The Ninja, a roller coaster at Six Flags over Georgia, has a height of 122 ft and a speed of 52 mi/h. The potential energy due to its height changes into kinetic energy of motion.
Objectives: After completing this module, you should be able to:

- Define kinetic energy and potential energy, along with the appropriate units in each system.
- Describe the relationship between work and kinetic energy, and apply the WORK-ENERGY THEOREM.
- Define and apply the concept of POWER, along with the appropriate units.
Energy

Energy is anything that can be converted into work; i.e., anything that can exert a force through a distance.

Energy is the capability for doing work.
Potential Energy

Potential Energy: Ability to do work by virtue of position or condition.

A suspended weight  A stretched bow
Example Problem: What is the potential energy of a 50-kg person in a skyscraper if he is 480 m above the street below?

**Gravitational Potential Energy**

What is the P.E. of a 50-kg person at a height of 480 m?

\[ U = mgh = (50 \text{ kg})(9.8 \text{ m/s}^2)(480 \text{ m}) \]

\[ U = 235 \text{ kJ} \]
**Kinetic Energy**

Kinetic Energy: Ability to do work by virtue of motion. (Mass with velocity)

- A speeding car
- or a space rocket
Examples of Kinetic Energy

What is the kinetic energy of a 5-g bullet traveling at 200 m/s?

\[ K = \frac{1}{2}mv^2 = \frac{1}{2}(0.005 \text{ kg})(200 \text{ m/s})^2 \]

\[ K = 100 \text{ J} \]

What is the kinetic energy of a 1000-kg car traveling at 14.1 m/s?

\[ K = \frac{1}{2}mv^2 = \frac{1}{2}(1000 \text{ kg})(14.1 \text{ m/s})^2 \]

\[ K = 99.4 \text{ J} \]
Work and Kinetic Energy

A resultant force changes the velocity of an object and does work on that object.

\[ W = Fx = (ma)x; \quad a = \frac{v_f^2 - v_0^2}{2x} \]

\[ W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \]
The Work-Energy Theorem

Work is equal to the change in kinetic energy as \( \frac{1}{2}mv^2 \) then we can state a very important physical principle:

**The Work-Energy Theorem:** The work done by a resultant force is equal to the change in kinetic energy that it produces.

\[
Work = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]
Example 1: A 20-g projectile strikes a mud bank, penetrating a distance of 6 cm before stopping. Find the stopping force $F$ if the entrance velocity is 80 m/s.

\[ Work = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \]

\[ F x = - \frac{1}{2} m v_o^2 \]

\[ F (0.06 \text{ m}) \cos 180^0 = - \frac{1}{2} (0.02 \text{ kg})(80 \text{ m/s})^2 \]

\[ F (0.06 \text{ m})(-1) = -64 \text{ J} \]

\[ F = 1067 \text{ N} \]

*Work to stop bullet = change in K.E. for bullet*
Example 2: A bus slams on brakes to avoid an accident. The tread marks of the tires are 80 m long. If $\mu_k = 0.7$, what was the speed before applying brakes?

\[ Work = 
\begin{align*}
\Delta K &= \frac{1}{2} mv_f^2 - \frac{1}{2} mv_o^2 \\
v_o &= \sqrt{2\mu_k gx}
\end{align*}
\]

Substituting the given values:

\[ v_o = \sqrt{2(0.7)(9.8 \text{ m/s}^2)(25 \text{ m})} \]

\[ v_o = 59.9 \text{ ft/s} \]
Example 3: A 4-kg block slides from rest at top to bottom of the $30^\circ$ inclined plane. Find velocity at bottom. ($h = 20$ m and $\mu_k = 0.2$)

Plan: We must calculate both the resultant work and the net displacement $x$. Then the velocity can be found from the fact that $\text{Work} = \Delta K$.

Resultant work = (Resultant force down the plane) $\times$ (the displacement down the plane)
**Example 3 (Cont.):** We first find the net displacement \( x \) down the plane:

From trig, we know that the \( \sin 30^\circ = \frac{h}{x} \) and:

\[
\sin 30^\circ = \frac{h}{x} \\
x = \frac{20 \text{ m}}{\sin 30^\circ} = 40 \text{ m}
\]
Example 3 (Cont.): Next we find the resultant work on 4-kg block. \((x = 40 \text{ m} \text{ and } \mu_k = 0.2)\)

**Draw free-body diagram to find the resultant force:**

\[ W_x = (4 \text{ kg})(9.8 \text{ m/s}^2)(\sin 30^0) = 19.6 \text{ N} \]
\[ W_y = (4 \text{ kg})(9.8 \text{ m/s}^2)(\cos 30^0) = 33.9 \text{ N} \]
Example 3(Cont.): Find the resultant force on 4-kg block. \((x = 40 \text{ m and } \mu_k = 0.2)\)

Resultant force down plane: \(19.6 \text{ N} - f\)

Recall that \(f_k = \mu_k n\)

\[\Sigma F_y = 0 \quad \text{or} \quad n = 33.9 \text{ N}\]

