Chapter 8A. Work

A PowerPoint Presentation by

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In this module, we will learn a measurable definition of work as the product of force and distance.
Objectives: After completing this module, you should be able to:

- Describe work in terms of force and displacement, using the definition of the scalar product.
- Solve problems involving concept of work.
- Distinguish between the resultant work and the work of a single force.
- Define the spring constant and calculate the work done by a varying spring force.
Three things are necessary for the performance of work:

- There must be an applied force $F$.
- There must be a displacement $x$.
- The force must have a component along the displacement.
If a force does not affect displacement, it does no work.

The force $F$ exerted on the pot by the man does work.

The earth exerts a force $W$ on pot, but does no work even though there is displacement.
**Definition of Work**

*Work is a scalar quantity equal to the product of the displacement $x$ and the component of the force $F_x$ in the direction of the displacement.*

\[
\text{Work} = \text{Force component} \times \text{displacement}
\]

\[
\text{Work} = F_x x
\]
Positive Work

Force $F$ contributes to displacement $x$.

Example: If $F = 40$ N and $x = 4$ m, then

$$Work = (40 \text{ N})(4 \text{ m}) = 160 \text{ N} \cdot \text{m}$$

$$Work = 160 \text{ J}$$

$1 \text{ N} \cdot \text{m} = 1 \text{ Joule (J)}$
Negative Work

The friction force $f$ opposes the displacement.

Example: If $f = -10$ N and $x = 4$ m, then

$Work = (-10 \text{ N})(4 \text{ m}) = -40 \text{ J}$
Resultant Work or Net Work

Resultant work is the algebraic sum of the individual works of each force.

Example: $F = 40 \text{ N}$, $f = -10 \text{ N}$ and $x = 4 \text{ m}$

$$\text{Work} = (40 \text{ N})(4 \text{ m}) + (-10 \text{ N})(4 \text{ m})$$

$$\text{Work} = 120 \text{ J}$$
Resultant Work (Cont.)

Resultant work is also equal to the work of the RESULTANT force.

Example: Work = (F - f) x

\[ \text{Work} = (40 - 10 \text{ N})(4 \text{ m}) \]

\[ \text{Work} = 120 \text{ J} \]
Work of a Force at an Angle

\[ \text{Work} = F_x x \]
\[ \text{Work} = (F \cos \theta) x \]

\[ \text{Work} = (70 \text{ N}) \cos 60^\circ (12 \text{ m}) = 420 \text{ J} \]

Only the \( x \)-component of the force does work!
Procedure for Calculating Work

1. Draw sketch and establish what is given and what is to be found.

2. Draw free-body diagram choosing positive x-axis along displacement.

\[ \text{Work} = (F \cos \theta) \times x \]

3. Find work of a single force from formula.

4. Resultant work is work of resultant force.
**Example 1:** A lawn mower is pushed a horizontal distance of 20 m by a force of 200 N directed at an angle of $30^0$ with the ground. What is the work of this force?

*Note: Work is positive since $F_x$ and $x$ are in the same direction.*

$$Work = (F \cos \theta) \times$$

$$Work = (200 \ N)(20 \ m) \ Cos \ 30^0$$

$$Work = 3460 \ J$$
Example 2: A 40-N force pulls a 4-kg block a horizontal distance of 8 m. The rope makes an angle of 35° with the floor and \( u_k = 0.2 \). What is the work done by each acting on block?

1. Draw sketch and find given values

   \( P = 40 \text{ N}; \quad x = 8 \text{ m}, \quad u_k = 0.2; \quad \theta = 35^0; \quad m = 4 \text{ kg} \)

2. Draw free-body diagram showing all forces. (Cont.)

\[
\text{Work} = (F \cos \theta) \times
\]

\[\begin{array}{c}
\text{mg} \\
\text{fk} \\
\text{n} \\
P \\
x
\end{array}\]

\[\begin{array}{c}
\text{40 N} \\
35^0 \\
+x \\
8 \text{ m}
\end{array}\]
Example 2 (Cont.): Find Work Done by Each Force.

\[ P = 40 \text{ N}; \ x = 8 \text{ m}, \ u_k = 0.2; \ \theta = 35^0; \ m = 4 \text{ kg} \]

4. First find work of \( P \).

\[ \text{Work} = (P \cos \theta) \ x \]

\[ \text{Work}_P = (40 \text{ N}) \cos 35^0 (8 \text{ m}) = 262 \text{ J} \]

