Objects. They Do Not Cancel Each Other!
Translational Equilibrium

• An object is said to be in Translational Equilibrium if and only if there is no resultant force.

• This means that the sum of all acting forces is zero.

In the example, the resultant of the three forces A, B, and C acting on the ring must be zero.
Visualization of Forces

Force diagrams are necessary for studying objects in equilibrium. Don’t confuse action forces with reaction forces.

Equilibrium: \[ \sum F = 0 \]

The action forces are each ON the ring.

- Force A: By ceiling on ring.
- Force B: By ceiling on ring.
- Force C: By weight on ring.
Now let’s look at the Reaction Forces for the same arrangement. They will be equal, but opposite, and they act on different objects.

**Reaction forces:**

- **Force $A_r$:** By ring on ceiling.
- **Force $B_r$:** By ring on ceiling.
- **Force $C_r$:** By ring on weight.

**Reaction forces are each exerted:** BY the ring.
Vector Sum of Forces

• An object is said to be in Translational Equilibrium if and only if there is no resultant force.

• The vector sum of all forces acting on the ring is zero in this case.

Vector sum: \( \Sigma F = A + B + C = 0 \)
A free-body diagram is a force diagram showing all the elements in this diagram: axes, vectors, components, and angles.
Free-body Diagrams:

- Read problem; draw and label sketch.
- Isolate a common point where all forces are acting.
- Construct force diagram at origin of x, y axes.
- Dot in rectangles and label x and y components opposite and adjacent to angles.
- Label all given information and state what forces or angles are to be found.
1. Isolate point.
2. Draw x, y axes.
3. Draw vectors.
4. Label components.
5. Show all given information.
Example 1. Draw a free-body diagram for the arrangement shown on the left. The pole is light and of negligible weight.

Careful:
The pole can only push or pull since it has no weight.

The force $B$ is the force exerted on the rope by the pole. Don’t confuse it with the reaction force exerted by the rope on the pole.
Translational Equilibrium

- The First Condition for Equilibrium is that there be no resultant force.
- This means that the sum of all acting forces is zero.

\[ \sum F_x = 0 \quad \sum F_y = 0 \]
Example 2. Find the tensions in ropes A and B for the arrangement shown.

The Resultant Force on the ring is zero:

\[ R = \Sigma F = 0 \]

\[ R_x = A_x + B_x + C_x = 0 \]

\[ R_y = A_y + B_y + C_y = 0 \]
Example 2. (Cont.) Finding components.

Recall trigonometry to find components:

\[ \text{Opp} = \text{Hyp} \times \sin \]
\[ A_y = A \sin 40^\circ \]
\[ \text{Adj} = \text{Hyp} \times \cos \]
\[ A_x = A \cos 40^\circ \]

The components of the vectors are found from the free-body diagram.

\[ B_y = 0 \]
\[ 200 \, \text{N} \]
\[ C_y = -200 \, \text{N} \]
A free-body diagram must represent all forces as components along x and y-axes. It must also show all given information.

**Components**

\[ A_x = A \cos 40^0 \]
\[ A_y = A \sin 40^0 \]
\[ B_x = B; \quad B_y = 0 \]
\[ C_x = 0; \quad C_y = W \]
Example 2. Continued . . .

\[ \sum F_x = A \cos 40^\circ - B = 0; \quad \text{or} \quad B = A \cos 40^\circ \]

\[ \sum F_y = A \sin 40^\circ - 200 \, \text{N} = 0; \quad \text{or} \quad A \sin 40^\circ = 200 \, \text{N} \]
Example 2. Continued . . .

Solve first for A

\[ A = \frac{200 \text{ N}}{\sin 40^\circ} = 311 \text{ N} \]

The tensions in A and B are

\[ A = 311 \text{ N}; \quad B = 238 \text{ N} \]

Solve Next for B

\[ B = A \cos 40^\circ = (311 \text{ N}) \cos 40^\circ; \quad B = 238 \text{ N} \]

Two equations; two unknowns

\[ A \sin 40^\circ = 200 \text{ N} \]

\[ B = A \cos 40^\circ \]
1. Draw a sketch and label all information.
2. Draw a free-body diagram.
3. Find components of all forces (+ and -).
4. Apply First Condition for Equilibrium:
   \[ \Sigma F_x = 0 ; \quad \Sigma F_y = 0 \]
5. Solve for unknown forces or angles.
Example 3. Find Tension in Ropes A and B.

1. Draw free-body diagram.
2. Determine angles.
3. Draw/label components.

Next we will find components of each vector.
Example 3. Find the tension in ropes A and B.