Resultant Force = \(19.6 \text{ N} - \mu_k n\); \(\mu_k = 0.2\)

Resultant Force = \(19.6 \text{ N} - (0.2)(33.9 \text{ N}) = 12.8 \text{ N}\)

Resultant Force Down Plane = 12.8 N
Example 3 (Cont.): The resultant work on a 4-kg block. \( (x = 40 \text{ m and } F_R = 12.8 \text{ N}) \)

\[
(\text{Work})_R = F_R x
\]

Net Work = \((12.8 \text{ N})(40 \text{ m})\)

Net Work = 512 J

Finally, we are able to apply the work-energy theorem to find the final velocity:

\[
\text{Work} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2
\]
Example 3 (Cont.): A 4-kg block slides from rest at top to bottom of the $30^0$ plane. Find velocity at bottom. ($h = 20 \text{ m}$ and $\mu_k = 0.2$)

Resultant Work $= 512 \text{ J}$

Work done on block equals the change in K. E. of block.

$$\frac{1}{2} mv_f^2 - \frac{1}{2} mv_o^2 = \text{Work} \quad \frac{1}{2} mv_f^2 = 512 \text{ J}$$

$$\frac{1}{2} (4 \text{ kg}) v_f^2 = 512 \text{ J}$$

$v_f = 16 \text{ m/s}$
Power

**Power** is defined as the rate at which work is done: \( P = \frac{dW}{dt} \)

\[
Power = \frac{Work}{time} = \frac{F \cdot r}{t}
\]

\[
P = \frac{mgr}{t} = \frac{(10\text{kg})(9.8\text{m/s}^2)(20\text{m})}{4\text{s}}
\]

\[
P = 490\text{ J/s} \quad \text{or} \quad 490\text{ watts (W)}
\]

**Power of 1 W is work done at rate of 1 J/s**
Units of Power

- One watt (W) is work done at the rate of one joule per second.

  \[ 1 \text{ W} = 1 \text{ J/s} \quad \text{and} \quad 1 \text{ kW} = 1000 \text{ W} \]

- One ft lb/s is an older (USCS) unit of power.

- One horsepower is work done at the rate of 550 ft lb/s. \( (1 \text{ hp} = 550 \text{ ft lb/s}) \)
What power is consumed in lifting a 70-kg robber 1.6 m in 0.50 s?

\[ P = \frac{Fh}{t} = \frac{mgh}{t} \]

\[ P = \frac{(70 \text{ kg})(9.8 \text{ m/s}^2)(1.6 \text{ m})}{0.50 \text{ s}} \]

**Power Consumed:**  \( P = 2220 \text{ W} \)
Example 4: A 100-kg cheetah moves from rest to 30 m/s in 4 s. What is the power?

Recognize that work is equal to the change in kinetic energy:

\[ \text{Work} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

\[ P = \frac{\text{Work}}{t} \]

\[ P = \frac{\frac{1}{2}mv_f^2}{t} = \frac{\frac{1}{2}(100 \text{ kg})(30 \text{ m/s})^2}{4 \text{ s}} \]

Power Consumed: \( P = 1.22 \text{ kW} \)
Power and Velocity

Recall that average or constant velocity is distance covered per unit of time \( v = \frac{x}{t} \).

\[
P = \frac{F \cdot x}{t} = F \cdot \frac{x}{t}
\]

If power varies with time, then calculus is needed to integrate over time. (Optional)

Since \( P = \frac{dW}{dt} \):

\[
Work = \int P(t) \, dt
\]
Example 5: What power is required to lift a 900-kg elevator at a constant speed of 4 m/s?

\[ P = F \bar{v} = mg \bar{v} \]

\[ P = (900 \text{ kg})(9.8 \text{ m/s}^2)(4 \text{ m/s}) \]

\[ P = 35.3 \text{ kW} \]
Example 6: What work is done by a 4-hp mower in one hour? The conversion factor is needed: 1 hp = 550 ft lb/s.

\[ 4\text{hp} \left( \frac{550\text{ft} \cdot \text{lb/s}}{1\text{hp}} \right) = 2200\text{ft} \cdot \text{lb/s} \]

\[ P = \frac{Work}{t} \quad ; \quad Work = Pt \]

\[ Work = (2200\text{ft} \cdot \text{lb/s})(60\text{s}) \]

\[ Work = 132,000\text{ ft lb} \]
The Work-Energy Theorem: The work done by a resultant force is equal to the change in kinetic energy that it produces.

**Summary**

**Potential Energy:** Ability to do work by virtue of position or condition.  \[ U = mgh \]

**Kinetic Energy:** Ability to do work by virtue of motion. (Mass with velocity)  \[ K = \frac{1}{2} m v^2 \]

**The Work-Energy Theorem:** The work done by a resultant force is equal to the change in kinetic energy that it produces.

\[ \text{Work} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 \]
Summary (Cont.)

Power is defined as the rate at which work is done: \( P = \frac{\text{Work}}{t} \)

\[
\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{F \cdot r}{t}
\]

Power of 1 W is work done at rate of 1 J/s
CONCLUSION: Chapter 8B
Work and Energy