5. Next consider normal force \( n \) and weight \( W \).

Each makes a \( 90^0 \) angle with \( x \), so that the works are zero. (\( \cos 90^0 = 0 \)):

\[ \text{Work}_n = 0 \]

\[ \text{Work}_P = 0 \]
Example 2 (Cont.):

$P = 40 \text{ N}; \ x = 8 \text{ m}, \ u_k = 0.2; \ \theta = 35^0; \ m = 4 \text{ kg}$

Work$_P = 262 \text{ J}$

Work$_n = Work_w = 0$

6. Next find work of friction. Recall: $f_k = \mu_k \ N$

$n + P \cos 35^0 - mg = 0; \ n = mg - P \cos 35^0$

$n = (4 \text{ kg})(9.8 \text{ m/s}^2) - (40 \text{ N})\sin 35^0 = 16.3 \text{ N}$

$f_k = \mu_k \ n = (0.2)(16.3 \text{ N}); \ f_k = 3.25 \text{ N}$
Example 2 (Cont.):  

\[ \text{Work}_n = \text{Work}_w = 0 \]

\[ \text{Work}_p = 262 \text{ J} \]

6. Work of friction (Cont.)

\[ f_k = 3.25 \text{ N}; \ x = 8 \text{ m} \]

\[ \text{Work}_f = (3.25 \text{ N}) \cos 180^0 (8 \text{ m}) = -26.0 \text{ J} \]

*Note work of friction is negative* \( \cos 180^0 = -1 \)

7. The resultant work is the sum of all works:

\[ 262 \text{ J} + 0 + 0 - 26 \text{ J} \]

\[ (\text{Work})_R = 236 \text{ J} \]
Example 3: What is the resultant work on a 4-kg block sliding from top to bottom of the 30° inclined plane? (h = 20 m and \( \mu_k = 0.2 \))

\[
\text{Net work} = \sum (\text{works})
\]

Find the work of 3 forces.

\[
\text{Work} = (F \cos \theta) x
\]

First find magnitude of \( x \) from trigonometry:

\[
\sin 30^0 = \frac{h}{x} \quad x = \frac{20 \text{ m}}{\sin 30^0} = 40 \text{ m}
\]
Example 3 (Cont.): What is the resultant work on 4-kg block? \( h = 20 \text{ m} \) and \( \mu_k = 0.2 \)

1. First find work of \( mg \).

\[
\text{Work} = mg(\cos \theta) \times 40 \text{ m}
\]

\[
\text{Work} = (4 \text{ kg})(9.8 \text{ m/s}^2)(40 \text{ m}) \cos 60^\circ
\]

Work done by weight \( mg \)

\[
\text{Work} = 784 \text{ J}
\]

Positive Work

2. Draw free-body diagram
Example 3 (Cont.): What is the resultant work on 4-kg block? \( (h = 20 \text{ m} \text{ and } \mu_k = 0.2) \)

3. Next find work of friction force \( f \) which requires us to find \( n \).

4. Free-body diagram:

\[ n = mg \cos 30^0 = (4)(9.8)(0.866) \]
\[ n = 33.9 \text{ N} \]
\[ f = \mu_k n \]
\[ f = (0.2)(33.9 \text{ N}) = 6.79 \text{ N} \]
Example 3 (Cont.): What is the resultant work on 4-kg block? \((h = 20 \text{ m} \text{ and } \mu_k = 0.2)\)

5. Find work of friction force \(f\) using free-body diagram

\[
Work = (f \cos \theta) x
\]

\[
Work = (6.79 \text{ N})(20 \text{ m})(\cos 180^\circ)
\]

\[
Work = (272 \text{ J})(-1) = -272 \text{ J}
\]

Note: Work of friction is Negative.

What work is done by the normal force \(N\)?

Work of \(N\) is 0 since it is at right angles to \(x\).
Example 3 (Cont.): What is the resultant work on 4-kg block? \(( h = 20 \text{ m} \text{ and } \mu_k = 0.2)\)

\[
\text{Net work} = \sum (\text{works}) \\
\text{Weight: } \text{Work} = +784 \text{ J} \\
\text{Friction: } \text{Work} = -272 \text{ J} \\
\text{Force n: } \text{Work} = 0 \text{ J}
\]

Resultant Work = 512 J

Note: Resultant work could have been found by multiplying the resultant force by the net displacement down the plane.
Assume that a constant force $F$ acts through a parallel displacement $\Delta x$.

