Objectives: After completing this module, you should be able to:

- Solve problems involving emf, terminal potential difference, internal resistance, and load resistance.
- Solve problems involving power gains and losses in a simple circuit containing internal and load resistances.
- Work problems involving the use of ammeters and voltmeters in dc circuits.
EMF and Terminal Potential Difference

The **emf** $\mathcal{E}$ is the open-circuit potential difference. The **terminal voltage** $V_T$ for closed circuit is reduced due to **internal resistance** $r$ inside source.

- **Open Circuit**: $\mathcal{E} = 1.5 \text{ V}$
- **Closed Circuit**: $V_T = 1.45 \text{ V}$

Applying Ohm’s law to battery $r$, gives:

$$V_T = \mathcal{E} - Ir$$
Ohm’s law: Current $I$ is the ratio of emf $\mathcal{E}$ to total resistance $R + r$:

$$I = \frac{\mathcal{E}}{R + r}$$

Cross multiplying gives:

$$IR + Ir = \mathcal{E};\quad V_T = IR$$

$$V_T = \mathcal{E} - Ir$$
Example 2. A 3-V battery has an internal resistance of 0.5 $\Omega$ and is connected to a load resistance of 4 $\Omega$. What current is delivered and what is the terminal potential difference $V_T$?

$$I = \frac{\varepsilon}{R + r} = \frac{3 \text{ V}}{4 \Omega + 0.5 \Omega}$$

$I = 0.667 \text{ A}$

$$V_T = \varepsilon - Ir$$

$$V_T = 3 \text{ V} - (0.667 \text{ A})(0.5 \Omega)$$

$V_T = 2.67 \text{ V}$
Recall that the definition of power is work or energy per unit of time. The following apply:

\[ P = VI; \quad P = I^2R; \quad P = \frac{V^2}{R} \]

The first of these is normally associated with the power gains and losses through emf’s; the latter two are more often associated with external loads.
Consider simple circuit:

Terminal Voltage

\[ V_T = \varepsilon - Ir \]

Multiply each term by \( I \):

\[ V_TI = \varepsilon I - I^2r \]

The power delivered to the external circuit is equal to the power developed in the emf less the power lost through internal resistance.
Example 3. The 3-V battery in Ex. 2 had an internal resistance of 0.5 Ω and a load resistance of 4 Ω. Discuss the power used in the circuit.

From Ex. 2, we found:

\[ I = 0.667 \text{ A} \quad V_T = 2.67 \text{ V} \]

Power developed in emf:

\[ \mathcal{E}I = (3.0 \text{ V})(0.667 \text{ A}) = 2.0 \text{ W} \]

Power lost in internal \( r \):

\[ I^2r = (0.667 \text{ A})^2(0.5 \text{ Ω}) = 0.222 \text{ W} \]
Example 3 (Cont.). Discuss the power used in the simple circuit below.

Power in emf: \( \varepsilon I = 2.00 \text{ W} \)

Power loss: \( I^2r = 0.222 \text{ W} \)

Power lost in external load \( R \):
\[
I^2R = (0.667)^2(4 \ \Omega) = 1.78 \text{ W}
\]

This power can also be found using \( V_T = 2.67 \text{ V} \)

\[
V_TI = (2.67)(0.667 \text{ A}) = 1.78 \text{ W}
\]

Actual power used externally.
Example 3 (Cont.). Discuss the power used in the simple circuit below.

Power in emf: \( \mathcal{E}I = 2.00 \, \text{W} \)

Power loss in internal \( r \): \( I^2r = 0.222 \, \text{W} \)

Power lost in external load \( R \): \( I^2R = V_TI = 1.78 \, \text{W} \)

\[ V_TI = \mathcal{E}I - I^2r \]

\[ 1.78 \, \text{W} = 2.00 \, \text{W} - 0.222 \, \text{W} \]
A Discharging EMF

When a battery is discharging, there is a gain in energy $\mathcal{E}$ as chemical energy is converted to electrical energy. At the same time, energy is lost through internal resistance $Ir$.

Discharging: $V_{BA} = \mathcal{E} - Ir$

12 V - (2 A)(1 Ω) = 12 V - 2 V = 10 V

If $V_B = 20$ V, then $V_A = 30$ V; Net Gain = 10 V
Charging: Reversing Flow Through EMF

When a battery is charged (current against normal output), energy is lost through chemical changes $\mathcal{E}$ and also through internal resistance $Ir$.

Charging: $V_{AB} = \mathcal{E} + Ir$

If $V_A = 20$ V, then $V_B = 6.0$ V; Net Loss = 14 V
When a battery is discharging, there is a **GAIN** in power $\mathcal{E}I$ as chemical energy is converted to electrical energy. At the same time, power is **L O S T** through internal resistance $I^2r$.

**Net Power Gain:**

$$V_{BA}I = \mathcal{E}I - I^2r$$

**Example:**

$$(12 \text{ V})(2 \text{ A}) - (2 \text{ A})^2(1 \text{ \Omega}) = 24 \text{ W} - 4 \text{ W} = 20 \text{ W}$$
When a battery is** charged** (current **against** normal output), **power is lost** through chemical changes \( E \) and through internal resistance \( I^2r \).

**Net Power Lost** = \( EI + I^2r \)

\[(12 \text{ V})(2 \text{ A}) + (2 \text{ A})^2(1 \text{ } \Omega) = 24 \text{ W} + 4 \text{ W} = 24 \text{ W}\]
**Example 4:** A 24-V generator is used to charge a 12-V battery. For the generator, $r_1 = 0.4 \, \Omega$ and for the battery $r_2 = 0.6 \, \Omega$. The load resistance is 5 $\Omega$.