First Condition for Equilibrium:

\[ \Sigma F_x = 0 ; \quad \Sigma F_y = 0 \]

4. Apply 1\textsuperscript{st} Condition for Equilibrium:

\[ \Sigma F_x = B_x - A_x = 0 \quad \Rightarrow \quad B_x = A_x \]

\[ \Sigma F_y = B_y + A_y - W = 0 \quad \Rightarrow \quad B_y + A_y = W \]
Example 3. Find the tension in ropes A and B.

\[ A_x = A \cos 30^0; \quad A_y = A \sin 30^0 \]
\[ B_x = B \cos 60^0 \]
\[ B_y = B \sin 60^0 \]
\[ W_x = 0; \quad W_y = -400 \text{ N} \]

Using Trigonometry, the first condition yields:

\[ B_x = A_x \quad \Rightarrow \quad B \cos 60^0 = A \cos 30^0 \]
\[ B_y + A_y = W \quad \Rightarrow \quad A \sin 30^0 + B \sin 60^0 = 400 \text{ N} \]
Example 3 (Cont.) Find the tension in A and B.

We will first solve the horizontal equation for \( B \) in terms of the unknown \( A \):

\[
B = \frac{A \cos 30^0}{\cos 60^0} = 1.73A
\]

We now solve for \( A \) and \( B \): Two Equations and Two Unknowns.

\[
B \cos 60^0 = B \cos 30^0
\]

\[
A \sin 30^0 + B \sin 60^0 = 400 \text{ N}
\]
Example 3 (Cont.) Find Tensions in A and B.

\[ A \sin 30^0 + B \sin 60^0 = 400 \text{ N} \]

\[ B = 1.732 \ A \]

Now apply Trig to:

\[ A_y + B_y = 400 \text{ N} \]

\[ A \sin 60^0 + B \sin 60^0 = 400 \text{ N} \]

\[ B = 1.732 \ A \]

\[ A \sin 30^0 + B \sin 60^0 = 400 \text{ N} \]

\[ 0.500 \ A + 1.50 \ A = 400 \text{ N} \]

\[ A = 200 \text{ N} \]
Example 3 (Cont.) Find $B$ with $A = 200$ N.

Rope tensions are:

$A = 200$ N

$B = 1.732 \times A$

$B = 1.732(400$ N$)$

$B = 346$ N

This problem is made much simpler if you notice that the angle between vectors $B$ and $A$ is $90^0$ and rotate the x and y axes (Continued)

Rope tensions are: $A = 200$ N and $B = 346$ N
Example 4. Rotate axes for same example.

We recognize that $A$ and $B$ are at right angles, and choose the $x$-axis along $B$ – not horizontally. The $y$-axis will then be along $A$—with $W$ offset.
Since $A$ and $B$ are perpendicular, we can find the new angle $\phi$ from geometry.

You should show that the angle $\phi$ will be $30^0$. We now only work with components of $W$. 
Recall $W = 400$ N. Then we have:

Apply the first condition for Equilibrium, and . . .

\[ W_x = (400 \text{ N}) \cos 30^\circ \]
\[ W_y = (400 \text{ N}) \sin 30^\circ \]

Thus, the components of the weight vector are:

\[ W_x = 346 \text{ N}; \quad W_y = 200 \text{ N} \]

$B - W_x = 0$ and $A - W_y = 0$
Example 4 (Cont.) We Now Solve for A and B:

\[ \sum F_x = B - W_x = 0 \]

\[
B = W_x = (400 \text{ N}) \cos 30^\circ
\]

\[
B = 346 \text{ N}
\]

\[ \sum F_y = A - W_y = 0 \]

\[
A = W_y = (400 \text{ N}) \sin 30^\circ
\]

\[
A = 200 \text{ N}
\]

Before working a problem, you might see if rotation of the axes helps.
• **Newton’s First Law:** An object at rest or an object in motion at constant speed will remain at rest or at constant speed in the absence of a resultant force.
Summary

- Second Law: Whenever a resultant force acts on an object, it produces an acceleration, an acceleration that is directly proportional to the force and inversely proportional to the mass.
Summary

• Third Law: To every action force there must be an equal and opposite reaction force.
Free-body Diagrams:

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Translational Equilibrium

- The First Condition for Equilibrium is that there be no resultant force.

- This means that the sum of all acting forces is zero.

\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \]
Problem Solving Strategy

1. Draw a sketch and label all information.
2. Draw a free-body diagram.
3. Find components of all forces (+ and -).
4. Apply First Condition for Equilibrium:
   \[ \sum F_x = 0 ; \quad \sum F_y = 0 \]
5. Solve for unknown forces or angles.
Conclusion: Chapter 4A
Translational Equilibrium