The area under the curve is equal to the work done.

$$Work = F(x_2 - x_1)$$

$$Work = F \Delta x$$
Example for Constant Force

What work is done by a constant force of 40 N moving a block from \( x = 1 \text{ m} \) to \( x = 4 \text{ m} \)?

\[
\text{Work} = F(x_2 - x_1)
\]

\[
\text{Work} = (40 \text{ N})(4 \text{ m} - 1 \text{ m})
\]

\[
\text{Work} = 120 \text{ J}
\]
Work of a Varying Force

Our definition of work applies only for a constant force or an average force.

What if the force varies with displacement as with stretching a spring or rubber band?
Hooke’s Law

When a spring is stretched, there is a **restoring** force that is proportional to the displacement.

\[ F = -kx \]

The spring constant \( k \) is a property of the spring given by:

\[ K = \frac{\Delta F}{\Delta x} \]
Work Done in Stretching a Spring

Work done ON the spring is positive; work BY the spring is negative.

From Hooke’s law: \( F = kx \)

Work = Area of Triangle

Area = \( \frac{1}{2} \) (base)(height) = \( \frac{1}{2} \) (x)(F_{avg}) = \( \frac{1}{2} \) x(kx)

Work = \( \frac{1}{2} \) kx^2
Compressing or Stretching a Spring
Initially at Rest:

Two forces are always present:
the outside force $F_{out}$ ON spring and
the reaction force $F_s$ BY the spring.

**Compression:** $F_{out}$ does positive work and $F_s$ does negative work (see figure).

**Stretching:** $F_{out}$ does positive work and $F_s$ does negative work (see figure).
Example 4: A 4-kg mass suspended from a spring produces a displacement of 20 cm. What is the spring constant?

The stretching force is the weight (\(W = mg\)) of the 4-kg mass:

\[ F = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N} \]

Now, from Hooke’s law, the force constant \(k\) of the spring is:

\[ k = \frac{\Delta F}{\Delta x} = \frac{39.2 \text{ N}}{0.2 \text{ m}} = 196 \text{ N/m} \]
Example 5: What work is required to stretch this spring \((k = 196 \text{ N/m})\) from \(x = 0\) to \(x = 30 \text{ cm}\)?

\[
\text{Work} = \frac{1}{2} kx^2
\]

\[
\text{Work} = \frac{1}{2} (196 \text{ N/m})(0.30 \text{ m})^2
\]

\[
\text{Work} = 8.82 \text{ J}
\]

Note: The work to stretch an additional 30 cm is greater due to a greater average force.
General Case for Springs:

If the initial displacement is not zero, the work done is given by:

\[
Work = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2
\]
**Summary**

\[
\text{Work} = F_x x
\]

\[
\text{Work} = (F \cos \theta) x
\]

*Work is a scalar quantity equal to the product of the displacement \(x\) and the component of the force \(F_x\) in the direction of the displacement.*
**Procedure for Calculating Work**

1. **Draw sketch and establish what is given and what is to be found.**

2. **Draw free-body diagram choosing positive x-axis along displacement.**

   \[ \text{Work} = (F \cos \theta) \times \]

3. **Find work of a single force from formula.**

4. **Resultant work is work of resultant force.**
Important Points for Work Problems:

1. Always draw a free-body diagram, choosing the positive x-axis in the same direction as the displacement.

2. Work is negative if a component of the force is opposite displacement direction.

3. Work done by any force that is at right angles with displacement will be zero (0).

4. For resultant work, you can add the works of each force, or multiply the resultant force times the net displacement.
**Summary For Springs**

**Hooke’s Law:**

\[ F = -kx \]

**Spring Constant:**

\[ k = \frac{F}{x} \]

The *spring constant* is the force exerted **by** the spring per unit change in its displacement. The spring force always opposes displacement. This explains the negative sign in Hooke’s law.
Summary (Cont.)

Work to Stretch a Spring:

\[ Work = \frac{1}{2} kx^2 \]

\[ Work = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2 \]
Springs: Positive/Negative Work

Two forces are always present: the outside force \( F_{\text{out}} \) on the spring and the reaction force \( F_s \) by the spring.

**Compression:** \( F_{\text{out}} \) does positive work and \( F_s \) does negative work (see figure).

**Stretching:** \( F_{\text{out}} \) does positive work and \( F_s \) does negative work (see figure).
CONCLUSION:
Chapter 8A - Work