First find current $I$:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{24 \, V - 12 \, V}{5 \, \Omega + 0.4 \, \Omega + 0.6 \, \Omega}$$

**Circuit current:** $I = 2.00 \, A$

What is the terminal voltage $V_G$ across the generator?

$$V_T = \mathcal{E} - Ir = 24 \, V - (2 \, A)(0.4 \, \Omega)$$

$V_G = 23.2 \, V$
Example 4: Find the terminal voltage $V_B$ across the battery.

Circuit current: $I = 2.00 \text{ A}$

$$V_B = \varepsilon + Ir = 12 \text{ V} + (2 \text{ A})(0.4 \Omega)$$

**Terminal** $V_B = 13.6 \text{ V}$

**Note:** The terminal voltage across a device in which the current is reversed is greater than its emf.

For a discharging device, the terminal voltage is less than the emf because of internal resistance.
Ammeters and Voltmeters

V

A

Emf

Rheostat

Voltmeter
Source of EMF
Ammeter
Rheostat
An ammeter is an instrument used to measure currents. It is always connected in series and its resistance must be small (negligible change in $I$).

The ammeter draws just enough current $I_g$ to operate the meter; $V_g = I_g r_g$
Galvanometer: A Simple Ammeter

The galvanometer uses torque created by small currents as a means to indicate electric current. A current $I_g$ causes the needle to deflect left or right. Its resistance is $R_g$.

The sensitivity is determined by the current required for deflection. (Units are in $\text{Amps/div.}$) Examples: 5 A/div; 4 mA/div.
**Example 5.** If 0.05 A causes full-scale deflection for the galvanometer below, what is its sensitivity?

\[
Sensitivity = \frac{0.05 \text{A}}{20 \text{ div}} = 2.50 \text{ mA/ div}
\]

Assume \( R_g = 0.6 \, \Omega \) and that a current causes the pointer to move to “10.” What is the voltage drop across the galvanometer?

\[
I = \frac{2.5 \text{ mA}}{\text{div}} \times (10 \text{ div}) = 25 \text{ mA}
\]

\[
V_g = (25 \text{ mA})(0.6 \, \Omega) = 15 \text{ mV}
\]
Operation of an Ammeter

The galvanometer is often the working element of both ammeters and voltmeters.

A shunt resistance in parallel with the galvanometer allows most of the current \( I \) to bypass the meter. The whole device must be connected in series with the main circuit.

\[
I = I_s + I_g
\]

The current \( I_g \) is negligible and only enough to operate the galvanometer. \([ I_s \gg I_g ]\)
**Shunt Resistance**

Current $I_g$ causes full-scale deflection of ammeter of resistance $R_g$. What $R_s$ is needed to read current $I$ from battery $V_B$?

**Junction rule at A:**

$$I = I_g + I_s$$

Or $$I_s = I - I_g$$

**Voltage rule for Ammeter:**

$$0 = I_g R_g - I_s R_s; \quad I_s R_s = I_g R_g$$

$$R_s = \frac{I_g R_g}{I - I_g}$$
Example 6. An ammeter has an internal resistance of 5 Ω and gives full-scale deflection for 1 mA. To read 10 A full scale, what shunt resistance $R_s$ is needed? (see figure)

\[ R_s = \frac{I_g R_g}{I - I_g} \]

\[ R_s = \frac{(0.001 A)(5 \Omega)}{10 A - (0.001 \Omega)} \]

\[ R_s = 5.0005 \times 10^{-4} \Omega \]

The shunt draws 99.999% of the external current.
Operation of an Voltmeter

The **voltmeter** must be connected in **parallel** and must have **high resistance** so as not to disturb the main circuit.

A **multiplier resistance** $R_m$ is added in series with the galvanometer so that very little current is drawn from the main circuit.

The voltage rule gives:

$$V_B = I_g R_g + I_g R_m$$
Multiplier Resistance

Current $I_g$ causes full-scale deflection of meter whose resistance is $R_g$. What $R_m$ is needed to read voltage $V_B$ of the battery?

$$V_B = I_g R_g + I_g R_m$$

Which simplifies to:

$$R_m = \frac{V_B - I_g R_g}{I_g}$$

$$R_m = \frac{V_B}{I_g} - R_g$$
Example 7. A voltmeter has an internal resistance of $5 \, \Omega$ and gives full-scale deflection for $1 \, \text{mA}$. To read $50 \, \text{V}$ full scale, what multiplier resistance $R_m$ is needed? (see figure)

\[
R_m = \frac{V_B}{I_g} - R_g
\]

\[
R_m = \frac{50 \, \text{V}}{0.001 \, \text{A}} - 5 \, \Omega
\]

\[
R_m = 49995 \, \Omega
\]

The high resistance draws negligible current in meter.
Summary of Formulas:

Discharging: \( V_T = \mathcal{E} - Ir \)

Power: \( V_T I = \mathcal{E}I - I^2r \)

Charging: \( V_T = \mathcal{E} + Ir \)

Power: \( V_T I = \mathcal{E}I + I^2r \)
\[ R_s = \frac{I_g R_g}{I - I_g} \]

\[ R_m = \frac{V_B}{I_g} - R_g \]
CONCLUSION: Chapter 28B
EMF and Terminal P.D.